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GRAVITATION AND COSMOLOGY

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1. CLASSICAL BLACK HOLES AS DISSIPATIVE BRANES
2. EXPERIMENTAL TESTS OF RELATIVISTIC GRAVITY
3. GRAVITATIONAL WAVES FROM COSMIC (SUPER)STRINGS
4. GRAVITATIONAL WAVES FROM COALESCING BLACK HOLES
5. CHAOS AND SYMMETRY IN 'STRING COSMOLOGY'

1. CLASSICAL BLACK HOLES AS DISSIPATIVE BRANES

- AIM : DERIVE THE VALUE OF THE (SURFACE) SHEAR VISCOSITY OF BLACK HOLES :

$$\boxed{\eta_{BH} = \frac{1}{16\pi G}} \quad (\text{WITH} \quad c = 1)$$

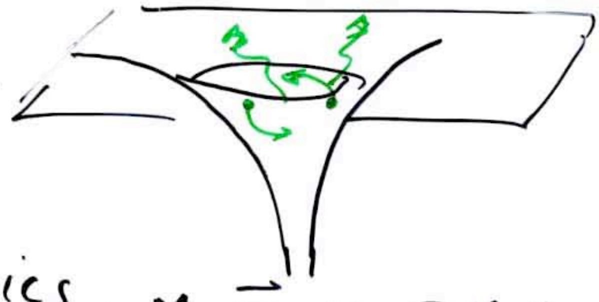
$$\Rightarrow \quad \frac{\eta_{BH}}{s_{BH}} = \frac{1/(16\pi G)}{1/(4\hbar G)} = \frac{\hbar}{4\pi}$$

ENTROPY DENSITY $= \frac{1}{4}\frac{1}{\hbar G}$

$\leftarrow$ OF RECENT INTEREST IN CONNECTION WITH AdS/CFT (Kovtun, Son, Starinets)

- EVOLVING VIEWS OF BLACK HOLES

PASSIVE GRAVITATIONAL WELLS



PHYSICAL OBJECTS : GLOBAL DYNAMICS : $M, \vec{J}, Q, \delta M, M_{irr} \ldots$

LOCAL DYNAMICS OF HORIZON : $\sim$ 'MEMBRANE' WITH DISSIPATIVE PROPERTIES

RESISTIVITY = 377 OHM = 4π

SHEAR VISCOSITY $= \frac{1}{16\pi G}$

QUANTUM OBJECTS : QUANTUM INSTABILITIES, PAIR CREATION

MICROSCOPIC ORIGIN OF BH ENTROPY

SPHERICALLY SYMMETRIC BLACK HOLES ($D = 3+1$)

$$ds^2 = -A(r)\,dT^2 + B(r)\,dr^2 + r^2\left(d\theta^2 + \sin^2\theta\, d\varphi^2\right)$$

$$A(r) = \frac{1}{B(r)} = 1 - \underbrace{\frac{2GM}{r}}_{\text{TOTAL MASS } M} + \underbrace{G\frac{Q^2}{r^2}}_{\text{TOTAL ELECTRIC CHARGE}} \qquad (c=1)$$

Reissner-Nordstrøm

SOLUTION OF $R_{\mu\nu} - \frac{1}{2} R\, g_{\mu\nu} = 8\pi G\, T_{\mu\nu}$

$$T_{\mu\nu} \leftarrow \frac{1}{4\pi}\left(F_\mu{}^\sigma F_{\nu\sigma} - \text{trace}\right)$$

$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$:

$$-A_0 = +V = +\frac{Q}{r}$$

$G=1$

$$A(r) = 1 - \frac{2M}{r} + \frac{Q^2}{r^2} = \frac{r^2 - 2Mr + Q^2}{r^2} = \frac{(r-r_+)(r-r_-)}{r^2}$$

(OUTER) HORIZON

$$\boxed{r_+ = M + \sqrt{M^2 - Q^2}} \qquad r_- = M - \sqrt{M^2 - Q^2}$$

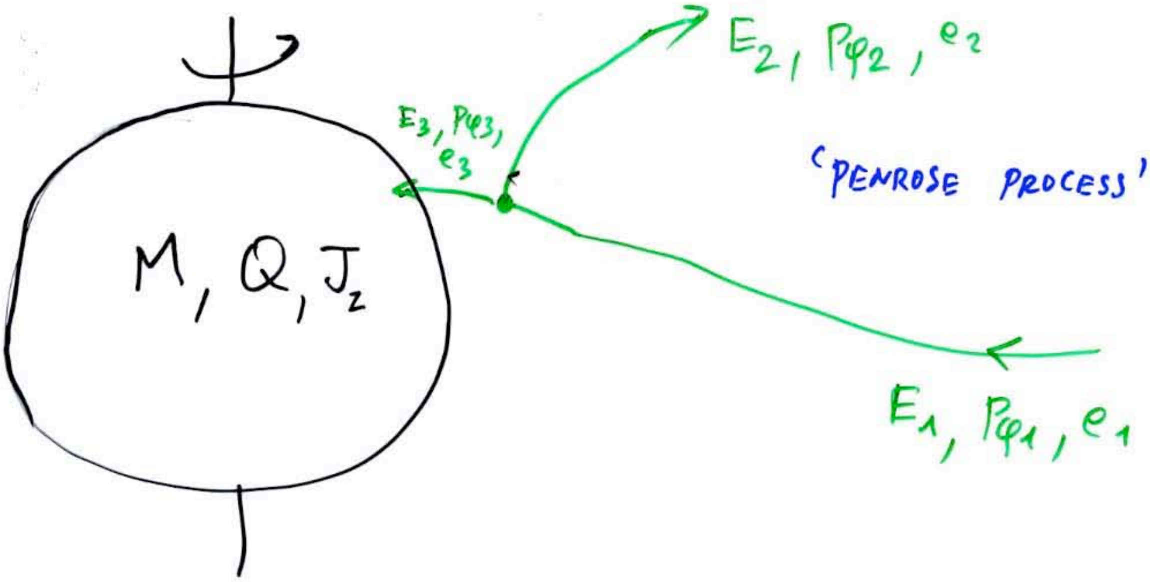
Kerr - Newman BH: M, J, Q

TOTAL ANGULAR MOMENTUM

$$r_+ = M + \sqrt{M^2 - Q^2 - a^2}$$

$a \equiv J/M$

INFINITESIMAL CHANGES IN M, J, Q OF BH



TEST-PARTICLE OF REST-MASS μ IN BH BACKGROUND

GEODESIC DYNAMICS : $S = -\int \mu \, ds$

HAMILTON-JACOBI EQ FOR $p_\mu = \frac{\partial S}{\partial x^\mu}$:

$$g^{\mu\nu}(p_\mu - eA_\mu)(p_\nu - eA_\nu) = -\mu^2$$

ELECTRIC CHARGE OF TEST PARTICLE

SYMMETRIES OF BH Background $\rightarrow$ CONSERVED QUANTITIES

CONSERVED ENERGY (OF TEST PARTICLE) : $E = -P_T = -P_0$

CONSERVED Z-COMPONENT OF ANGULAR MOMENTUM : P_φ

+ CONSERVED ELECTRIC CHARGE e

PENROSE PROCESS: IN and OUT at ∞ AT and AFTER SPLITTING

CHANGE IN TOTAL ENERGY OF SPACE-TIME

$$\delta M = E_1 - E_2 = E_3$$

$$\delta J = p_{\varphi 1} - p_{\varphi 2} = p_{\varphi 3}$$

$$\delta Q = e_1 - e_2 = e_3$$

SOLVE THE MASS-SHELL CONDITION

$$A_0 = -\frac{Q}{r}$$

$$-\mu^2 = g^{\mu\nu}(p_\mu - eA_\mu)(p_\nu - eA_\nu) = -\frac{(p_0 - eA_0(r))^2}{A(r)} + A(r)\, p_r^2 + \frac{1}{r^2}\left(p_\theta^2 + \frac{p_\varphi^2}{\sin^2\theta}\right)$$

$$p^r \equiv g^{rr} p_r = A(r)\, p_r$$

CONSERVED $L^2 = p_\theta^2 + \frac{p_\varphi^2}{\sin^2\theta}$

$$\boxed{-p_0 = E = \frac{eQ}{r} + \sqrt{(p^r)^2 + A(r)\left(\mu^2 + \frac{L^2}{r^2}\right)}}$$

CHRISTODOULOU, CHRISTODOULOU-RUFFINI '71

INFINITESIMAL VARIATION OF BH MASS: APPLY TO E_3 ON THE HORIZON

$E = E_3 = \delta M$; $e = e_3 = \delta Q$

$$\boxed{\delta M = \frac{Q\,\delta Q}{r_+(M,Q)} + |p^r|_{r_+} \geq \frac{Q\,\delta Q}{r_+(M,Q)}}$$

INEQUALITY $\Rightarrow$ IRREVERSIBILITY IN BH PHYSICS

CHRISTODOULOU-RUFFINI

WHEN ADDING ANGULAR MOMENTUM:

$$\delta M - \frac{a\delta J + r_+ Q\delta Q}{r_+^2 + a^2} = \frac{r_+^2 + a^2\cos^2\theta}{r_+^2+a^2}|p^r| \geq 0$$

$\rightarrow$ CHRISTODOULOU-RUFFINI MASS FORMULA

$$\boxed{M^2 = \left(M_{IRR} + \frac{Q^2}{4M_{IRR}}\right)^2 + \frac{J^2}{4M_{IRR}^2}}$$

$$\boxed{\delta M_{IRR} \geq 0}$$

M_{IRR} = INTEGRATION CONSTANT ALONG 'REVERSIBLE' TRANSFORMATIONS:

$$\delta M = \frac{a\delta J + r_+ Q\delta Q}{r_+^2 + a^2}$$

$\rightarrow$ CLASSICALLY EXTRACTABLE ENERGY OF A BH : $M - M_{IRR}$
- UP TO 29% M IN ROTATIONAL ENERGY
- UP TO 50% M IN COULOMB EN.

GIVEN INEQUALITY

$$a^2 + Q^2 \leq M^2$$

GEOMETRICAL MEANING OF M_{IRR} : $\boxed{A \equiv \underset{\text{HORIZON}}{\text{AREA}} = 16\pi M_{IRR}^2}$

GENERAL RESULT OF HAWKING '71

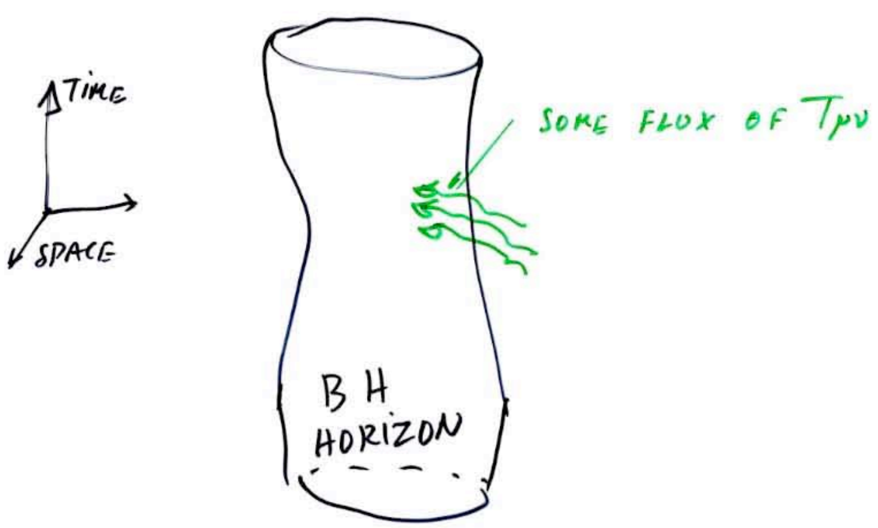
$$\boxed{\delta A \geq 0}$$

VARIOUS FACETS OF BH IRREVERSIBILITY:

CLASSICALLY $\delta A \nearrow$ CAN BE INTERPRETED AS: BH ~ DISSIPATIVE MEMBRANE

QUANTUM MECHANICALLY BEKENSTEIN SUGGESTED $A \propto$ BH ENTROPY

LET US STUDY MORE IN DETAIL WHAT HAPPENS WHEN ONE IS 'THROWING STUFF IN A BLACK HOLE':

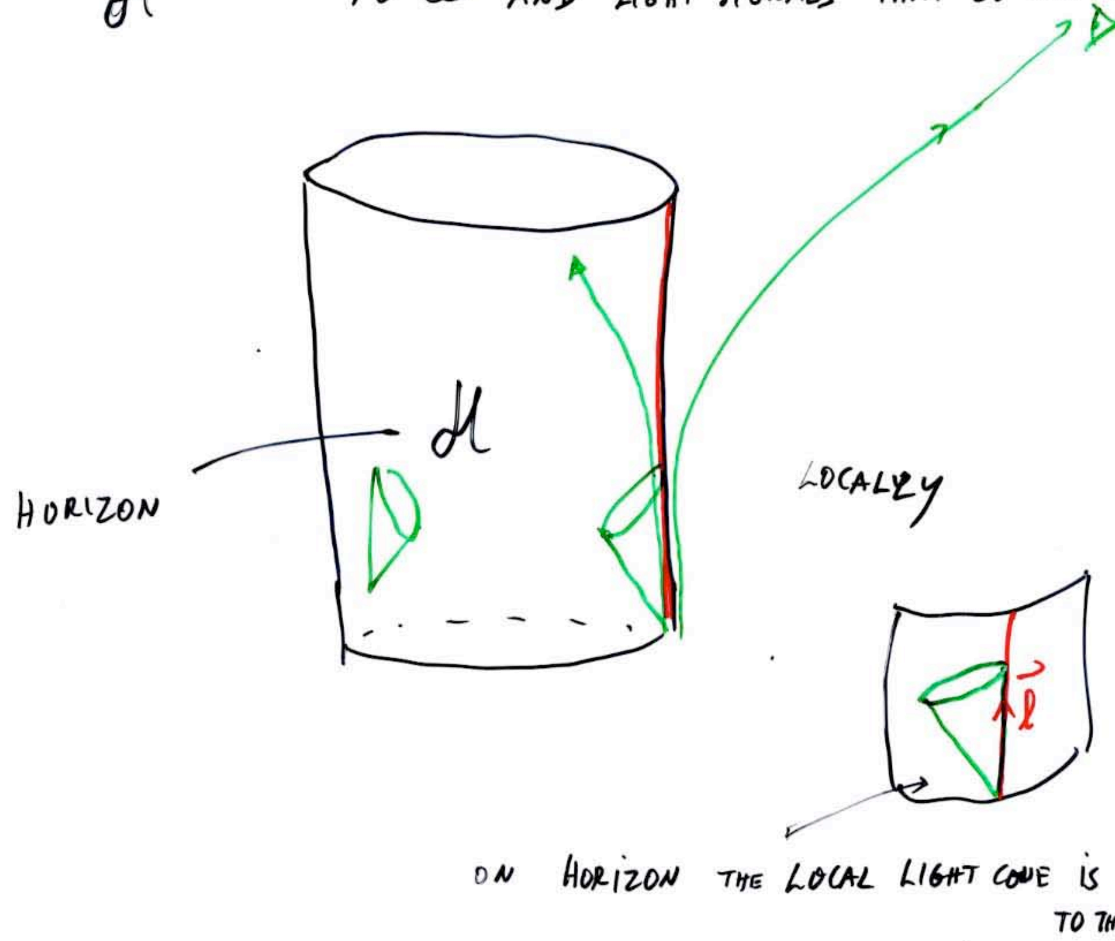


→ DERIVE AN EQUATION FOR THE DYNAMICS OF THE BH 'SURFACE' WHICH IS CLOSELY ANALOGOUS TO THE NAVIER-STOKES EQ. OF A VISCOUS FLUID (Damour '79)

(1) DEFINITION OF BH HORIZON (OR BH 'SURFACE') IN A GENERAL TIME-DEPENDENT SITUATION

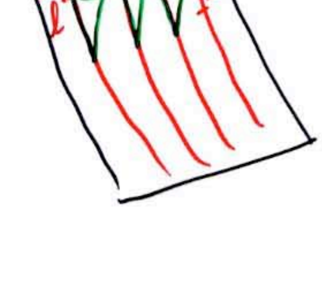
PENROSE, HAWKING, HAWKING-ELLIS

BH HORIZON $\mathcal{H}$ = BOUNDARY BETWEEN LIGHT SIGNALS THAT ESCAPE TO ∞ AND LIGHT SIGNALS THAT DO NOT ESCAPE



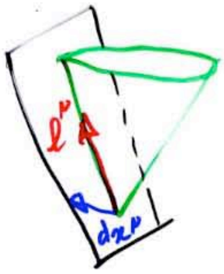
ON HORIZON THE LOCAL LIGHT CONE IS TANGENT TO THE HORIZON

$\rightarrow$ HORIZON = NULL HYPERSURFACE



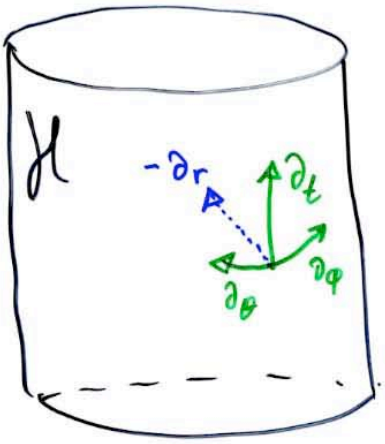
$\vec{\ell}$ = PARTICULAR LIGHT-LIKE DIRECTION ALONG WHICH $\mathcal{H}$ IS TANGENT TO LIGHTCONE

NB $\vec{l} = l^\mu \frac{\partial}{\partial x^\mu}$ IS BOTH TANGENT AND NORMAL TO $\mathcal{H}$



$l_\mu l^\mu = 0$ NULL $l_\mu dx^\mu = 0$ WITHIN $\mathcal{H}$

USE AN ADAPTED COORDINATE SYSTEM WHICH IS REGULAR ON $\mathcal{H}$



i.e. SIMILAR TO USUAL 'INGOING EDDINGTON-FINKELSTEIN COORDINATES' OF SCHWARZSCHILD (OR REISSNER-NORDSTRØM)

SINGULAR ON $\mathcal{H}$: $A(r_+) = 0$

$$ds^2 = -A(r)\, dT^2 + \frac{dr^2}{A(r)} + r^2 d\Omega^2$$

INTRODUCE 'TORTOISE' r_*

$$= -A(r)\left[dT^2 - \left(\frac{dr}{A(r)}\right)^2\right] + r^2 d\Omega^2$$

$$\boxed{r_* \equiv \int \frac{dr}{A(r)}}$$

$$= -A(r)\,(dT - dr_*)(dT + dr_*) + r^2 d\Omega^2$$

AND THE NEW TIME

$$= -A(r)\,(dt - 2dr_*)\,dt + r^2 d\Omega^2$$

$$\boxed{t = T + r_*}$$

$$\boxed{ds^2 = -A(r)\, dt^2 + 2\, dt\, dr + r^2(d\theta^2 + \sin^2\theta\, d\varphi^2)}$$

$A(r) = \frac{(r-r_+)(r-r_-)}{r^2}$ VANISHES ON $\mathcal{H}$ BUT ds^2 REMAINS REGULAR IN COORDS (t, r, θ, φ)

$\mathcal{H}$: $r = \text{const}$: $\frac{\partial}{\partial r}$ IS TRANSVERSE, WHILE $\frac{\partial}{\partial t}, \frac{\partial}{\partial \theta}, \frac{\partial}{\partial \varphi}$ ARE TANGENT TO $\mathcal{H}$

FOR ARBITRARY, TIME-DEPENDENT HORIZON LET US CHOOSE COORDINATES S.T.

- $\mathcal{H}$ IS LOCATED AT $x^1 = 0$ (i.e. $x^1 \sim r - r_+$)

- THE 3-d HYPERSURFACE $\mathcal{H}$ IS PARAMETRIZED BY x^0, x^2, x^3 ($\sim t, \theta, \varphi$)

$\partial_1 \equiv \frac{\partial}{\partial x^1}$ IS TRANSVERSE TO $\mathcal{H}$, WHILE $\partial_0 = \frac{\partial}{\partial x^0}$, ∂_2, ∂_3 ARE TANGENT

- NORMALIZE $\vec{l} = l^\mu \partial_\mu$ S.T. $\underbrace{\boxed{\vec{l} = \partial_0 + v^A \partial_A}}_{\text{ON } \mathcal{H}}$, $A = 2,3$

EXPLOIT $\quad g_{\mu\nu} l^\mu l^\nu = \vec{l} \cdot \vec{l} = 0 \quad$ (ON $\mathcal{H}$)

$\rightarrow$ FOR ANY TANGENT VECTOR $\vec{t}$ TO $\mathcal{H}$ $\quad 0 = \nabla_{\vec{t}}(\vec{l} \cdot \vec{l}) = 2\, \vec{l} \cdot \underbrace{\nabla_{\vec{t}} \vec{l}}$

NORMAL TO $\mathcal{H}$ MUST BE TANGENT TO $\mathcal{H}$

ONE DEFINES THE FOLLOWING SPECIAL QUANTITIES

$\vec{e}_A \equiv \partial_A \equiv \frac{\partial}{\partial x^A}$

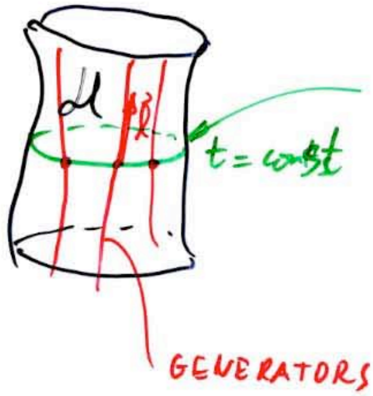
$$\boxed{\begin{aligned} \nabla_{\vec{l}} \vec{l} &= g\, \vec{l} \\ \nabla_{\vec{e}_A} \vec{l} &= -8\pi G\, \Pi_A\, \vec{l} + D_A^{\ B}\, \vec{e}_B \end{aligned}}$$

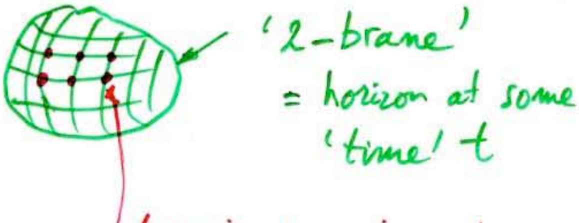
g is called SURFACE GRAVITY $\quad g^{\text{Schwarzschild}} = \dfrac{GM}{r_+^2}$

$D_A^{\ B}$ = DEFORMATION TENSOR OF the horizon geometry:

$$ds^2|_{\mathcal{H}} = \gamma_{AB}(t,x^A)(dx^A - v^A dt)(dx^B - v^B dt)$$

METRIC TENSOR OF A $t=\text{const}$ SLICE OF $\mathcal{H}$





'CONVECTIVE DERIVATIVE' OF γ_{AB}:

$$D_{AB} \equiv \frac{1}{2}\frac{D}{dt}\gamma_{AB} = \frac{1}{2}\mathcal{L}_{\vec{\ell}}\gamma_{AB} = \frac{1}{2}\left(\partial_t\gamma_{AB} + v^C\partial_C\gamma_{AB} + \partial_A v^C\gamma_{CB} + \partial_B v^C\gamma_{AC}\right)$$

$$D_{AB} = \gamma_{BC}D_A^{\ C} \qquad = \frac{1}{2}\left(\partial_t\gamma_{AB} + v_{A|B} + v_{B|A}\right)$$

AS IN USUAL FLUID: RATE OF DEFORMATION OF FLUID ELEMENTS

AS USUAL, DECOMPOSE

$$\boxed{D_{AB} = \sigma_{AB} + \theta\,\gamma_{AB}}$$

SHEAR TENSOR: trace-free part

EXPANSION RATE

$$\theta = D_A^{\ A} = \frac{1}{2}\gamma^{AB}\partial_t\gamma_{AB} + v^A_{\ |A}$$

? MEANING OF π_A AND EXPLANATION OF COEFFICIENT $\chi = 8\pi G$

TOTAL ANGULAR MOMENTUM OF ANY AXISYMMETRIC SPACETIME:

KILLING VECTOR $m^\mu \partial_\mu = \partial/\partial\varphi$

$$J_\infty = -\frac{1}{\chi}\oint_{S_\infty}\frac{1}{2}\nabla^\nu m^\mu d^2S_{\mu\nu} = -\frac{1}{\chi}\oint_{\mathcal{H}}\frac{1}{2}\nabla^\nu m^\mu d^2S_{\mu\nu} + \int m^\mu T^\nu{}_\mu d^3\Sigma_\nu$$

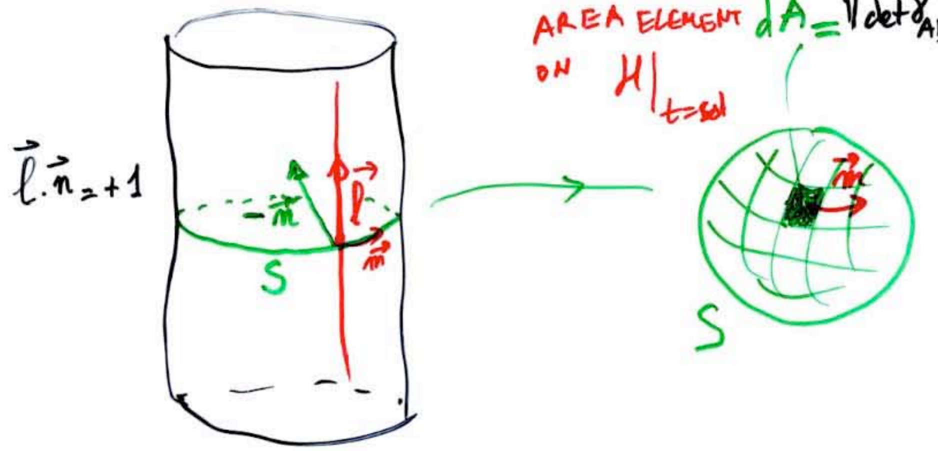
DEFINES THE ANG. MOMENTUM OF B.H.

ANG. MOMENTUM OF MATTER, AFTER USING $R_{\mu\nu} - \frac{1}{2}R g_{\mu\nu} = \chi T_{\mu\nu}$

USING $d^2S^{\mathcal{H}}_{\mu\nu} = (n_\mu \ell_\nu - n_\nu \ell_\mu)\, dA$ AND $\ell^\nu \nabla_\nu m^\mu = m^\nu \nabla_\nu \ell^\mu$

$$\boxed{J_{BH} = -\frac{1}{8\pi G}\oint_{\mathcal{H}} n_\mu m^\nu \nabla_\nu \ell^\mu dA = \oint_S m^A \pi_A \, dA = \oint_S \pi_\varphi \, dA}$$

AREA ELEMENT $dA = \sqrt{\det\gamma_{AB}}\, dx^2 \wedge dx^3$ ON $\mathcal{H}|_{t=\mathrm{sol}}$



$\rightarrow$ $\boxed{\pi_A = \text{'SURFACE DENSITY OF LINEAR MOMENTUM OF B.H.'}}$

NAVIER-STOKES-LIKE EQUATION OF BH (Damour '79)

BY PROJECTING EINSTEIN'S EQS $R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu} = 8\pi G\, T_{\mu\nu}$ ALONG $\ell^\mu e_A^\nu$ (USING THE FACT THAT (g, π_A, D_A^B) GENERALIZES THE 2ND FUNDAMENTAL FORM (Weingarten map), AND USING A NULL generalization of the Gauss-Codazzi eqs) ONE FINDS

$$\boxed{\frac{D}{dt}\pi_A = -\frac{\partial}{\partial x^A}\left(\frac{g}{8\pi G}\right) + \frac{1}{8\pi G}\sigma_{A\,|B}^{B} - \frac{1}{16\pi G}\partial_A\theta - \ell^\mu T_{\mu A}}$$

CONVECTIVE ∂, i.e. Lie derivative
$\frac{D\pi_A}{dt} = (\partial_t + \theta)\pi_A + v^B \pi_{A|B} + v^B_{\,|A}\pi_B$

SHEAR RATE

EXPANSION RATE
$\theta = \frac{\partial_t \sqrt{\gamma}}{\sqrt{\gamma}} + v^A_{\,|A}$

RATE OF FLUX OF MOMENTUM THROUGH THE HORIZON

NEARLY IDENTICAL TO USUAL, NON-RELATIVISTIC NAVIER-STOKES EQ. FOR A (2-d) VISCOUS FLUID

$$\boxed{\frac{D'}{dt}\pi_i = -\frac{\partial}{\partial x^i} p + 2\eta\, \sigma_{i\,,k}^{k} + \zeta\, \partial_i\theta + f_i}$$

Newtonian convective ∂
$(\partial_t + \theta)\pi_i + v^k \pi_{i,k}$

PRESSURE

SHEAR VISCOSITY

BULK VISCOSITY

FORCE DENSITY

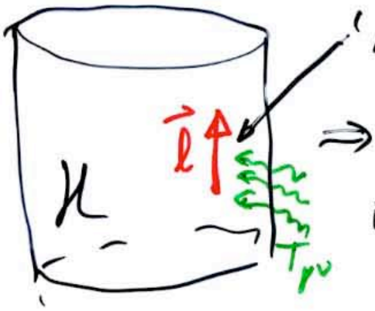
$\Rightarrow$

$p_{BH} = \frac{g}{8\pi G}$	$\eta_{BH} = \frac{1}{16\pi G}$	$\zeta_{BH} = -\frac{1}{16\pi G}$

gives $\frac{\eta_{BH}}{s_{BH}} = \frac{(16\pi G)^{-1}}{(4\hbar G)^{-1}} = \frac{\hbar}{4\pi}$

Bekenstein-Hawking

IRREVERSIBLE THERMODYNAMICS OF BH's



'NORMAL' THROUGH HORIZON IS $\vec{l}$

$\Rightarrow$ FLUX OF ENERGY-MOMENTUM THROUGH $\mathcal{H}$

IS $l^\mu T_{\mu\nu}$

NAVIER-STOKES FOLLOWS FROM $\nu = A$: "SPATIAL AND TANGENT TO $\mathcal{H}$"

ANOTHER USEFUL RESULT FOLLOWS FROM "$\nu = 0$" 'TIME AND TANGENT'

NAMELY $l^\mu l^\nu T_{\mu\nu}$ ('Raychauduri eq.')

FOLLOWING BEKENSTEIN-HAWKING ATTRIBUTE AN

ENTROPY $s = \hat{\alpha}\, dA$ TO EACH BH SURFACE ELEMENT

$$\boxed{\frac{d}{dt}s - \tau \frac{d^2}{dt^2}s = \frac{1}{T_{BH}}\Big[2\eta_{BH}\sigma_{AB}\sigma^{AB} + \zeta_{BH}\,\theta^2 + \rho_{BH}\underbrace{\left(\vec{K} - \sigma_H \vec{v}\right)^2}_{\text{"JOULE'S LAW"}}\Big] dA}$$

$\tau = \frac{1}{g}$

$T_{BH} = \frac{g}{8\pi G\hat{\alpha}}$

BH SURFACE RESISTIVITY $4\pi = 377$ OHM

ALSO, THERE $\exists$ BH OHM'S LAW (Damour, Znajek '78)

$$\boxed{\vec{E} + \vec{v} \times \vec{B} = \rho_{BH}\left(\vec{K} - \sigma_H \vec{v}\right)}$$

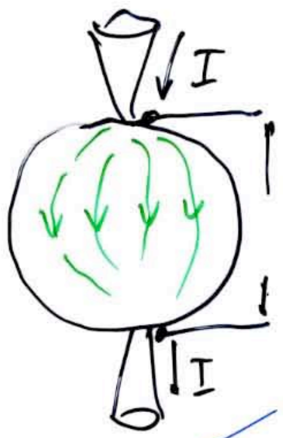
THIS RESULT CONFIRMS THE CONSISTENCY OF INTERPRETING A BH AS A VISCOUS MEMBRANE, ENDOWED WITH ELECTRICAL RESISTIVITY ρ_{BH}

$V = R_{BH}\, I$

WITH R_{BH} COMPUTED FROM ρ_{BH}

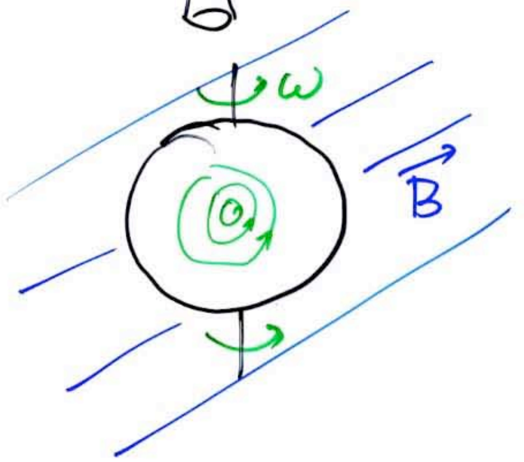
EXAMPLES OF DISSIPATIVE PROCESSES

CONSEQUENCES OF $\rho_H = 4\pi = 377\Omega \neq 0$



$V = R\,I$ AND $\frac{dQ}{dt} = T\frac{dS}{dt} = R\,I^2$

$R \sim 30\,\Omega$

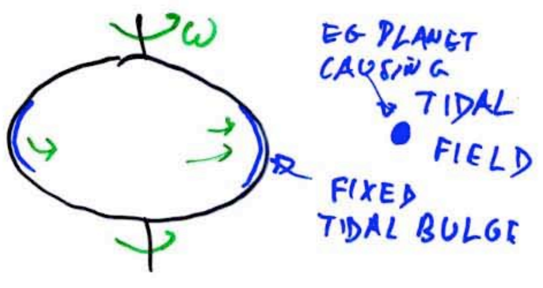


→ EDDY CURRENTS → DISSIPATION AND TORQUE ALIGNING $\vec{\omega}$ TOWARD $\vec{B}$

CONSEQUENCES OF $\eta_{BH} = \frac{1}{16\pi G} \neq 0$

BH EQUILIBRIUM STATES : $\sigma_{AB} = \theta = 0$, $\partial_t = 0$ → UNIFORM 'PRESSURE' ⇒ g UNIFORM ON $\mathcal{H}$

TIDAL BULGE → DISSIPATION (Hartle)



→ DISSIPATION → $\frac{d\omega}{dt} < 0$

+ VALIDITY OF 'MINIMUM ENTROPY PRODUCTION PRINCIPLE'

à la Prigogine

BEKENSTEIN's VIEW OF WHY $T_{BH} = \frac{g}{8\pi G \hat{\alpha}}$

SURFACE GRAVITY

RECALL CHRISTODOULOU-RUFFINI (SPHERICAL SYMM.)

$$\delta M = \underbrace{\frac{Q\delta Q}{r_+(M,Q)}}_{\text{REVERSIBLE (WORK)}} + \underbrace{|g^{rr} p_r|_{r_+}}_{\text{IRREVERSIBLE (HEAT)}}$$

$$\delta E = \delta W + T\delta S$$

TO REACH REVERSIBILITY ONE WOULD NEED

TO FIX BOTH $r = r_+$ AND $p_r = 0$ SIMULTANEOUSLY

CONTRARY TO HEISENBERG's $\delta r\, \delta p_r \geq \frac{1}{2}\hbar$

$$\Rightarrow (r - r_+)\, p_r \geq \frac{1}{2}\hbar$$

USING $g^{rr} = A(r) = \frac{(r-r_+)(r-r_-)}{r^2} \simeq \left(\frac{\partial A}{\partial r}\right)_{r_+}(r - r_+)$

$$g^{rr} p_r = A(r)\, p_r \simeq \left(\frac{\partial A}{\partial r}\right)_{r_+} \underbrace{(r-r_+)\, p_r}_{\text{Heisenberg}} \geq \frac{1}{2}\hbar \left(\frac{\partial A}{\partial r}\right)_{r_+}$$

$$\Rightarrow T\delta S \geq \frac{1}{2}\hbar \left(\frac{\partial A}{\partial r}\right)_{r_+}$$

WHEN ABSORBING ONE PARTICLE

ONE CHECKS $\boxed{\left.\frac{\partial A(r)}{\partial r}\right|_{r_+} = 2g}$

EG FOR SCHWARZSCHILD $A(r) = 1 - 2\frac{GM}{r}$

$$\Rightarrow \frac{\partial A}{\partial r} = 2\frac{GM}{r^2}$$

SO THAT

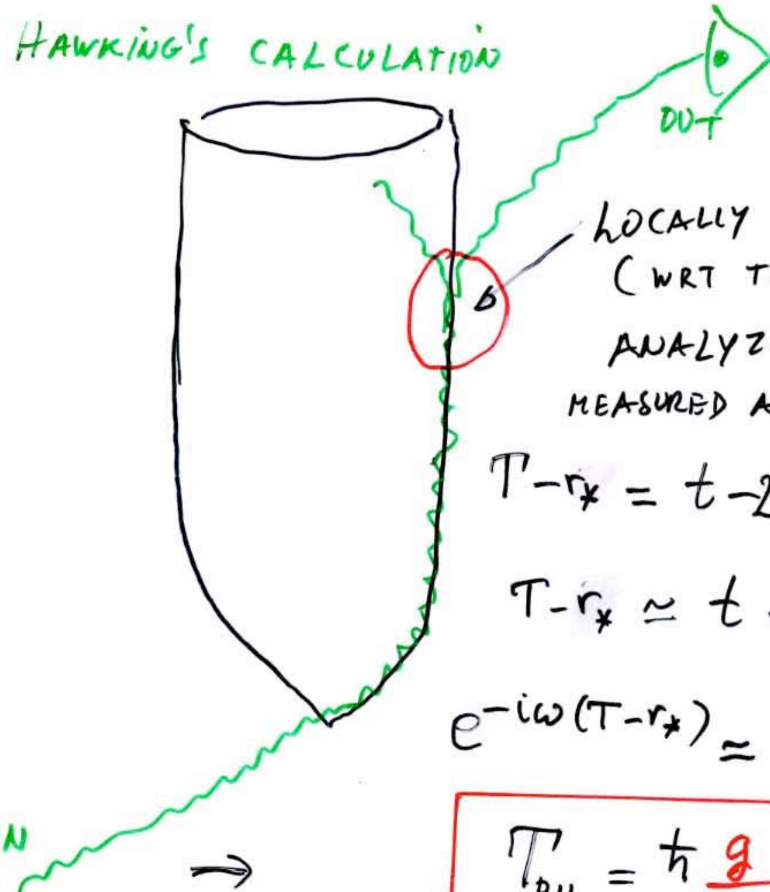
$$T\,\delta S \geqslant \hbar g$$

UPON ABSORBING ONE PARTICLE

BEKENSTEIN ARGUED $\delta S \gtrsim 1$ ('ONE BIT OF INFORMATION LOST')

$$\Rightarrow \quad T_{BH} \sim \hbar g$$

HAWKING'S CALCULATION



LOCALLY NEGATIVE-FREQUENCY MODE (WRT TO LOCALLY REGULAR COORDS t, r, θ, φ) ANALYZED WRT GLOBAL TIME MEASURED AT ∞,

$A(r) \simeq 2g(r - r_+)$

$$T - r_* = t - 2r_* = t - 2\int \frac{dr}{A(r)}$$

$$T - r_* \simeq t - \frac{1}{g}\ln(r - r_+)$$

$$e^{-i\omega(T - r_*)} \simeq e^{-i\omega t}\exp\left(i\frac{\omega}{g}\ln(r - r_+)\right)$$

$$\rightarrow \quad \boxed{T_{BH} = \hbar\frac{g}{2\pi}}$$

3

GRAVITATIONAL WAVES

FROM

COSMIC (SUPER) STRINGS

VARIOUS COSMOLOGICAL SCENARIOS

- STANDARD COSMOLOGICAL "SCENARIO" TO EXPLAIN WHY UNIVERSE SO LARGE, SO HOMOGENEOUS, + $\frac{\delta\rho}{\rho} \sim 5\times10^{-5}$

GR IS VALID

$\exists$ 'INFLATON' φ WITH $V(\varphi)$ VERY FLAT

$\epsilon \sim M_P^2 \left(\frac{V'}{V}\right)^2 \ll 1$ AND $\eta \sim M_P^2 \frac{V''}{V} \ll 1$

QUANTUM FLUCT. $\hat{\delta\varphi} \Rightarrow$ ADIABATIC GAUSSIAN $\frac{\delta\rho}{\rho}$

$\frac{\delta\rho}{\rho} \sim 5\times10^{-5} \Leftrightarrow \exists$ SMALL PARAMETER

$V(\varphi) = \lambda\varphi^4$ or $\frac{1}{2}m^2\varphi^2$ $\quad \lambda \sim 10^{-13}$, $m^2 \sim 10^{-12} m_P^2$

- STRING THEORY CHALLENGES

- FIND A NATURAL CANDIDATE FOR THE INFLATON FIELD φ

E.G. DILATON (Veneziano, Gasperini ...)

SEPARATION OF D-BRANES (Dvali, Tye; Burgess...Quevedo; KKLT, KKLMMT, ...)

- GET GR WITHOUT DILATON-MODULI EFFECTS WHICH "KILL" INFLATION BY INTRODUCING "STEEP" DIRECTIONS IN $V(\varphi, \Phi, \ldots)$

E.G. WARPED FLUX COMPACTIFICATIONS (Giddings, Kachru, Polchinski; Kachru et al...

- ARRANGE EXISTENCE OF SLOW-ROLL REGIONS OF $V(\varphi)$

E.G. LARGE BRANE SEPARATION: (Dvali, Tye; ...; KKLMMT

OR SYMMETRIC CONFIGURATIONS (Trivedi, ...

OR HIGH-DERIVATIVE TERMS $\sim -g(\phi)\sqrt{1+f(\phi)g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi} - V(\phi)$ (Silverstein, Tong)

à la k-inflation (Armendariz-Picon, Damour, Mukhanov; Garriga, Mukhanov)

- TUNE-IN SOME SMALL PARAMETER ($\lambda \sim 10^{-13}$) TO ARRANGE $\frac{\delta\rho}{\rho} \sim 5\times10^{-5}$
- INCORPORATE STANDARD MODEL, AND ARRANGE FOR REHEATING
- ? ARRANGE INITIAL CONDITIONS, OR USE "ANTHROPIC-LIKE" ARGUMENTS

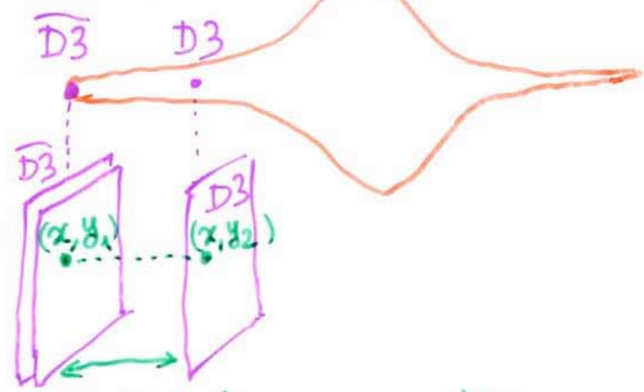
COSMIC SUPERSTRINGS?

Witten '85; ... Dvali, Tye; Tye, ...; KKLMMT; Copeland, Myers, Polchinski; Dvali, Vilenkin

10 dim spacetime:

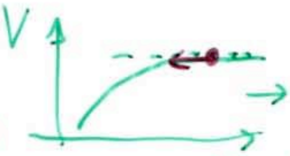
$X^M = (x^\mu, y^a)$

4   6 COMPACT



$r \sim |y_1 - y_2|$

$$V(r) = A - \frac{B}{r^4}$$

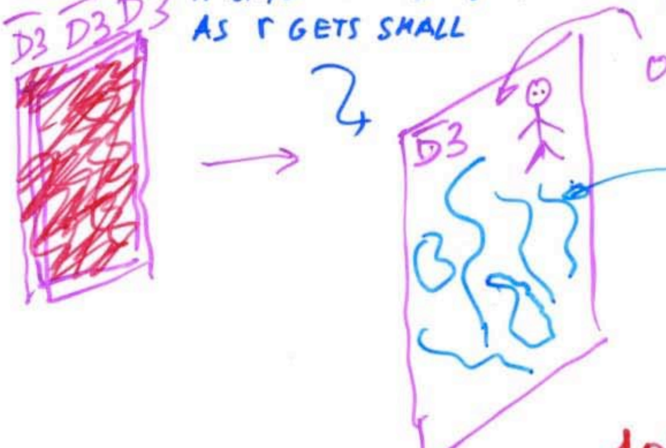


SLOW ROLL INFLATION

$\varphi(x) = r(x)$

TACHYONIC MODE $-\partial T \partial \bar{T} + |m^2| \bar{T} T$ AS r GETS SMALL

HEAT OF HOT BIG BANG



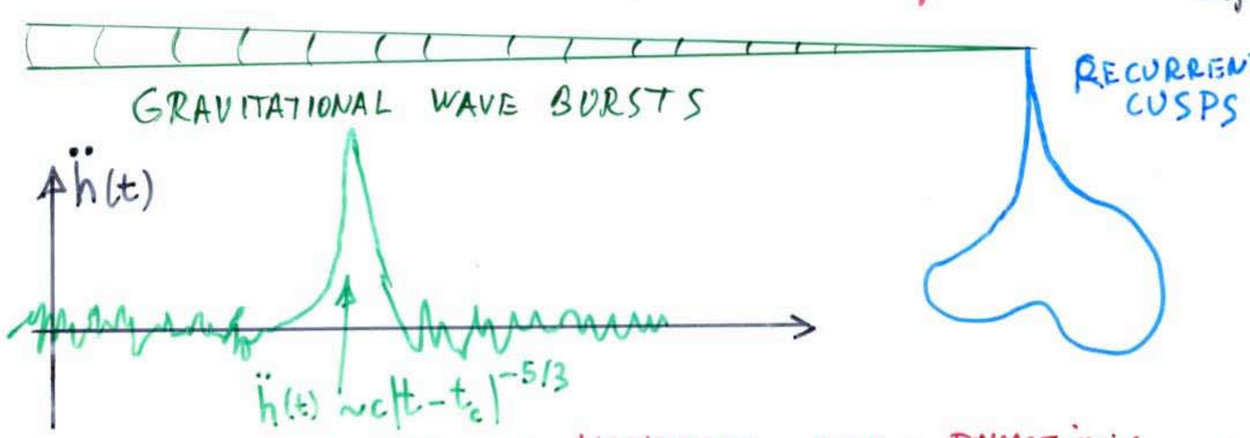
OUR WORLD

COSMOLOGICAL NETWORK OF MASSIVE (F OR D) STRINGS

WITH STRING TENSION

$10^{-11} \lesssim G\mu \lesssim 10^{-6}$ Tye

$G\mu \sim 10^{-8} - 10^{-9}$ KKLMMT Copeland MP



GRAVITATIONAL WAVE BURSTS

RECURRENT CUSPS

$\ddot{h}(t) \sim c|t - t_c|^{-5/3}$

POTENTIALLY DETECTABLE IN LIGO/VIRGO/...; LISA; PULSARTIMING

Damour, Vilenkin

STRING DYNAMICS: $X^\mu(\tau,\sigma)$

$$S_{\text{NAMBU}} = -\mu \int d\tau\, d\sigma \sqrt{(\dot{X}\cdot X')^2 - (\dot{X})^2 (X')^2}$$

$\dot{X} = \partial_\tau X^\mu$, $X' = \partial_\sigma X^\mu$, $\dot{X}\cdot X' \equiv g_{\mu\nu}(X)\dot{X}^\mu X'^\nu$

$$S_{\text{POLYAKOV}} = -\frac{1}{2}\mu \int d\tau\, d\sigma \sqrt{h}\, h^{ab} g_{\mu\nu}(X)\, \partial_a X^\mu \partial_b X^\nu$$

CONFORMAL GAUGE: $\sqrt{h}\, h^{ab} = \eta^{ab}$

$\Rightarrow$ CONSTRAINTS $\dot{X}^2 + X'^2 = 0 \;;\; \dot{X}\cdot X' = 0$

EQ. OF MOTION: $\boxed{\ddot{X}^\mu - X''^\mu + \Gamma^\mu_{\alpha\beta}(X)\,(\dot{X}^\alpha \dot{X}^\beta - X'^\alpha X'^\beta) = 0}$

IN FLAT SPACE: $\ddot{X}^\mu - X''^\mu = 0$

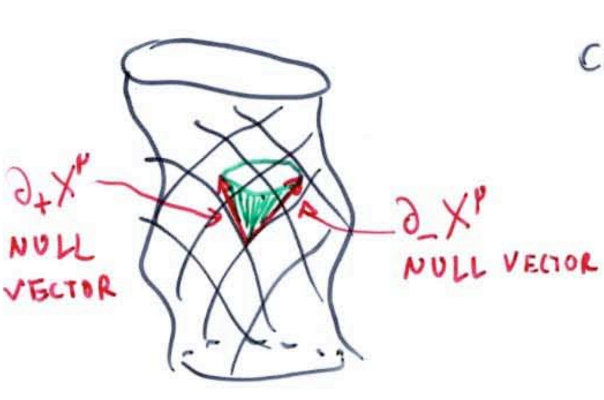
NULL WORLD-SHEET COORDS:

$$\sigma_\pm = \tau \pm \sigma$$

EOM: $\frac{\partial}{\partial\sigma_+}\frac{\partial}{\partial\sigma_-} X^\mu = 0$

LEFT-MOVING RIGHT-MOVING

$$\boxed{X^\mu(\tau,\sigma) = \frac{1}{2}\left[X^\mu_+(\sigma_+) + X^\mu_-(\sigma_-)\right]}$$



CONSTRAINTS: $\boxed{\begin{aligned}(\partial_+ X^\mu_+)^2 &= 0\\ (\partial_- X^\mu_-)^2 &= 0\end{aligned}}$

CUSPS

TIME GAUGE : $X^0(\tau,\sigma) = \tau = \frac{1}{2}[\sigma_+ + \sigma_-]$

IN CENTER-OF-MASS FRAME : $X^i_\pm(\sigma_\pm) =$ PERIODIC $\Rightarrow \langle \dot{X}^i_\pm(\sigma_\pm) \rangle = 0$

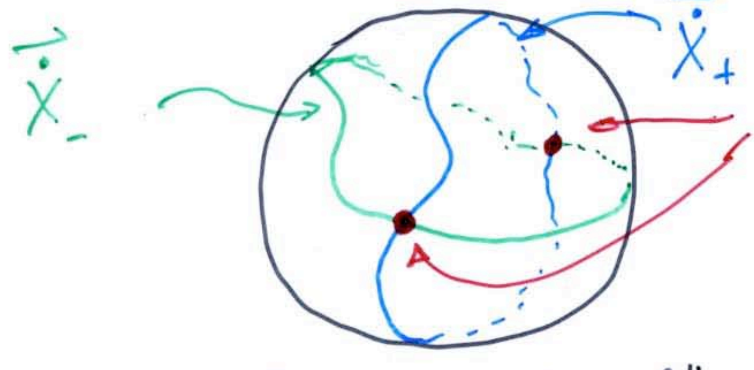
CONSTRAINTS : $(\partial_\pm X^\mu_\pm)^2 = -(\partial_\pm \underbrace{X^0_\pm}_{\sigma_\pm})^2 + (\partial_\pm X^i_\pm)^2 = 0$

$$-1 + (\dot{X}^i_\pm)^2 = 0$$

$$(\vec{\dot{X}}_+)^2 = 1 = (\vec{\dot{X}}_-)^2$$

$\vec{\dot{X}}_+$ AND $\vec{\dot{X}}_-$ ARE PERIODIC (WITH ZERO AVERAGE) ON UNIT SPHERE

Kibble, Turok '82



GENERICALLY EXPECT

$\exists$ 2 INTERSECTIONS

Turok '84

INTERSECTION : $\partial_+ X^\mu_+ = \partial_- X^\mu_- = \ell^\mu$

LIGHT-CONE TANGENT TO WORLD-SHEET

IN SPACE :

GW BURSTS FROM CUSPY STRINGS

$$g_{\mu\nu}(x) = \eta_{\mu\nu} + h_{\mu\nu}(x)$$

$$\bar{h}_{\mu\nu} \equiv h_{\mu\nu} - \frac{1}{2} h\, \eta_{\mu\nu}$$

$$\Box \bar{h}_{\mu\nu} = -16\pi\, G\, T_{\mu\nu} \qquad \partial^\nu \bar{h}_{\mu\nu} = 0$$

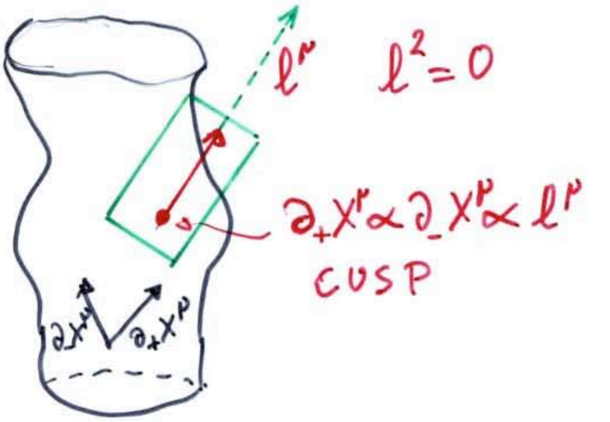
HARMONIC GAUGE

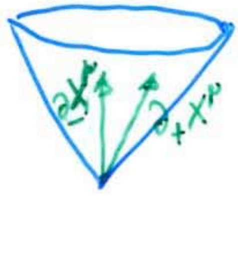
STRING STRESS-ENERGY TENSOR

$$T^{\mu\nu}(x^\lambda) = \mu \int d\tau\, d\sigma \left(\dot{X}^\mu \dot{X}^\nu - X'^\mu X'^\nu \right) \delta^{(4)}\!\left(x^\lambda - X^\lambda(\tau,\sigma)\right)$$

- USE LEFT _ RIGHT DECOMPOSITION : $\boxed{\sigma_\pm \equiv \tau \pm \sigma}$

$$\boxed{X^\mu(\tau,\sigma) = \frac{1}{2}\left[X_+^\mu(\sigma_+) + X_-^\mu(\sigma_-) \right] \qquad \begin{array}{l} (\partial_+ X_+^\mu)^2 = 0 \\ (\partial_- X_-^\mu)^2 = 0 \end{array}}$$





- USE FOURIER TRANSFORM

$$\boxed{T^{\mu\nu}(k^\lambda) = \frac{\mu}{T_\ell} \int_{\Sigma_\ell} d\tau\, d\sigma\, \dot{X}_+^{(\mu} \dot{X}_-^{\nu)} e^{-\frac{i}{2} k \cdot (X_+ + X_-)}}$$

- STAY POINCARÉ COVARIANT

GW AMPLITUDE FROM STRINGS

$$\boxed{\bar{h}_{\mu\nu}(t,\vec{x}) = \frac{\kappa_{\mu\nu}(t-r,\vec{n})}{r} + \mathcal{O}\left(\frac{1}{r^2}\right)}$$

$$\kappa_{\mu\nu}(t-r,\vec{n}) = \sum_{\omega = \pm m \frac{2\pi}{T_\ell}} 4G e^{-i\omega(t-r)} T_{\mu\nu}(\omega, \vec{k}=\omega\vec{n})$$

$T_\ell = \frac{\ell}{2}$ ∧ invariant length $\ell = \frac{E_0}{\mu}$

$$\boxed{T^{\mu\nu}(\vec{k}_m, \omega_m) = \frac{\mu}{\ell} I_+^{(\mu} I_-^{\nu)}}$$

$$\boxed{I_\pm^\mu \equiv \int_0^\ell d\sigma_\pm \dot{X}_\pm^\mu(\sigma_\pm) e^{-\frac{i}{2} k_m \cdot X_\pm}}$$

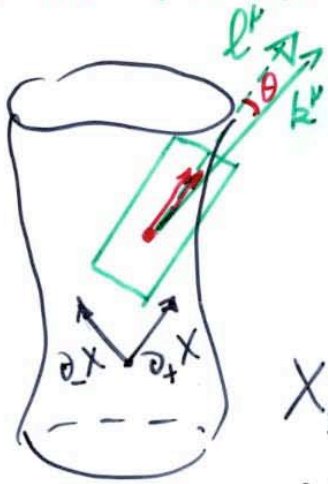
LOGARITHMIC FOURIER TRANSFORM OF WAVEFORM (HIGH-FREQ PART)

$$\kappa^{\mu\nu}(f,\vec{n}) \equiv |f| \int dt\, e^{2\pi i f t} \kappa^{\mu\nu}(t-r,\vec{n})$$

$$\boxed{\kappa^{\mu\nu}(f,\vec{n}) = 2G\mu |f| I_+^{(\mu}(\omega,\omega\vec{n}) I_-^{\nu)}(\omega,\omega\vec{n})}$$

LEFT-RIGHT FACTORIZATION OF GW AMPLITUDE $\kappa^{\mu\nu}(k)$

GW AMPLITUDE FROM CUSPS



AT CUSP

$$\partial_+ X^\mu \propto \partial_- X^\mu \propto \ell^\mu \quad \text{NULL VECTOR}$$

NEAR CUSP

$$X^\mu_\pm(\sigma_\pm) = X^\mu_c + \ell^\mu \sigma_\pm + \frac{1}{2}\ddot{X}^\mu_\pm \sigma_\pm^2 + \frac{1}{6}\dddot{X}^\mu_\pm \sigma_\pm^3 + \dots$$

$$\dot{X}^\mu_\pm(\sigma_\pm) = \ell^\mu + \ddot{X}^\mu_\pm \sigma_\pm + \frac{1}{2}\dddot{X}^\mu_\pm \sigma_\pm^2 + \dots$$

$$\kappa^{\mu\nu}(f) \propto I_+^{(\mu} I_-^{\nu)} \qquad \omega_\ell = \frac{2\pi}{T_\ell} = \frac{4\pi}{\ell}$$

$$I^\mu_\pm = \int_{\sigma_0}^{\sigma_0+\ell} d\sigma_\pm \left(\ell^\mu + \ddot{X}^\mu_\pm \sigma_\pm + \dots\right) e^{+\frac{i}{12} m \omega_\ell \ddot{X}_\pm^2 \sigma_\pm^3 + \dots}$$

CAN BE GAUGED AWAY (WHEN $\theta = 0$)

$m \to \pm\infty$

$\sim \theta \sigma_\pm^2 + \theta^2 \sigma_\pm$ WHEN $\theta \neq 0$

$$\boxed{\kappa^{\mu\nu}(f, \vec{n}_{cusp}) = -C \frac{G\mu}{(2\pi |f|)^{1/3}} e^{2\pi i f t_c} A_+^{(\mu} A_-^{\nu)} + \text{GAUGE}}$$

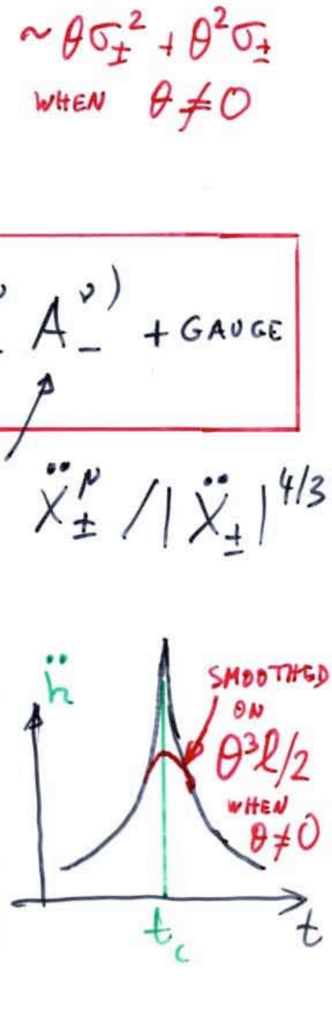
i.e. FOR $\theta = 0$

$$C = 4\pi (12)^{4/3} / \left[3\Gamma(\tfrac{1}{3})\right]^2 \qquad A^\mu_\pm = \ddot{X}^\mu_\pm / |\ddot{X}_\pm|^{4/3}$$

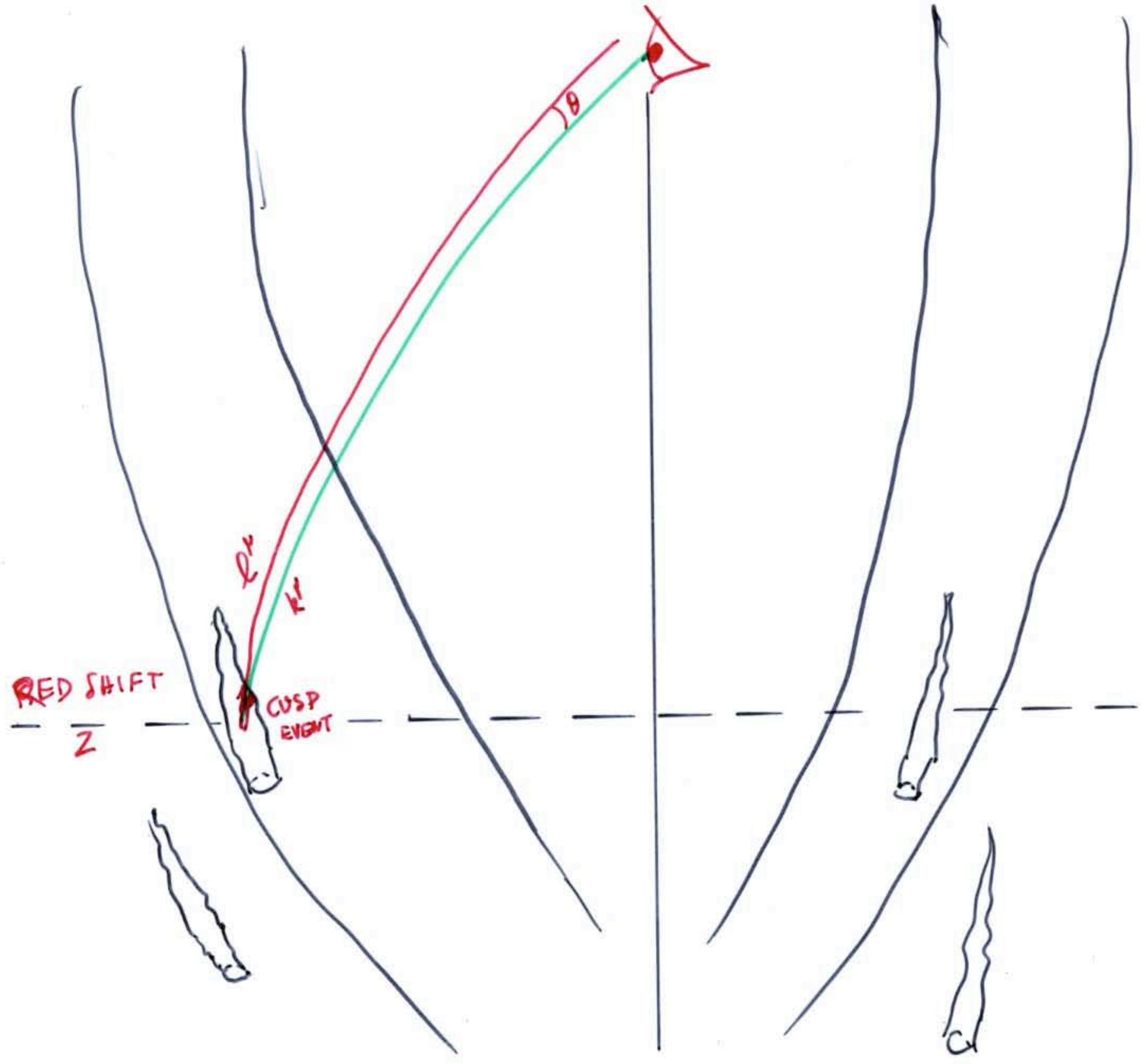
TIME DOMAIN

SIGNAL ROBUST UNDER $\exists$ SMALL-SCALE WIGGLES (Siemens, Olum '03)

$$\boxed{\begin{aligned} \kappa(t) &\propto |t - t_c|^{1/3} \\ \ddot{\kappa}(t) &\propto |t - t_c|^{-5/3} \end{aligned}}$$



GW BURSTS FROM COSMOLOGICAL STRING NETWORK



- EFFECT OF COSMOLOGICAL EXPANSION ON $\tilde{h}_{\mu\nu}(f)$ — ON FREQUENCY / ON AMPLITUDE
- NUMBER OF CUSP EVENTS PER UNIT SPACE-TIME VOLUME

$$\boxed{\nu(t) \sim c\, n_\ell(t)/(\ell/2)}$$

$c \equiv$ # cusp events per loop period

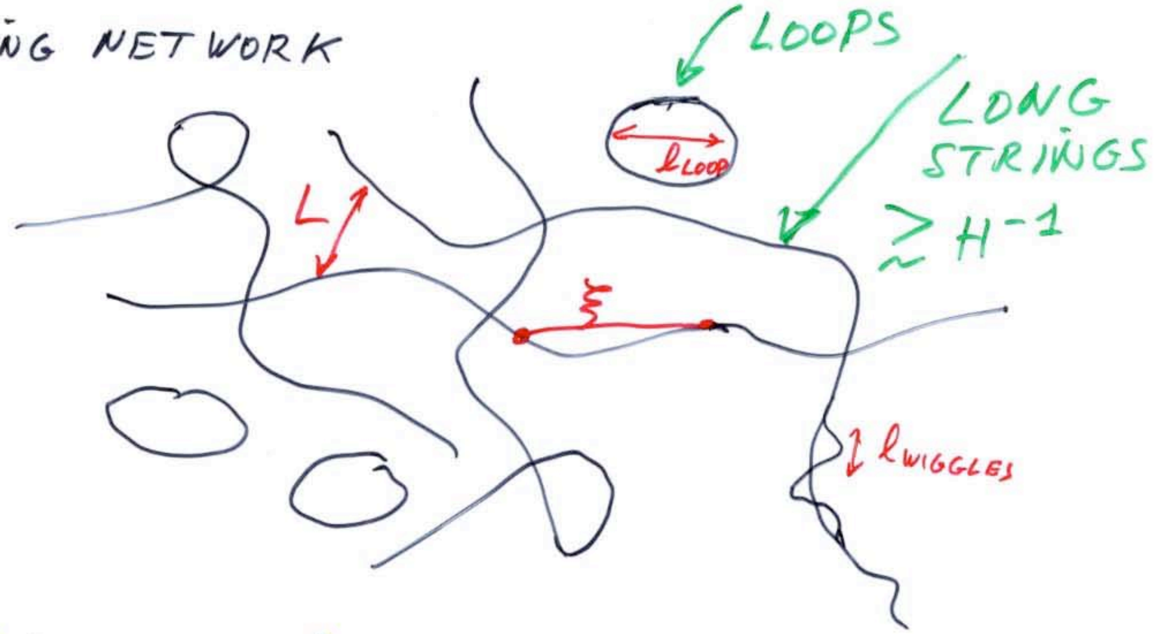
$n_\ell(t) =$ loop density

- BEAMING FRACTION WITHIN $\boxed{\theta_m \equiv \left[(1+z) f \ell/2\right]^{-1/3}}$

LOOP NUMBER DENSITY $n_\ell(t)$?

SEVERAL COMPETING PHENOMENA AT WORK:

- STRING NETWORK



COSMOLOGICAL EXPANSION: → TENDS TO STRAIGHTEN OUT STRINGS

STRING INTERACTIONS: INTERSECTIONS, RECONNECTIONS (WITH PROBA. P), SELF-RECONNECTIONS
→ CREATES LOOPS AND SMALL-SCALE STRUCTURE

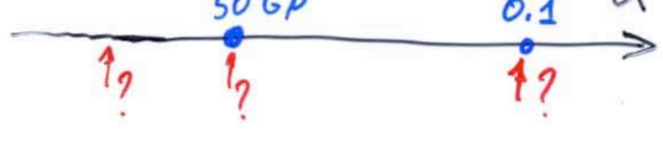
- VARIOUS SCALES: ξ, L, ℓ_{LOOP}, $\ell_{WIGGLES}$

- NUMERICAL SIMULATIONS INDICATE SCALING BUT $\exists$ LARGE UNCERTAINTY IN FINAL SCALING STATE AND PARAMETERS

PROBABLY: $\xi(t) \sim t$, $\boxed{L(t) \sim p^{1/2}\, t}$, $\boxed{\ell_{WIGGLES} \sim \ell_{LOOP\ \text{JUST FORMED}} \sim \alpha\, t}$

GOOD NEWS $p \ll 1$
INCREASES THE DENSITY OF LONG STRINGS AND LOOPS

CRUCIAL DIMENSIONLESS PARAMETER α IS VERY UNCERTAIN



α DETERMINES $\ell_{\substack{LOOP \\ \text{JUST FORMED}}} \sim \alpha t \rightarrow$ LOOP LIFE-TIME $\tau \sim \frac{\alpha}{50 G\mu} t$

$\Rightarrow$ LOOP DENSITY

$$\boxed{n_\ell \sim \overbrace{\frac{1}{p\, 50 G\mu t^3}}^{\text{redshifts } z \lesssim 1} + \overbrace{n_\ell^{z>1}}^{\text{high-redshift}}}$$

DOMINATES IF $\alpha \lesssim 50\, G\mu$ (considered by Damour Vilenkin 05)

DOMINATES IF $\alpha \gg 50\, G\mu$ (considered by Hogan06)

$\exists$ LARGE RANGE OF VALUES OF $G\mu$ WHERE GW BURSTS WOULD BE OBSERVABLE BY LIGO OR LISA. IN ADDITION, AT LOW GW FREQUENCIES CONFUSION NOISE $\rightarrow$ STOCHASTIC GW BACKGROUND OBSERVABLE BY PULSAR TIMING

OBSERVABLE RANGE OF $G\mu$ VALUES IS LARGER BUT THE POPULATION OF HIGH z LOOPS CREATE A LARGER CONFUSION NOISE (i.e. STOCHASTIC BACKGRND) WHICH MAKES MORE DIFFICULT TO SEE INDIVIDUAL BURST EVENTS

LIGO COULD DETECT $G\mu \gtrsim 10^{-12}$

LISA COULD DETECT $G\mu \gtrsim 10^{-14}$

LISA COULD DETECT $G\mu \gtrsim 10^{-15}$

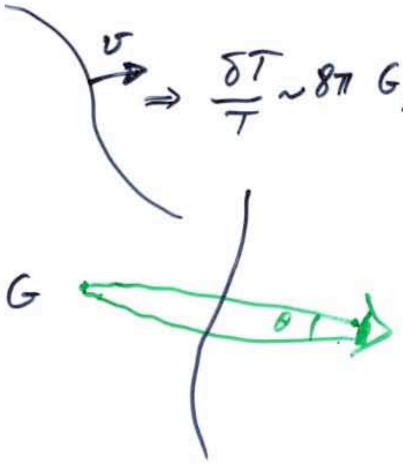
BUT, PULSAR TIMING GIVE ALREADY STRINGENT LIMITS ON THE EXISTENCE OF A STOCHASTIC BACKGRND OF GW's (Jenet et al. 06) WHICH (PROBABLY) ALREADY SETS SEVERE LIMITS ON COSMIC (SUPER) STRINGS

? $G\mu \lesssim 10^{-9}$ OR EVEN $G\mu \lesssim 10^{-10}$?

! WHICH IS ALREADY $\sim$ THE KKLMMT LEVEL !

NOTE THE VARIOUS POSSIBLE OBSERVABLE SIGNALS FROM COSMIC (SUPER)-STRINGS

CMB

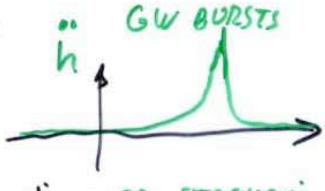


$\Rightarrow \frac{\delta T}{T} \sim 8\pi G\mu v \gamma$ CURRENT BOUND $G\mu \lesssim 5\times 10^{-7}$

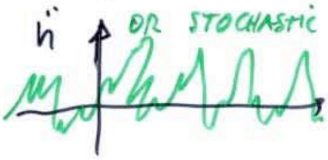
LENSING ONLY SENSITIVE TO $G\mu \gtrsim 10^{-7}$

SEVERAL GW DETECTORS

LIGO / VIRGO / GEO
ADVANCED LIGO
LISA



POTENTIALLY SENSITIVE DOWN TO $G\mu \gtrsim 10^{-12}$



$G\mu \gtrsim 10^{-14}$ OR 10^{-15}

PULSAR TIMING

$$H_{ij}(t) = \int dt\, h_{ij}(t)$$

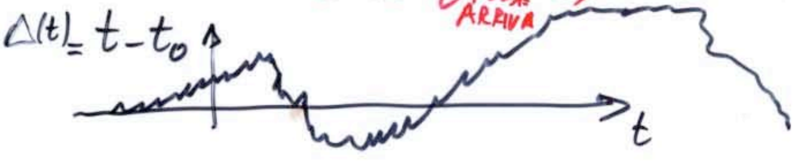
STOCHASTIC SUPERPOSITION OF GW's

$\vec{m}$ = DIRECTION OF GW

$$\Rightarrow t_{\text{PULSAR} \to \text{EARTH}} = t_0 + \frac{1}{2}\frac{1}{1-\vec{m}\cdot\vec{e}}\, e^i e^j \left[H_{ij}(t) - H_{ij}\left(t - (1-\vec{m}\cdot\vec{e})t_0\right)\right]$$

$\vec{e}$ = DIRECTION OF ELM WAVE

STOCHASTIC FLUCTUATION ADDED TO $t_{\text{PULSE ARRIVAL}}$ → RED NOISE



FREQUENCY ANALYSIS OF $\Delta(t)$: $\Delta(f) = \frac{H_0^2}{8\pi^4} \frac{\Omega_{GW}(f)}{f^4}$ ← ENERGY DENSITY OF GW

- EXCITING POSSIBILITY OF DETECTING COSMIC (SUPER) STRINGS
- FRUSTRATING UNCERTAINTIES IN NETWORK PROPERTIES (see Polchinski)