Introduction to hadronic collisions: theoretical concepts and practical tools for the LHC

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    - jets
    - top quark
    - W+multijets
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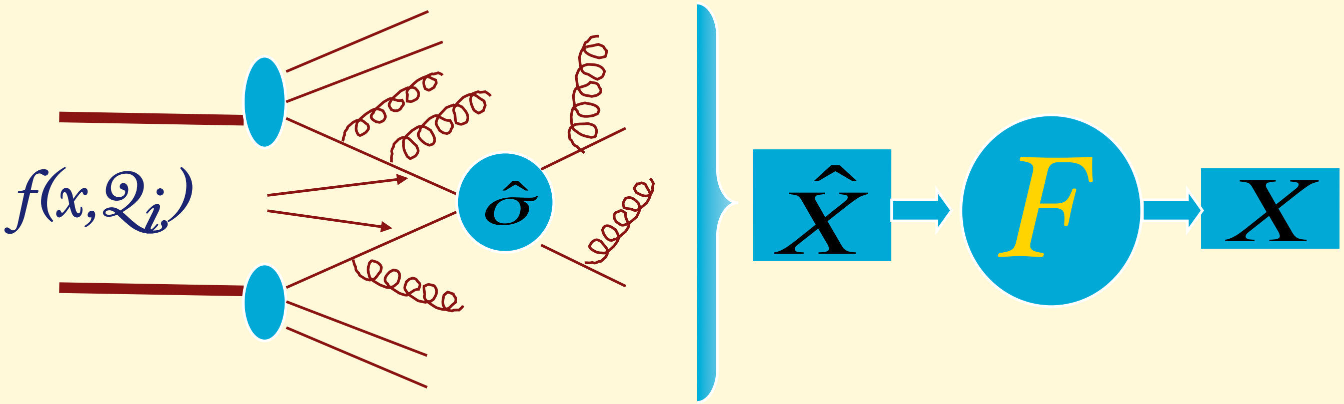
Factorization Theorem

$$\frac{d\sigma}{dX} = \sum_{j,k} \int_{\hat{X}} f_j(x_1,Q_i)\, f_k(x_2,Q_i)\, \frac{d\hat{\sigma}_{jk}(Q_i,Q_f)}{d\hat{X}}\, F(\hat{X} \to X; Q_i,Q_f)$$



$f_j(x,Q)$ Parton distribution functions (PDF)

- sum over all initial state histories leading, at the scale Q, to:

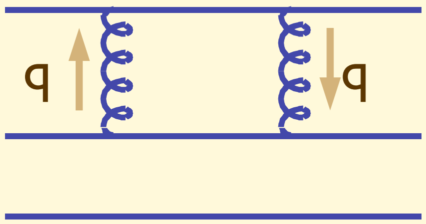
$$\vec{p}_j = x\,\vec{P}_{proton}$$

$F(\hat{X} \to X; Q_i,Q_f)$

- transition from partonic final state to the hadronic observable (hadronization, fragm. function, jet definition, etc)
- Sum over all histories with *X* in them

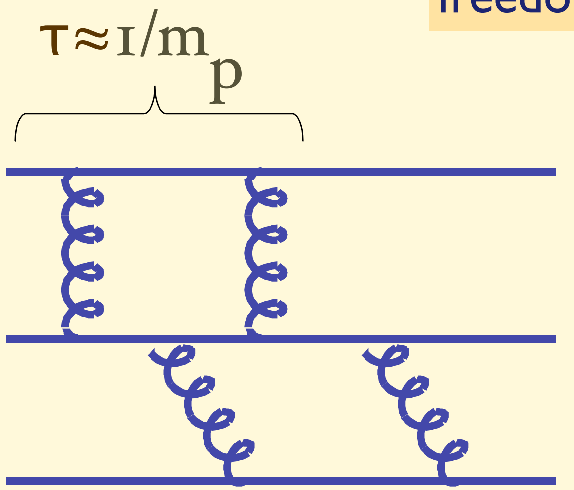
Universality of parton densities and factorization, an intuitive view

1) Exchange of **hard gluons** among quarks inside the proton is suppressed by powers of $(m_p/Q)^2$



$$\stackrel{q>Q}{\sim} \int_Q^\infty \frac{d^4q}{q^6} \sim \frac{1}{Q^2}$$

Assuming asymptotic freedom!

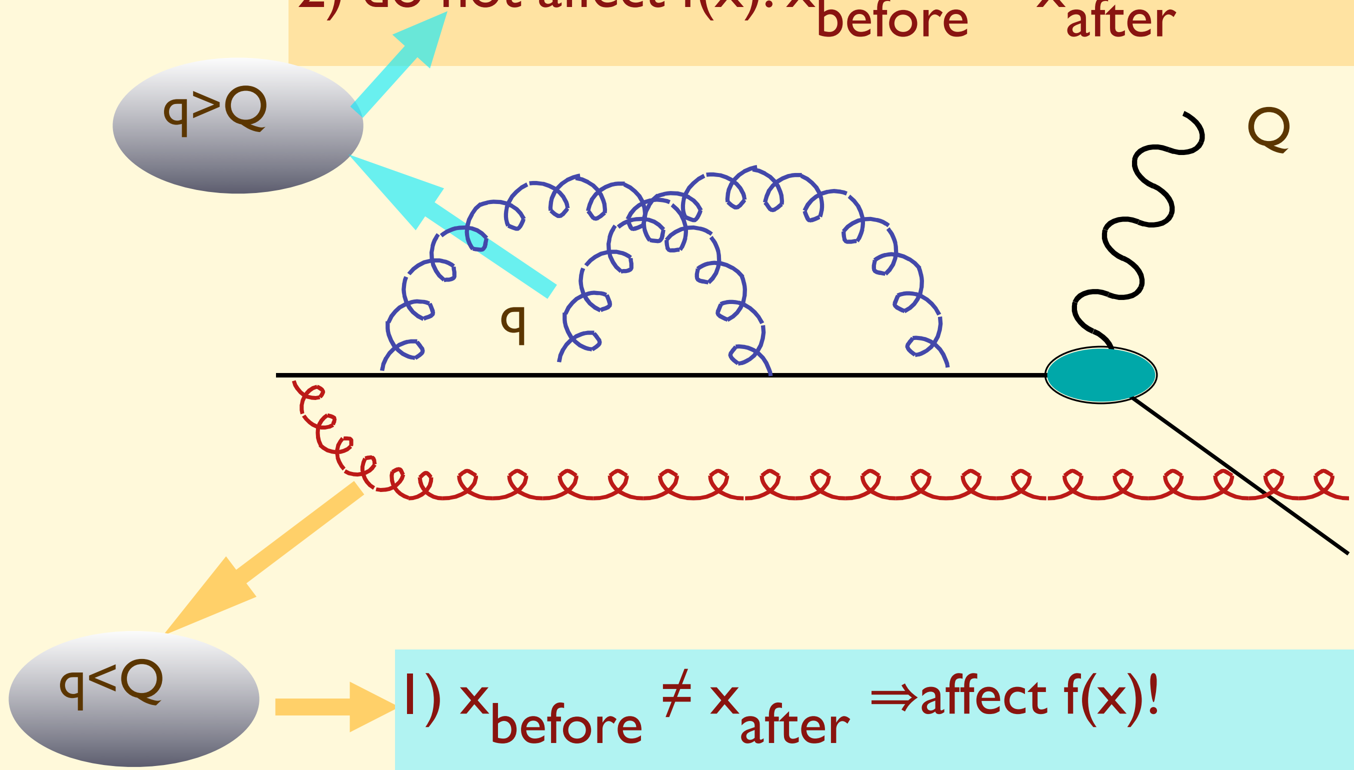


2) Typical time-scale of interactions binding the proton is therefore of $O(1/m_p)$ (in a frame in which the proton has energy E, $\tau=\gamma/m_p = E/m_p^2$)

3) If a hard probe ($Q>>m_p$) hits the proton, on a time scale $=1/Q$, there is no time for quarks to negotiate a coherent response. The struck quark receives no feedback from its pals, and acts as a free particle

As a result, to study inclusive processes at large Q it is sufficient to consider the interactions between the external probe and a single parton:

1) calculable in perturbative QCD (pQCD)
2) do not affect f(x): $x_{before} = x_{after}$



This gluon cannot be reabsorbed because the quark is gone

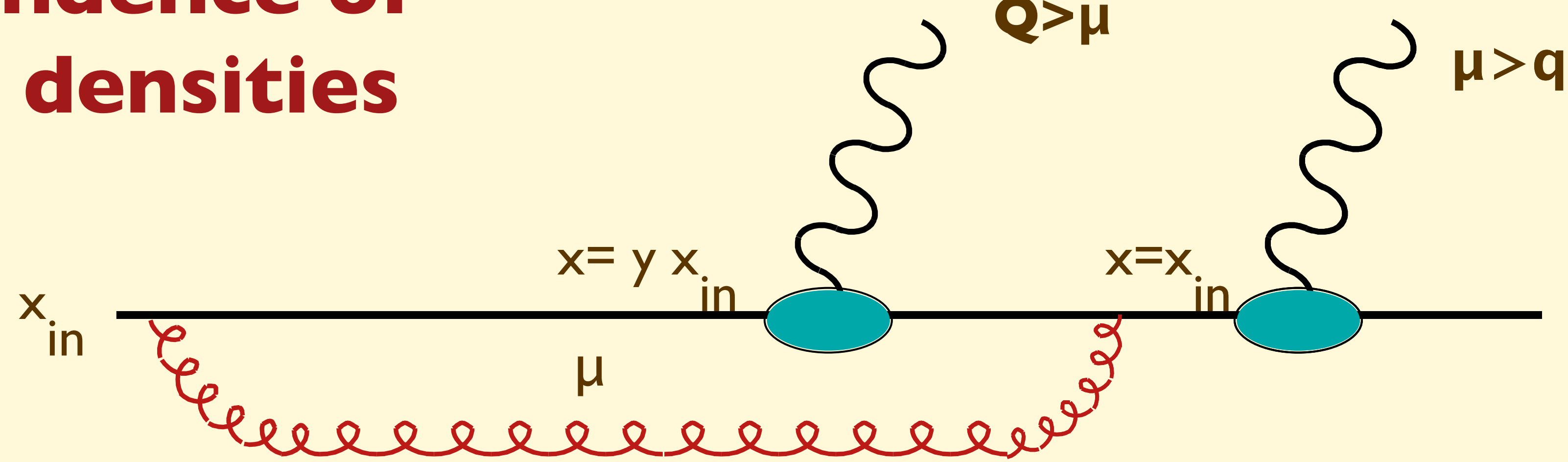
1) $x_{before} \neq x_{after}$ ⇒affect f(x)!

2) for q≈1 GeV **not** calculable in pQCD

However, since τ(q≈1GeV)>>1/Q, the emission of low-virtuality gluons will take place long before the hard collision, and therefore cannot depend on the detailed nature of the hard probe. While it is not calculable in pQCD, f(q<<Q) can be measured using a reference probe, and used elsewhere

➡ Universality of f(x)

Q dependence of parton densities



The larger is Q, the more gluons will **not** have time to be reabsorbed

PDF's depend on Q!

$$f(x,Q) = f(x,\mu) + \int_x^1 dx_{in}\, f(x_{in},\mu) \int_\mu^Q dq^2 \int_0^1 dy\, P(y,q^2)\, \delta(x - y x_{in})$$

$$f(x,Q) = f(x,\mu) + \int_x^1 dx_{in} f(x_{in},\mu) \int_\mu^Q dq^2 \int_0^1 dy P(y,q^2) \delta(x - yx_{in})$$

f(x,Q) should be independent of the intermediate scale μ considered:

$$\frac{df(x,Q)}{d\mu^2} = 0 \quad \Rightarrow \frac{df(x,\mu)}{d\mu^2} = \int_x^1 \frac{dy}{y} f(y,\mu) P(x/y,\mu^2)$$

One can prove that:

calculable in pQCD

$$P(x,Q^2) = \frac{\alpha_s}{2\pi} \frac{1}{Q^2} P(x)$$

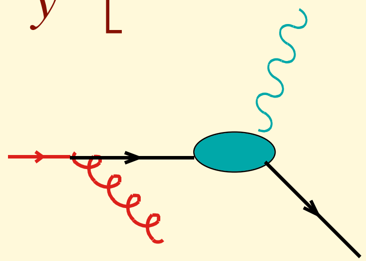
and therefore (Altarelli-Parisi equation):

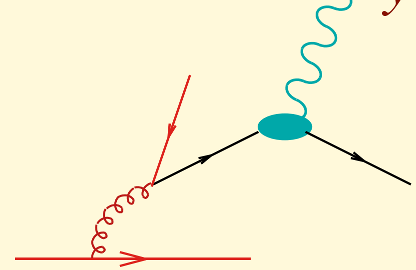
$$\frac{df(x,\mu)}{d\log\mu^2} = \frac{\alpha_s}{2\pi} \int_x^1 \frac{dy}{y} f(y,\mu) P(x/y)$$

More in general, one should consider additional processes which lead to the evolution of partons at high Q ($t=\log Q^2$):

$$[g(x)]_+ : \quad \int_0^1 dx\, f(x)\, g(x)_+ \equiv \int_0^1 [f(x) - f(1)]\, g(x)\, dx$$

$$\frac{dq(x,Q)}{dt} = \frac{\alpha_s}{2\pi} \int_x^1 \frac{dy}{y} \left[q(y,Q)\, P_{qq}(\frac{x}{y}) + g(y,Q)\, P_{qg}(\frac{x}{y}) \right]$$

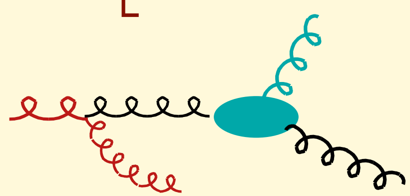


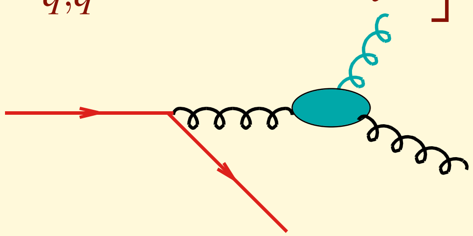


$$P_{qq}(x) = C_F \left(\frac{1+x^2}{1-x} \right)_+$$

$$P_{qg}(x) = \frac{1}{2} \left[x^2 + (1-x)^2 \right]$$

$$\frac{dg(x,Q)}{dt} = \frac{\alpha_s}{2\pi} \int_x^1 \frac{dy}{y} \left[g(y,Q)\, P_{gg}(\frac{x}{y}) + \sum_{q,\bar{q}} q(y,Q)\, P_{gq}(\frac{x}{y}) \right]$$

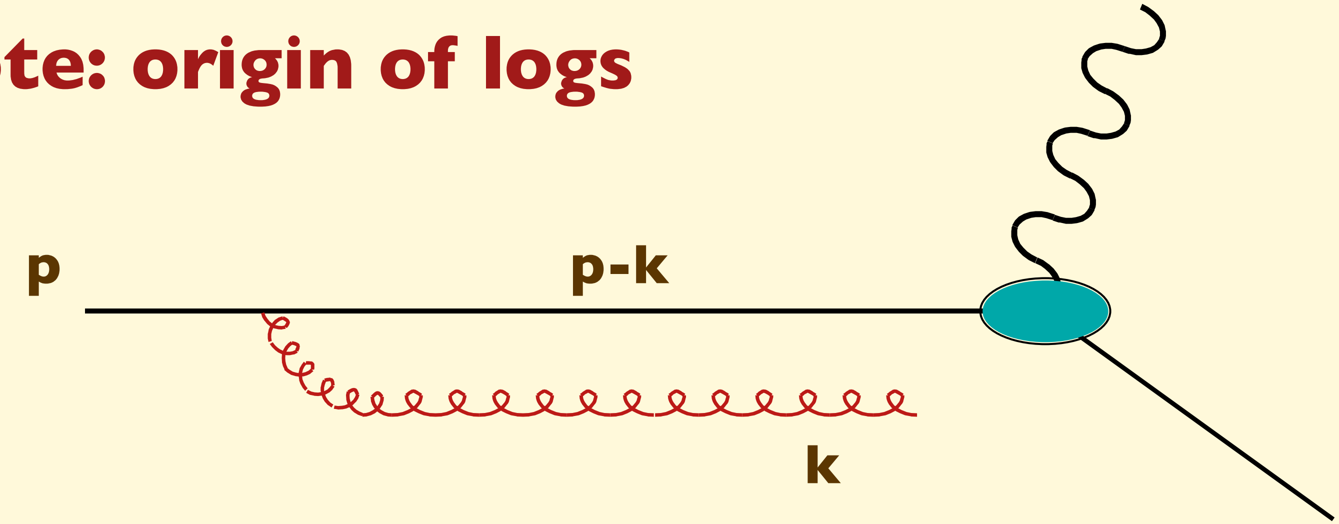




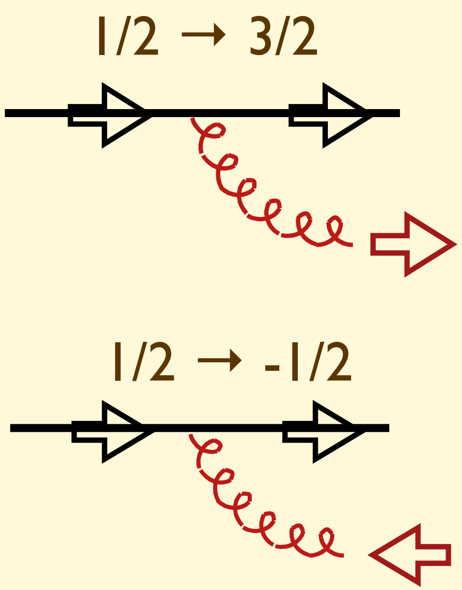
$$P_{gq}(x) = C_F \left(\frac{1+(1-x)^2}{x} \right)$$

$$P_{gg}(x) = 2N_c \left[\frac{x}{(1-x)_+} + \frac{1-x}{x} + x(1-x) \right] + \delta(1-x) \left(\frac{11N_c - 2n_f}{6} \right)$$

Note: origin of logs



$$(p-k)^2 = -2\,p^0 k^0 (1-\cos\theta_{pk})$$



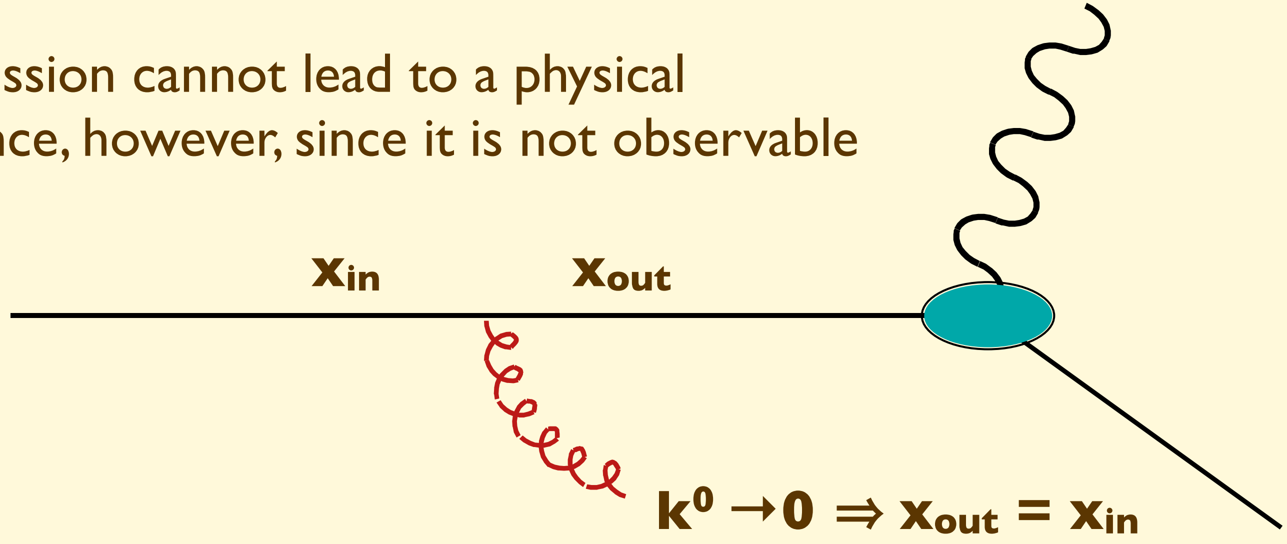
Helicity conservation
~ p · k

$$|M|^2 \sim \left[\frac{1}{(p-k)^2}\right]^2 \times (p\cdot k) \quad \rightarrow \quad \frac{1}{p^0}\frac{dk^0}{k^0}\frac{d\theta}{\theta}$$

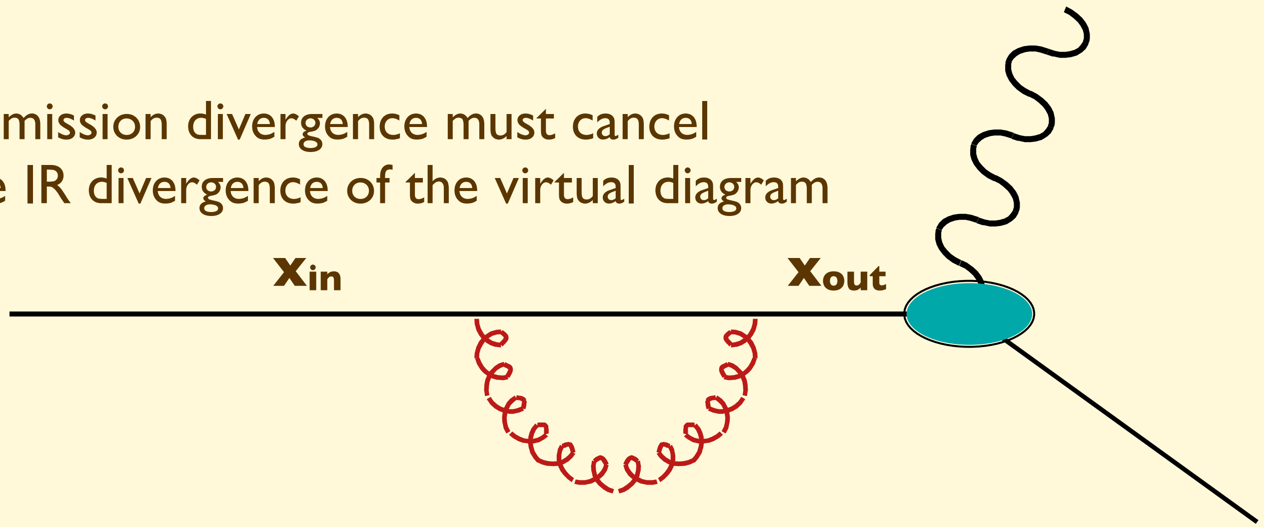
Soft divergence ($\frac{dk^0}{k^0}$)

Collinear divergence ($\frac{d\theta}{\theta}$)

Soft emission cannot lead to a physical divergence, however, since it is not observable

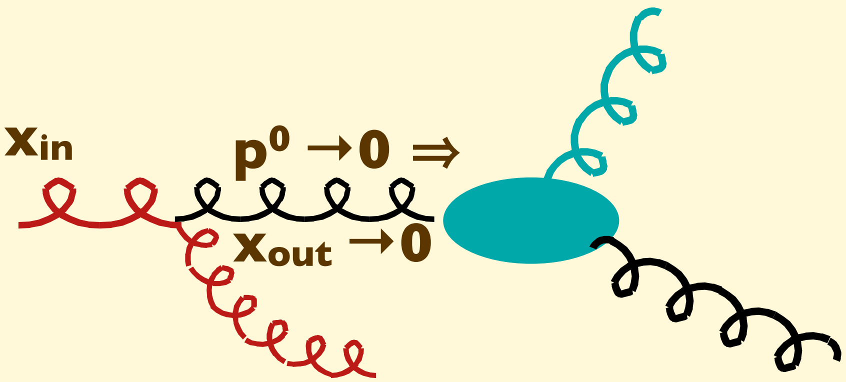


The soft-emission divergence must cancel against the IR divergence of the virtual diagram



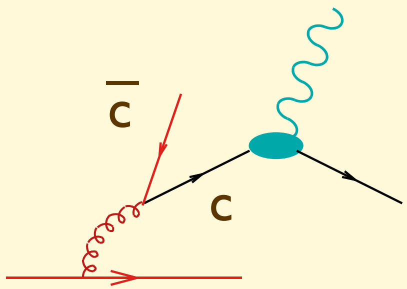
The cancellation cannot take place in the case of collinear divergence, since $x_{out} \neq x_{in}$, so virtual and real configurations are not equivalent

Things are different if $\mathbf{p^0 \to 0}$. In this case, again, $\mathbf{x_{out} \neq x_{in}}$, no virtual-real cancellation takes place, and an extra singularity due to the $\mathbf{1/p^0}$ pole appears



These are called **small-x** logarithms. They give rise to the double-log growth of the number of gluons at small $\mathbf{x}$ and large $\mathbf{Q}$

Example: charm in the proton



$$\frac{dc(x,Q)}{dt} = \frac{\alpha_s}{2\pi}\int_x^1 \frac{dy}{y} g(y,Q)\, P_{qg}(\frac{x}{y})$$

Assuming a typical behaviour of the gluon density: $g(x,Q) \sim A/x$

and using $P_{qg}(x) = \frac{1}{2}\left[x^2 + (1-x)^2\right]$ we get:

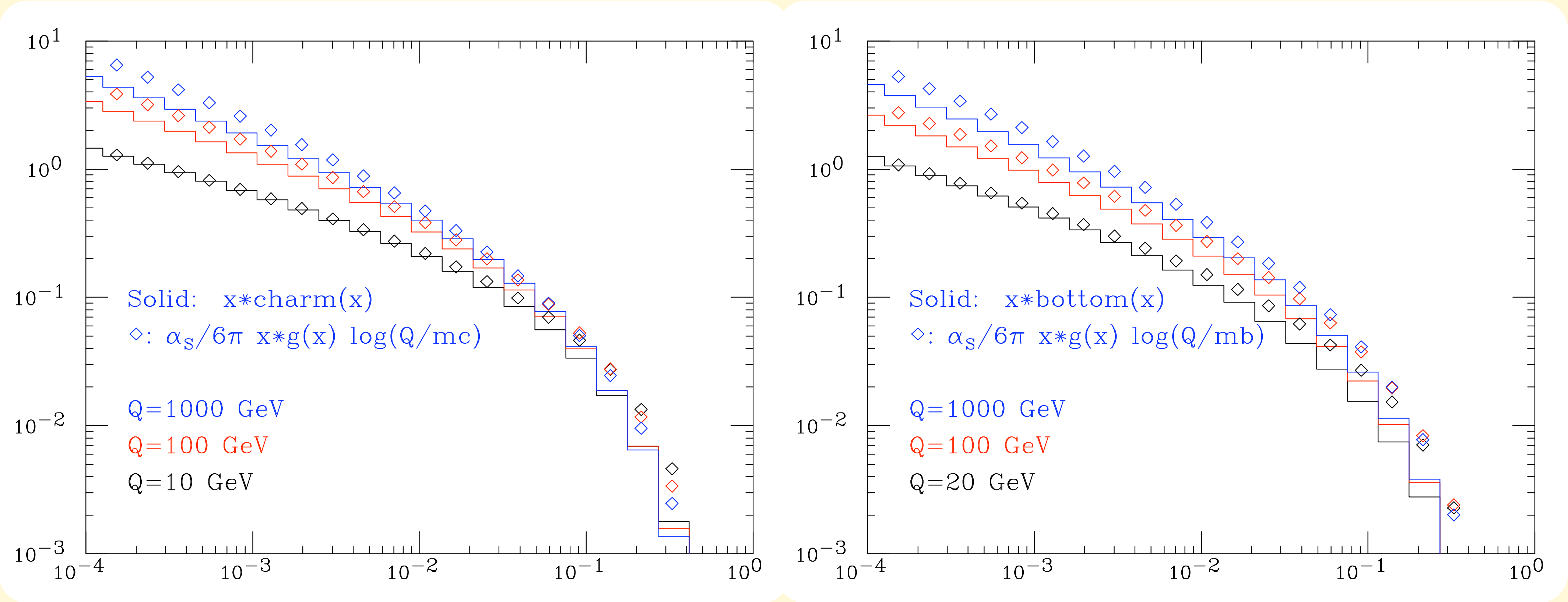
$$\frac{dc(x,Q)}{dt} = \frac{\alpha_s}{2\pi}\int_x^1 \frac{dy}{y} g(x/y,Q)\, P_{qg}(y) = \frac{\alpha_s}{2\pi}\int_x^1 dy \frac{A}{x}\frac{1}{2}[y^2 + (1-y)^2] = \frac{\alpha_s}{6\pi}\frac{A}{x}$$

and therefore:

$$c(x,Q) \sim \frac{\alpha_s}{6\pi} \log(\frac{Q^2}{m_c^2})\, g(x,Q)$$

Corrections to this simple formula will arise due to the Q dependence of g(x) and of α_s

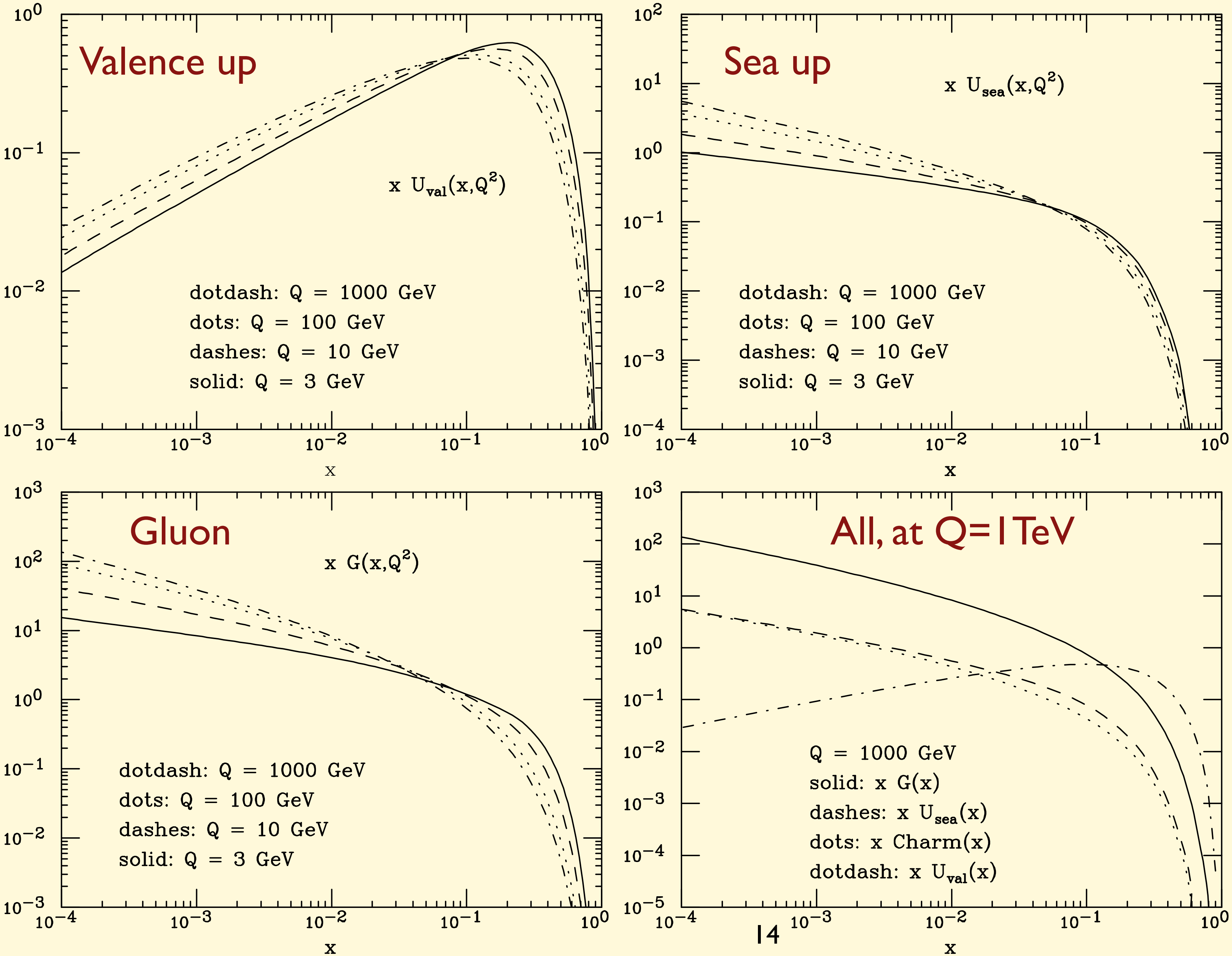
Numerical example



Excellent agreement, given the simplicity of the approximation!

Can be improved by tuning the argument of the log (threshold onset), including a better parameterization of g(x), etc....

Examples of PDFs and their evolution

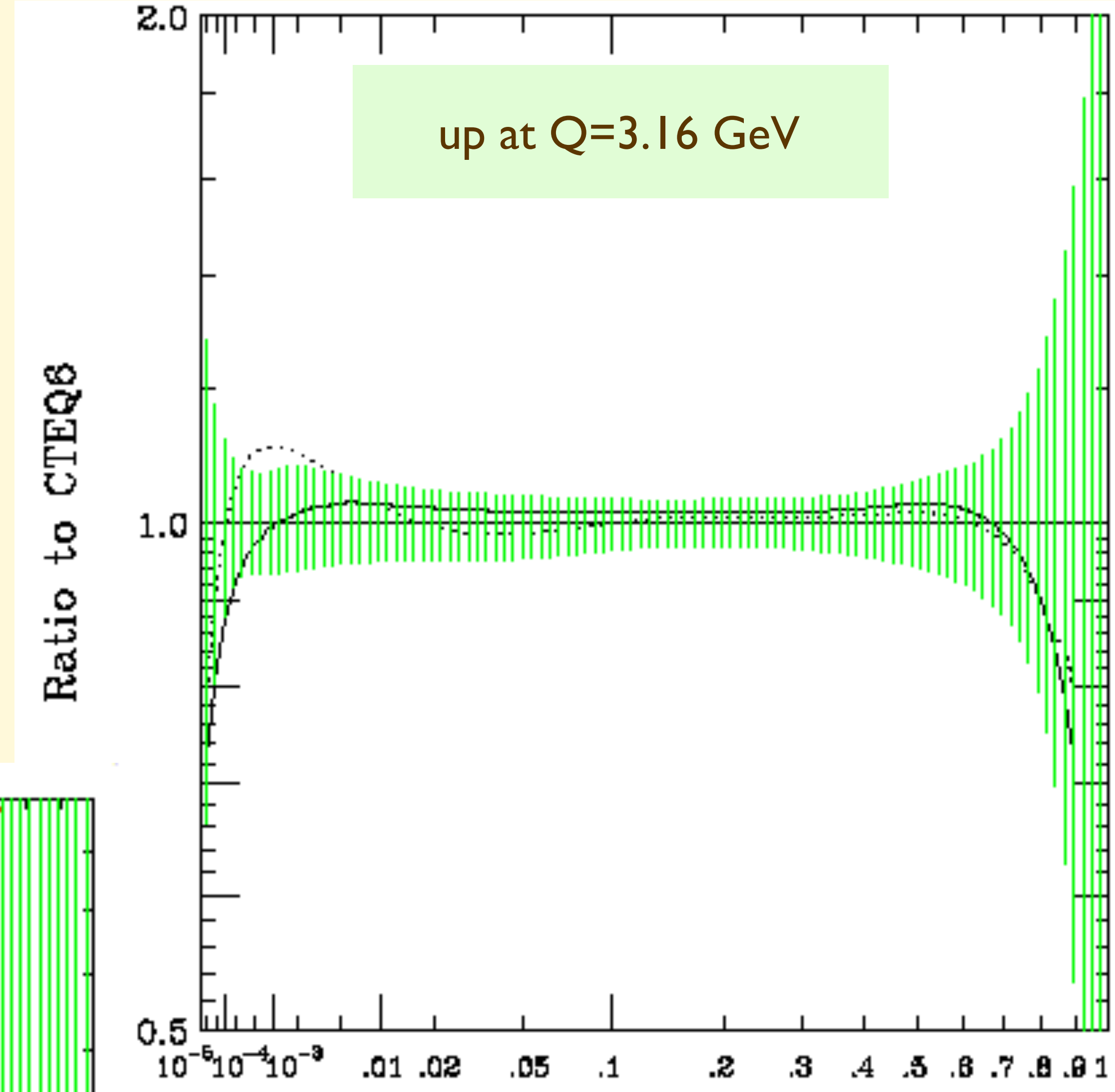


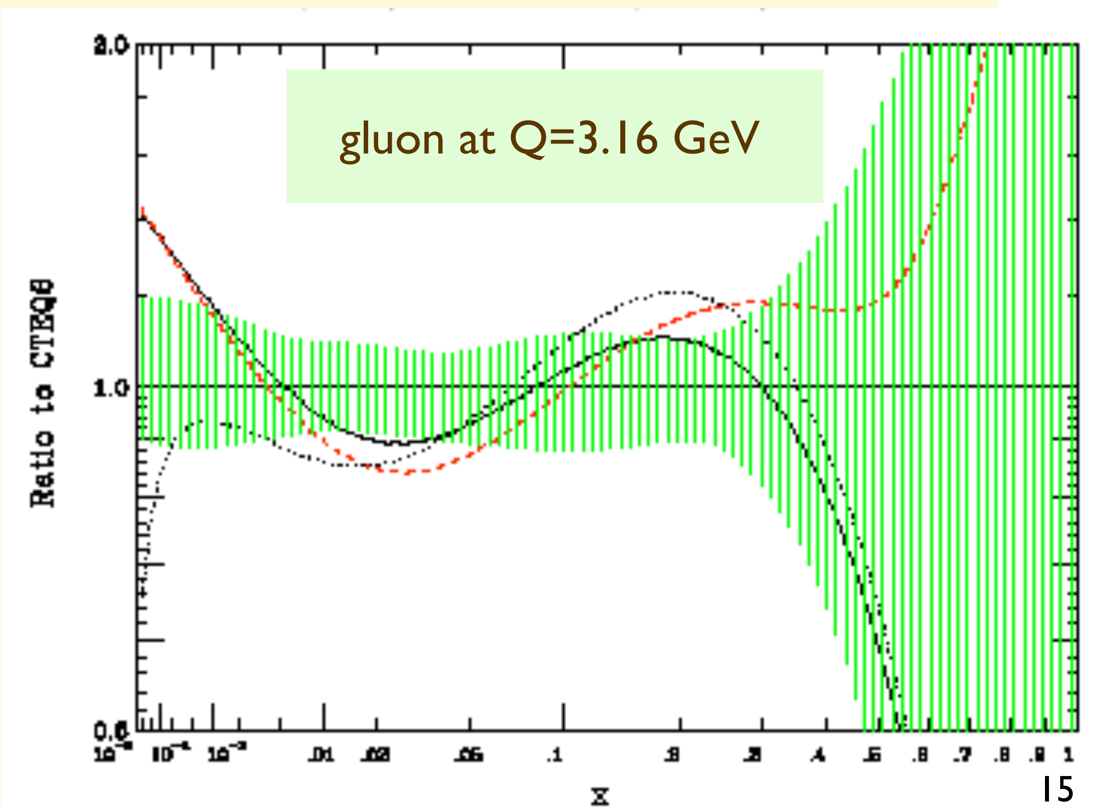
Note:
sea ≈10% glue

Note:
charm≈up at
high Q

PDF uncertainties

Green bands represent the convolution of theoretical and experimental systematics in the determination of PDFs

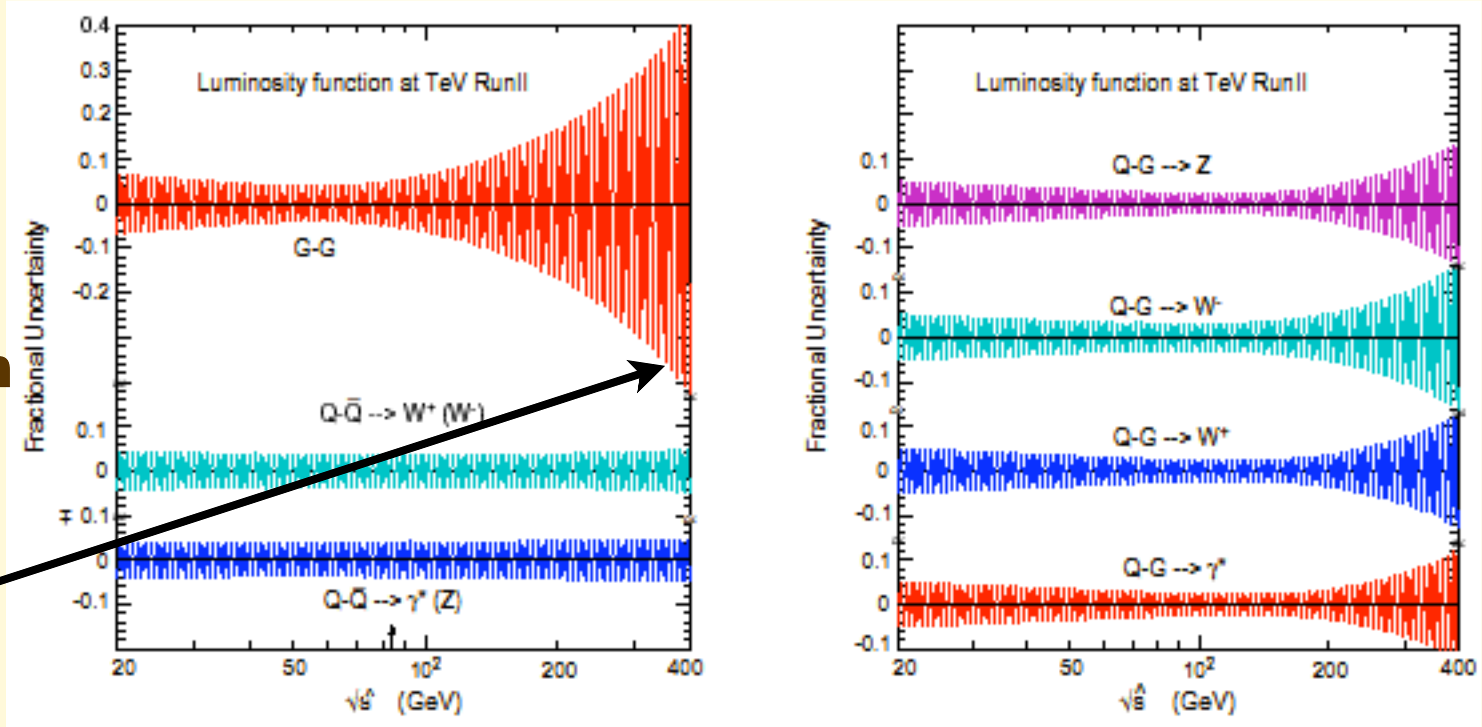




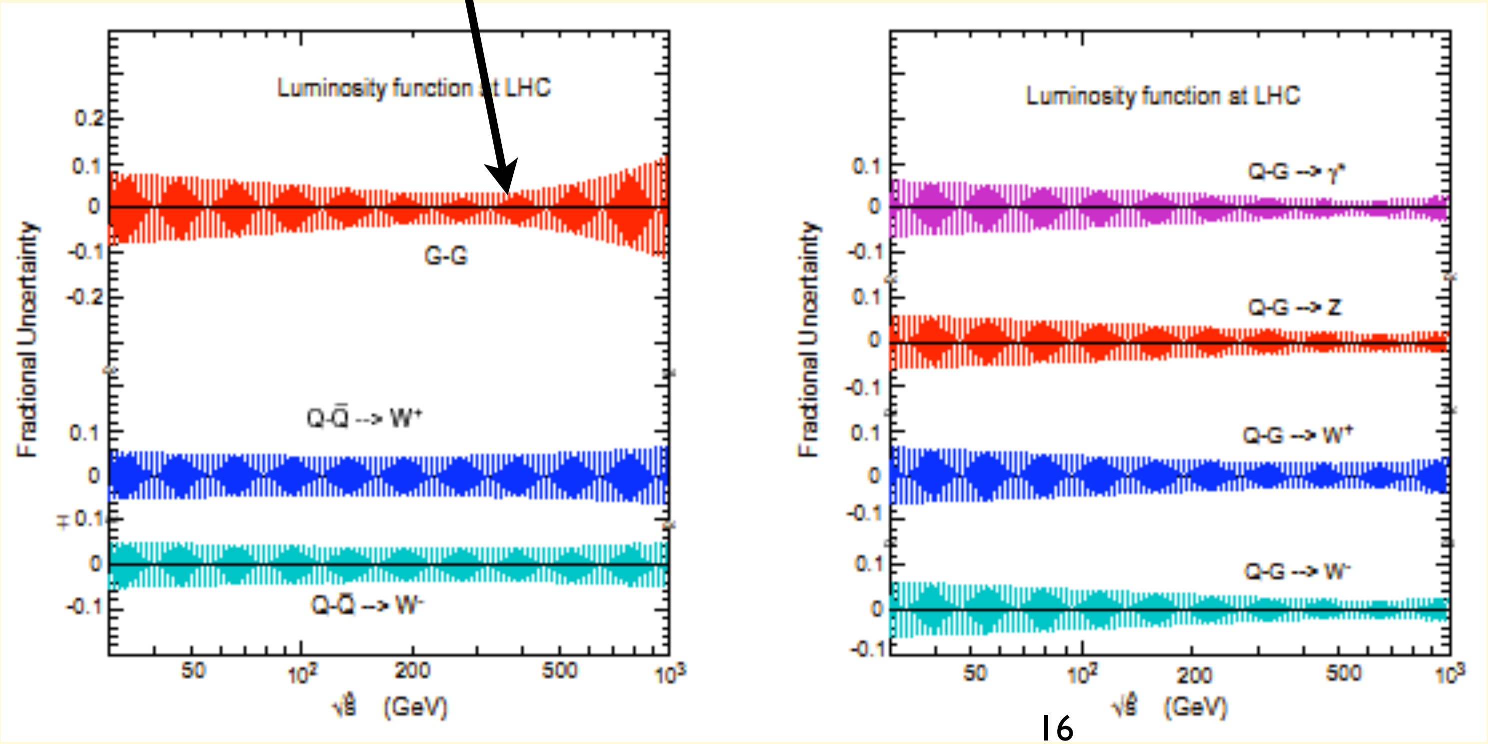
Proton PDFs known to 10-20% for $10^{-3}<x<0.3$, with uncertainties getting smaller at larger Q

PDF luminosity uncertainties

At the Tevatron

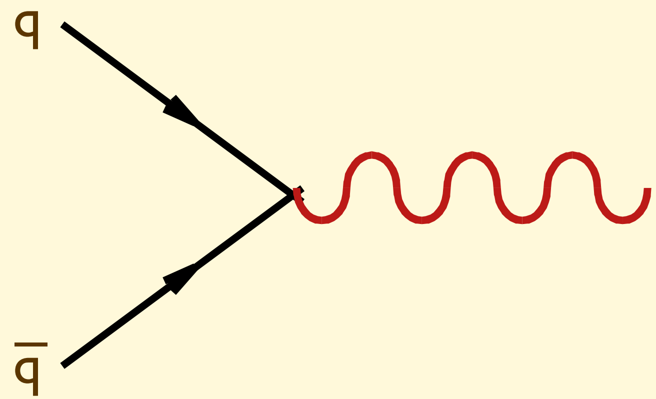


tt production, smaller uncertainty at the LHC!



At the LHC

Example: Drell-Yan processes



$W \to \ell\nu$

$Z \to \ell^+\ell^-$

Properties/Goals of the measurement:

- Clean final state (no hadrons from the hard process)
- Tests of QCD: σ(W,Z) known up to NNLO (2-loops)
- Measure m(W) (➙ constrain m(H))
- constrain PDFs (e.g. $f_{up}(x)/f_{down}(x)$)
- search for new gauge bosons: $q\bar{q} \to W', Z'$
- Probe contact interactions: $q\bar{q}\ell^+\ell^-$

Some useful relations and definitions

Rapidity: $y = \frac{1}{2}\log\frac{E_W + p_W^z}{E_W - p_W^z}$

Pseudorapidity: $\eta = -\log(\tan\frac{\theta}{2})$

where:

$\tan\theta = \frac{p_T}{p^z}$ and $p_T = \sqrt{p_x^2 + p_y^2}$

Exercise: prove that for a massless particle rapidity=pseudorapidity:

Exercise: using $\tau = \frac{\hat{s}}{S} = x_1 x_2$ and

$$\begin{cases} E_W = (x_1 + x_2)\, E_{\text{beam}} \\ p_W^z = (x_1 - x_2)\, E_{\text{beam}} \end{cases} \Rightarrow y = \frac{1}{2}\log\frac{x_1}{x_2}$$

prove the following relations:

$$x_{1,2} = \sqrt{\tau}\, e^{\pm y} \qquad dx_1\, dx_2 = dy\, d\tau$$

$$dy = \frac{dx_1}{x_1} \qquad d\tau\,\delta(\hat{s} - m_W^2) = \frac{1}{S}$$

LO Cross-section calculation

$$\sigma(pp \to W) = \sum_{q,q'} \int dx_1 dx_2 \, f_q(x_1, Q) \, f_{\bar{q}'}(x_2, Q) \, \frac{1}{2\hat{s}} \int d[PS] \overline{\sum_{spin,col}} |M(q\bar{q}' \to W)|^2$$

where:

$$\overline{\sum_{spin,col}} |M(q\bar{q}' \to W)|^2 = \frac{1}{3}\frac{1}{4} 8 g_W^2 \, |V_{qq'}|^2 \hat{s} = \frac{2}{3} \frac{G_F \, m_W^2}{\sqrt{2}} |V_{qq'}|^2 \hat{s}$$

$$d[PS] = \frac{d^3 p_W}{(2\pi)^3 p_W^0} (2\pi)^4 \, \delta^4(P_{in} - p_W)$$
$$= 2\pi \, d^4 p_W \, \delta(p_W^2 - m_W^2) \, \delta^4(P_{in} - p_W) = 2\pi \, \delta(\hat{s} - m_W^2)$$

leading to:

$$\sigma(pp \to W) = \sum_{ij} \frac{\pi A_{ij}}{m_W^2} \, \tau \int_\tau^1 \frac{dx}{x} f_i(x, Q) \, f_j(\frac{\tau}{x}, Q) \equiv \sum_{ij} \frac{\pi A_{ij}}{m_W^2} \, \tau L_{ij}(\tau)$$

where:

$$\frac{\pi A_{u\bar{d}}}{m_W^2} = 6.5\text{nb} \quad \text{and} \quad \tau = \frac{m_W^2}{S}$$

Exercise: Study the function $\tau L(\tau)$

Assume, for example, that $$f(x) \sim \frac{1}{x^{1+\delta}}, \quad 0 < \delta < 1$$

Then: $$L(\tau) = \int_\tau^1 \frac{dx}{x} \frac{1}{x^{1+\delta}} \left(\frac{x}{\tau}\right)^{1+\delta} = \frac{1}{\tau^{1+\delta}} \log\left(\frac{1}{\tau}\right)$$

and: $$\sigma_W = \sigma_W^0 \left(\frac{S}{m_W^2}\right)^\delta \log\left(\frac{S}{m_W^2}\right)$$

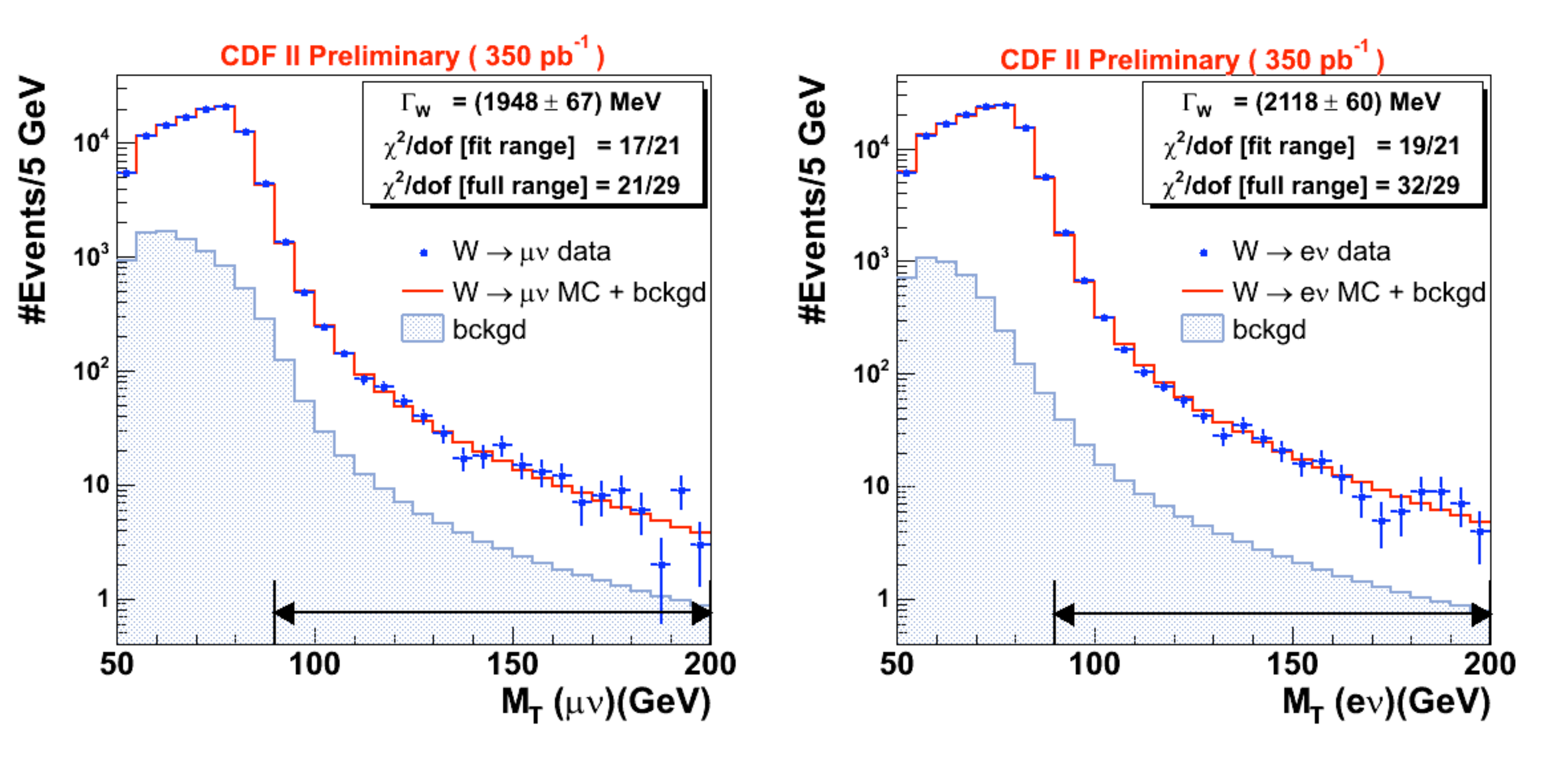
Therefore the W cross-section grows at least logarithmically with the hadronic CM energy. This is a typical behavior of cross-sections for production of fixed-mass objects in hadronic collisions, contrary to the case of e+e- collisions, where cross-sections tend to decrease with CM energy.

Note also the following relation, which allows the measurement of the total width of the W boson from the determination of the leptonic rates of W and Z bosons,

$$\Gamma_W = \frac{N(e^+e^-)}{N(e^\pm\nu)} \left(\frac{\sigma_{W^\pm}}{\sigma_Z}\right) \left(\frac{\Gamma^W_{e\nu}}{\Gamma^Z_{e^+e^-}}\right) \Gamma_Z$$

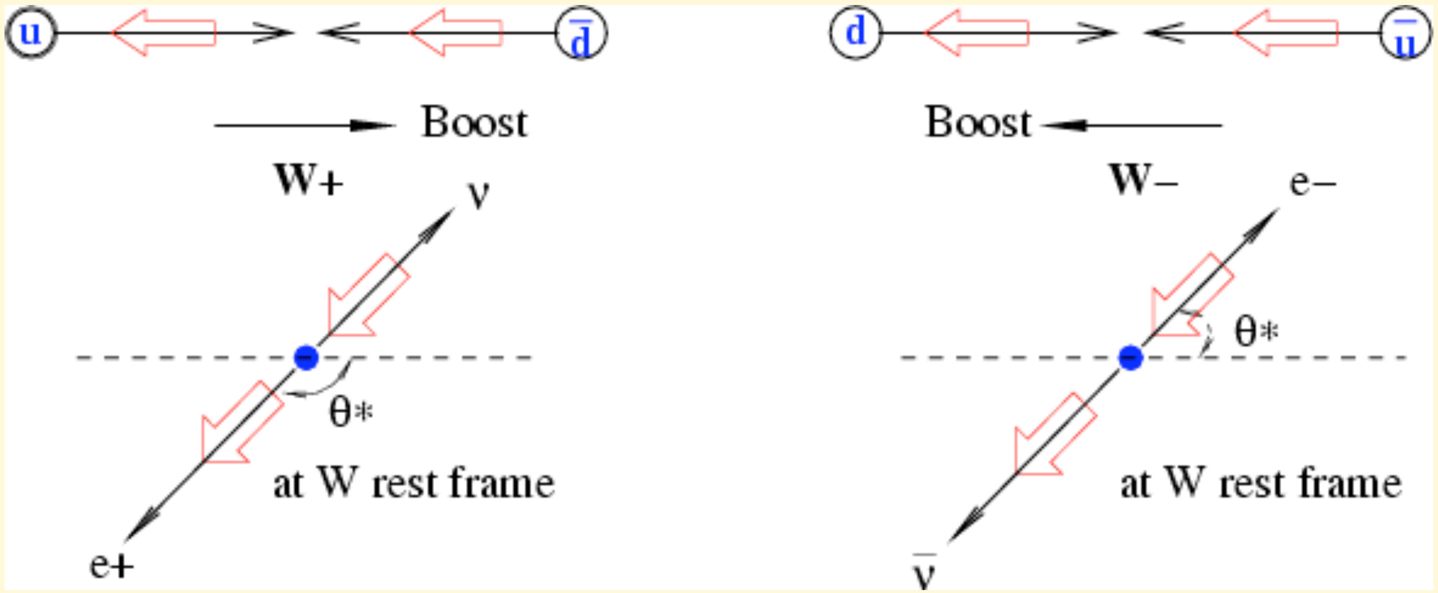
LHC data — theory — LEP/SLC

Again on the W width



$\Gamma_W = 2032 \pm 73$ MeV/c²

Example: W rapidity asymmetry



$$\frac{d\sigma_{W^+}}{dy} \propto f_u^p(x_1)\ f_{\bar{d}}^{\bar{p}}(x_2) + f_{\bar{d}}^p(x_1) f_u^{\bar{p}}(x_2)$$

$$\frac{d\sigma_{W^-}}{dy} \propto f_{\bar{u}}^p(x_1)\ f_d^{\bar{p}}(x_2) + f_d^p(x_1) f_{\bar{u}}^{\bar{p}}(x_2)$$

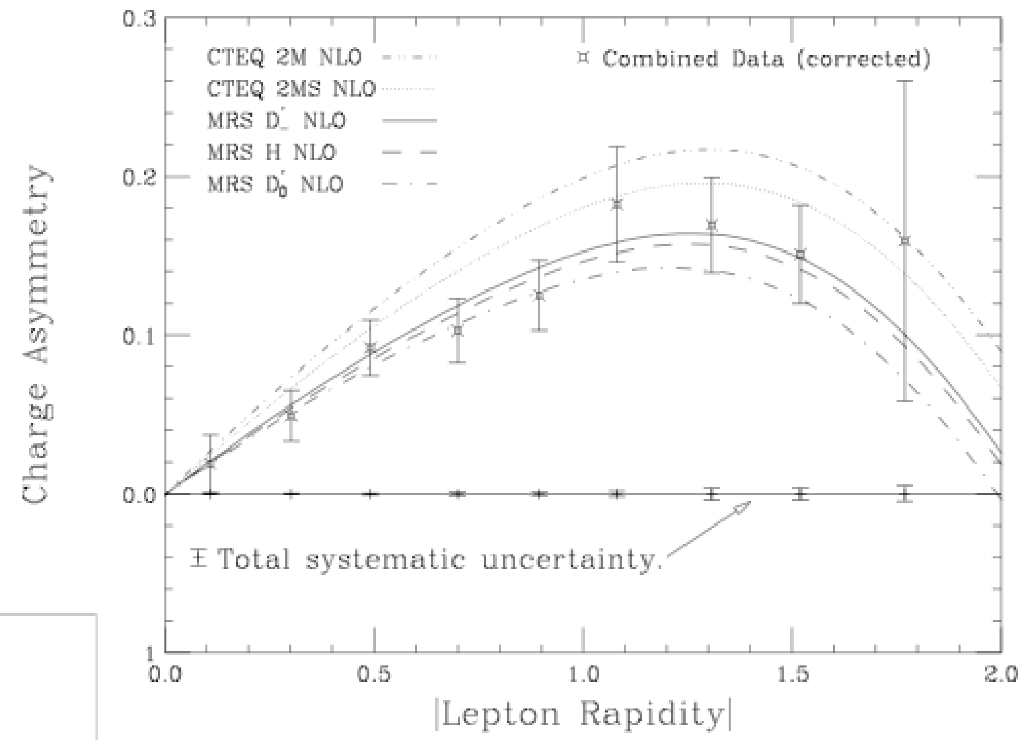
$$f_d(x) = f_u(x)\ R(x)$$

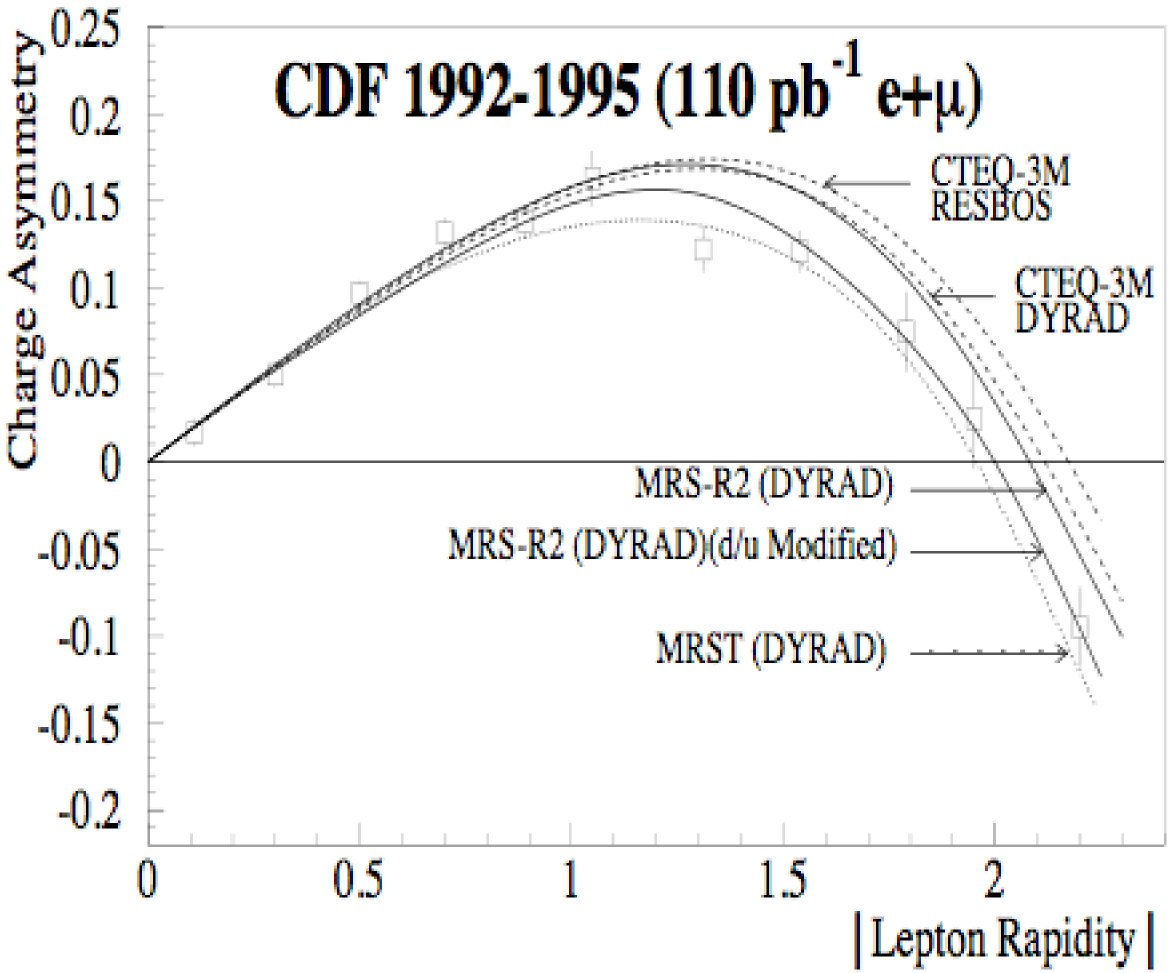
(Assuming dominance of valence contributions)

$$A(y) = \frac{\frac{d\sigma_{W^+}}{dy} - \frac{d\sigma_{W^-}}{dy}}{\frac{d\sigma_{W^+}}{dy} + \frac{d\sigma_{W^-}}{dy}} = \frac{f_u^p(x_1)\ f_d^p(x_2) - f_d^p(x_1)\ f_u^p(x_2)}{f_u^p(x_1)\ f_d^p(x_2) + f_d^p(x_1)\ f_u^p(x_2)} = \frac{R(x_2) - R(x_1)}{R(x_2) + R(x_1)}$$

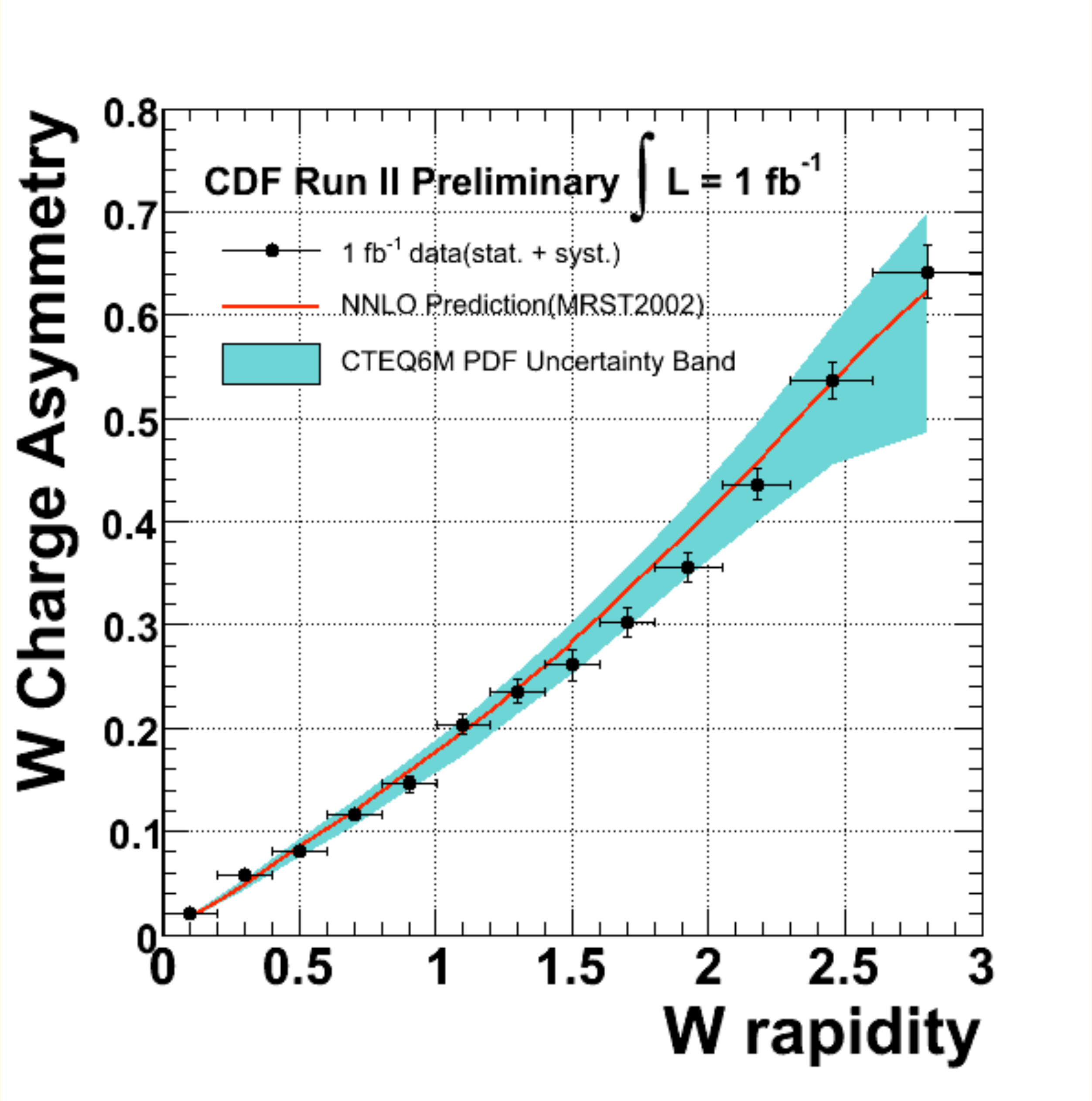
Run 1 comparisons of leptonic charge asymmetry with previous PDF parameterizations

Early data, no statistical power





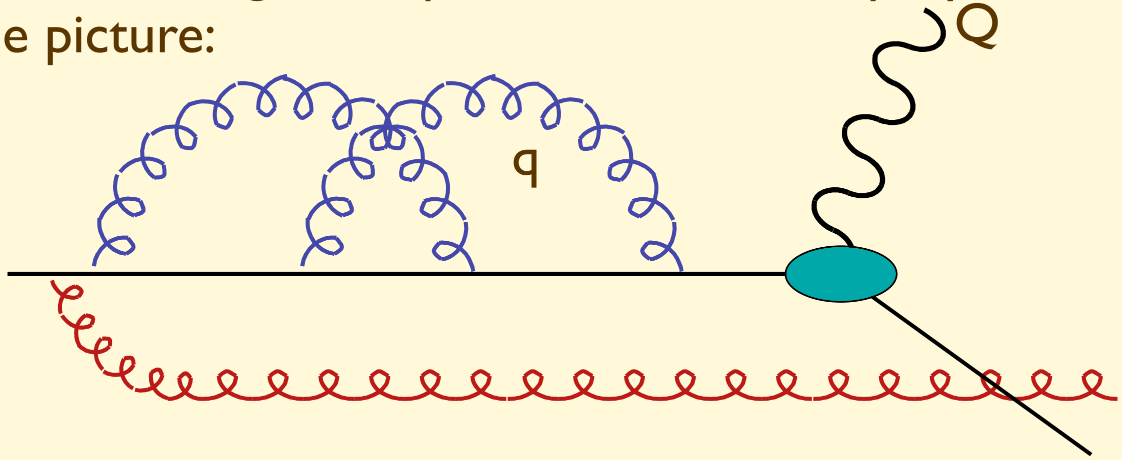
Full dataset, good discrimination



Run II comparison of W charge asymmetry with current PDF parameterizations

Comment

The parton densities are inclusive quantities, namely they say nothing about the number and spectrum of gluons/quarks which accompany the struck parton, like the red gluon in the picture:



Cross-sections obtained with matrix element calculations can therefore only represent inclusive observables. To fully describe, on an event-by-event basis, the multiplicity and kinematics of the emitted radiation requires the so-called parton-shower Monte Carlos.

Occasionally, the gluons emitted during the evolution of the parton towards its hard scattering can themselves be hard, and give rise to what are called "initial state radiation (ISR) jets". Since these are hard objects, with scales comparable or larger than Q, interference effects with the final state are relevant, and their description in the factorized approximation is not correct.

The separation between these two regimes of ISR amounts to a factorization prescription choice. Reducing the dependence of the prescription and guaranteeing a continuity of distributions across this boundary is the subject of intensive study

Introduction to hadronic collisions: theoretical concepts and practical tools for the LHC

Lecture 2

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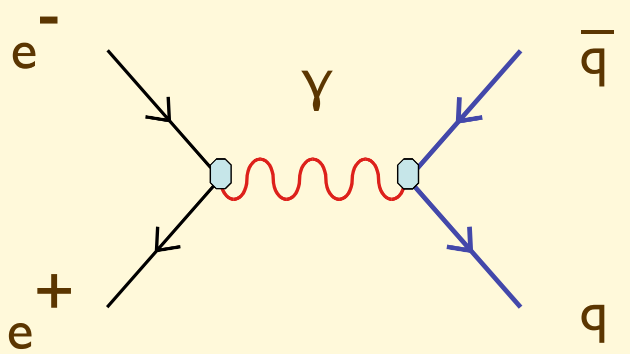
Evolution of hadronic final states

Asymptotic freedom implies that at E_{CM} >> 1 GeV

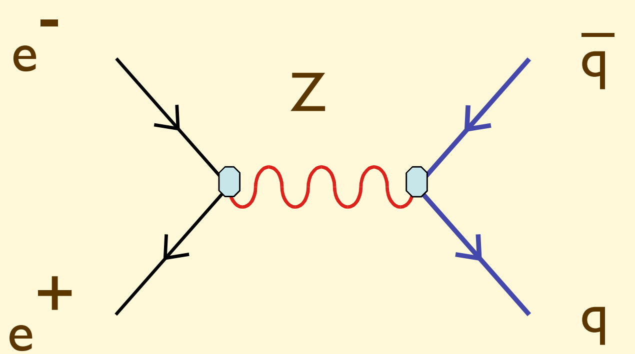
$\sigma(e^+ e^- \rightarrow \text{hadrons})$ ⟷ $\sigma(e^+ e^- \rightarrow \text{quarks/gluons})$

At the Leading Order (LO) in PT:



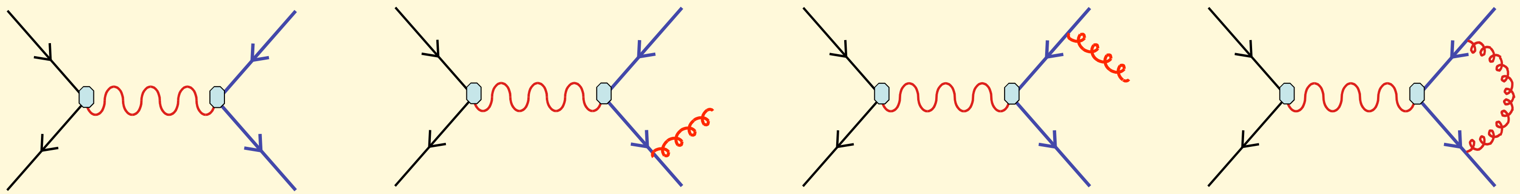
$$\sigma_0(e^+e^- \rightarrow q\bar{q}) = \frac{4\pi\alpha^2}{9s} N_c \sum_{f=u,d,\ldots} e_{q_f}^2$$

$$\frac{\sigma_0(e^+e^- \rightarrow q\bar{q})}{\sigma_0(e^+e^- \rightarrow \mu^+\mu^-)} = N_c \sum_{f=u,d,\ldots} e_{q_f}^2$$



$$\frac{\sigma_0(e^+e^- \rightarrow Z \rightarrow q\bar{q})}{\sigma_0(e^+e^- \rightarrow Z \rightarrow \mu^+\mu^-)} = N_c \frac{\sum_{f=u,d,\ldots} \left(v_{q_f}^2 + a_{q_f}^2\right)}{\left(v_\mu^2 + a_\mu^2\right)}$$

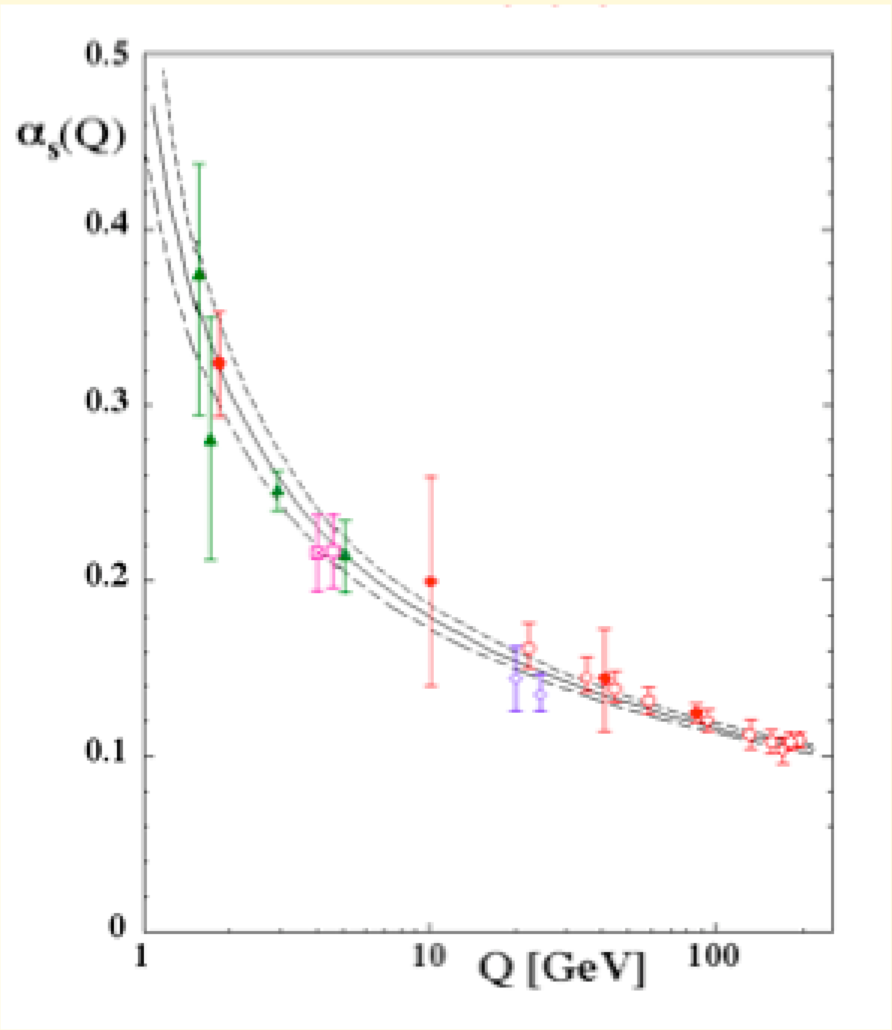
Adding higher-order perturbative terms:



+ ≥2-gluon emissions

$$\sigma_1(e^+e^- \to q\bar{q}(g)) = \sigma_0(e^+e^- \to q\bar{q}) \left(1 + \frac{\alpha_s(E_{CM})}{\pi} + O(\alpha_s^2)\right)$$

O(3%) at M_Z

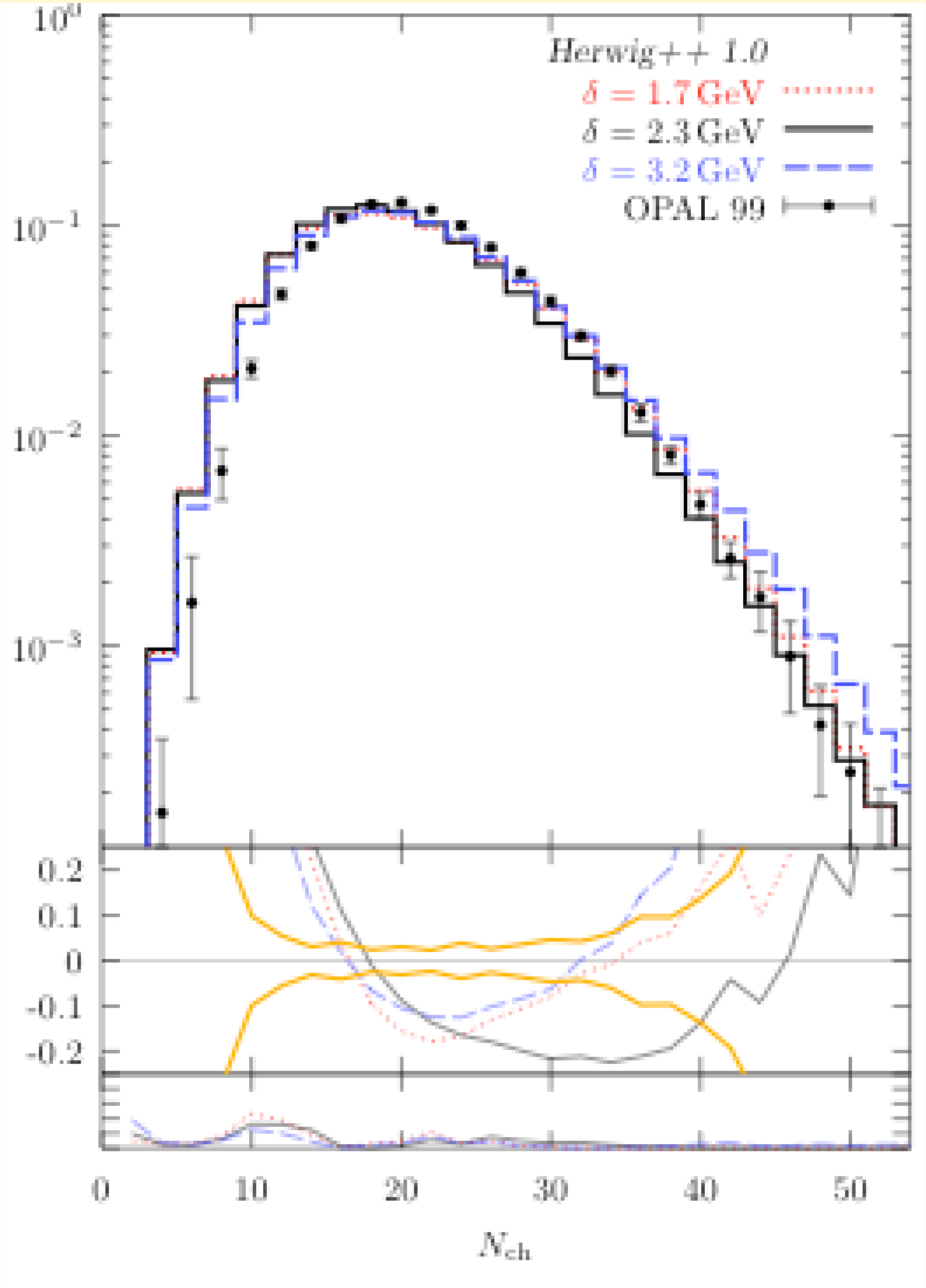


Excellent agreement with data,
provided N_c=3
Extraction of αs consistent with the Q evolution predicted by QCD

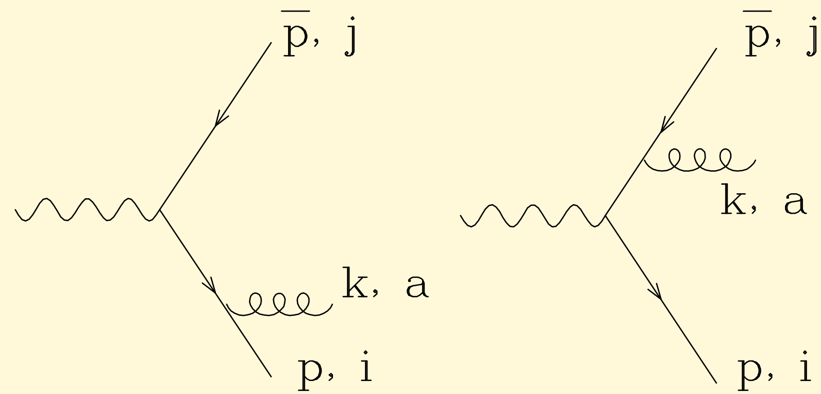
Experimentally, the final states contain a large number of particles, not the 2 or 3 which apparently saturate the perturbative cross-section.

Experimental multiplicity distribution

$\langle n_{charged} \rangle = 20.9$



Soft gluon emission



$$A = \bar{u}(p)\epsilon(k)(ig)\frac{-i}{\not{p}+\not{k}}\Gamma^\mu v(\bar{p})\ \lambda^a_{ij} + \bar{u}(p)\Gamma^\mu \frac{i}{\not{\bar{p}}+\not{k}}(ig)\epsilon(k)v(\bar{p})\ \lambda^a_{ij}$$

$$= \left[\frac{g}{2p\cdot k}\bar{u}(p)\epsilon(k)(\not{p}+\not{k})\Gamma^\mu v(\bar{p}) - \frac{g}{2\bar{p}\cdot k}\bar{u}(p)\Gamma^\mu(\not{\bar{p}}+\not{k})\epsilon(k)v(\bar{p})\right]\lambda^a_{ij}$$

$p\cdot k = p_0\, k_0\, (1-\cos\theta) \Rightarrow$ singularities for collinear ($\cos\theta \to 1$) or soft ($k_0 \to 0$) emission

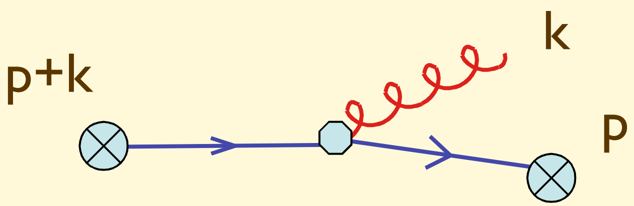
Collinear emission does not alter the global structure of the final state, since its preserves its "pencil-like-ness". **Soft emission** at large angle, however, could spoil the structure, and leads to strong interferences between emissions from different legs. So soft emission needs to be studied in more detail.

In the soft ($k_0 \to 0$) limit the amplitude simplifies and factorizes as follows:

$$A_{soft} = g\lambda^a_{ij}\left(\frac{p\cdot\varepsilon}{p\cdot k} - \frac{\bar{p}\cdot\varepsilon}{\bar{p}\cdot k}\right) A_{Born}$$

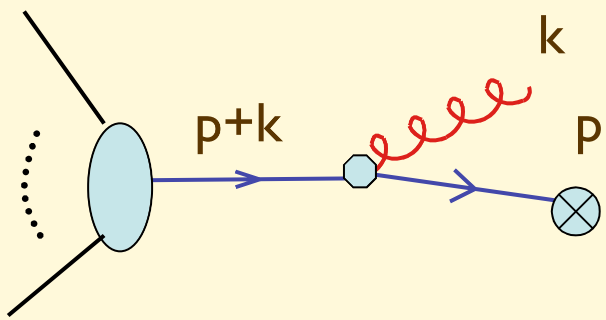
Factorization: it is the expression of the independence of long-wavelength (soft) emission on the nature of the hard (short-distance) process.

Another simple derivation of soft-gluon emission rules



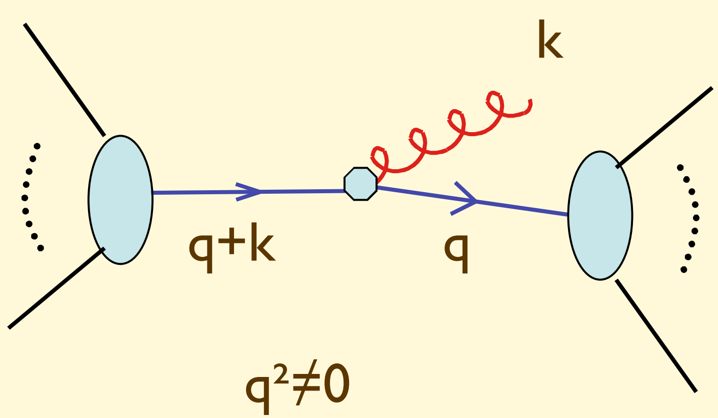
charge current of a free fermion

$$\bar{\psi}(p)\,\gamma_\mu \psi(p+k)\,\varepsilon^\mu(k) \quad \stackrel{k\to 0}{\longrightarrow} \quad \bar{\psi}(p)\,\gamma_\mu \psi(p)\varepsilon^\mu(k) \;=\; 2p\cdot\varepsilon$$



$$\frac{1}{\not{p}+\not{k}}\gamma_\mu \psi(p)\,\varepsilon^\mu(k) \quad \stackrel{k\to 0}{\longrightarrow}$$

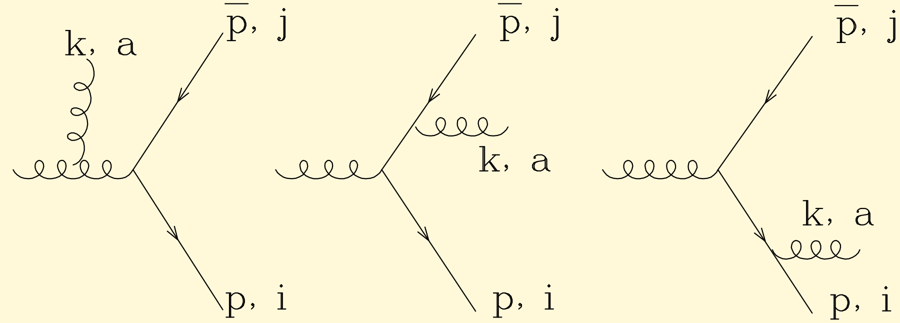
$$\frac{1}{2p\cdot k}\not{p}\gamma_\mu \psi(p)\varepsilon^\mu(k) \;=\; \frac{p\cdot\varepsilon}{p\cdot k}$$



$$\frac{1}{\not{q}+\not{k}}\gamma_\mu \frac{1}{\not{q}}\,\varepsilon^\mu(k) \quad \stackrel{q^2\neq 0,\, k\to 0}{\longrightarrow} \quad \frac{1}{q^2}\not{q}\gamma_\mu \not{q}\frac{1}{q^2}\varepsilon^\mu(k)$$

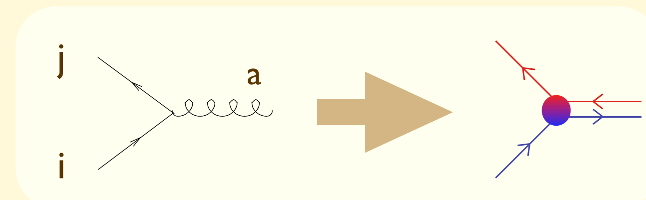
=> finite

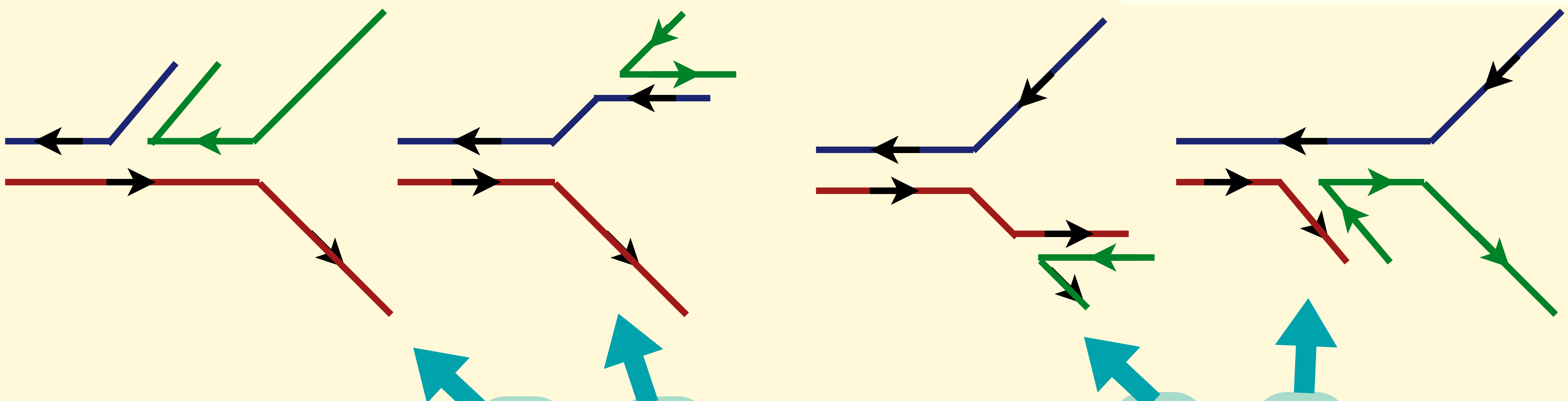
Similar, but more structured, result in the case of a fully coloured process:



$$A_{soft} = g\,(\lambda^a\lambda^b)_{ij}\left[\frac{Q\varepsilon}{Qk} - \frac{\bar{p}\varepsilon}{\bar{p}k}\right] + g\,(\lambda^b\lambda^a)_{ij}\left[\frac{p\varepsilon}{pk} - \frac{Q\varepsilon}{Qk}\right]$$

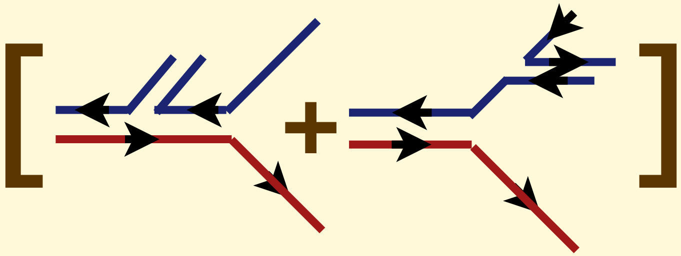
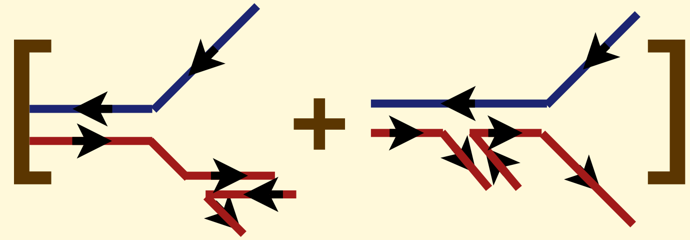
The four terms correspond to the two possible ways colour can flow, and to the two possible emissions for each colour flow:





$$A_{soft} = g\,(\lambda^a\lambda^b)_{ij}\left[\frac{Q\varepsilon}{Qk} - \frac{\bar{p}\varepsilon}{\bar{p}k}\right] + g\,(\lambda^b\lambda^a)_{ij}\left[\frac{p\varepsilon}{pk} - \frac{Q\varepsilon}{Qk}\right]$$

The interference between the two colour structures

 $\propto (\lambda^a\lambda^b)_{ij}$  $\propto (\lambda^b\lambda^a)_{ij}$

is suppressed by $1/N_c^2$:

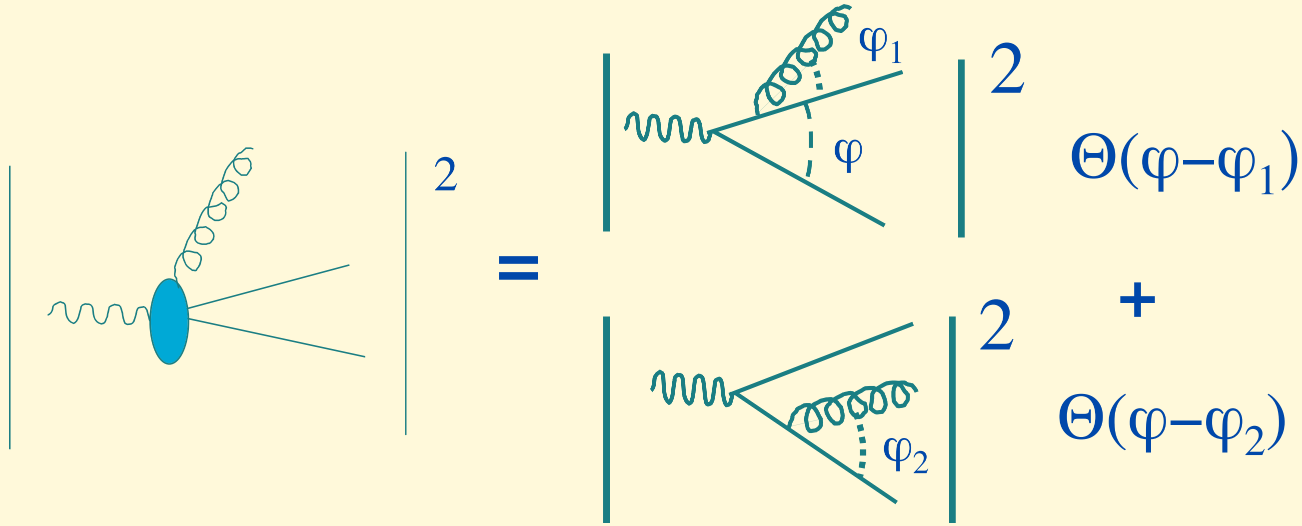
$$\sum_{a,b,i,j} |(\lambda^a\lambda^b)_{ij}|^2 = \sum_{a,b} tr\ (\lambda^a\lambda^b\lambda^b\lambda^a) = \frac{N^2-1}{2} C_F = O(N^3)$$

$$\sum_{a,b,i,j} (\lambda^a\lambda^b)_{ij}[(\lambda^b\lambda^a)_{ij}]^* = \sum_{a,b} tr(\lambda^a\lambda^b\lambda^a\lambda^b) = \frac{N^2-1}{2} \underbrace{(C_F - \frac{C_A}{2})}_{-\frac{1}{2N}} = O(N)$$

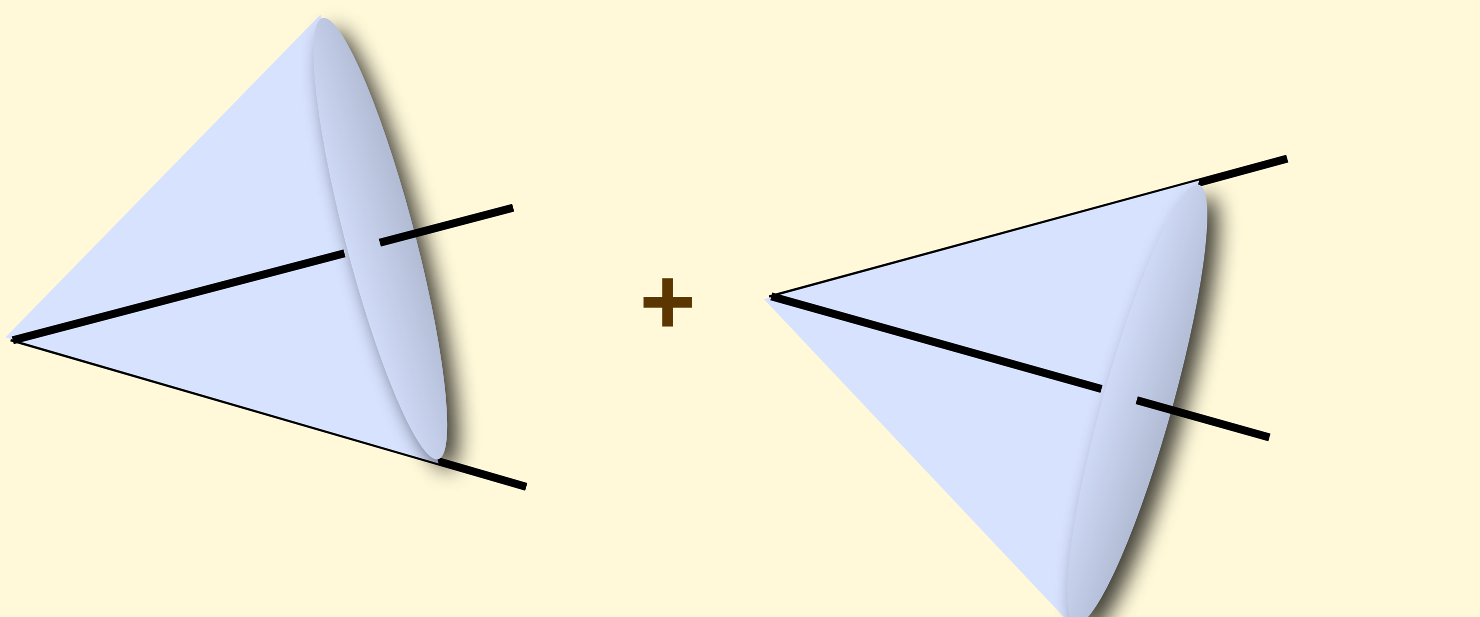
As a result, the emission of a soft gluon can be described, to the leading order in $1/N_c^2$, as the incoherent sum of the emission from the two colour currents

What about the interference between the two diagrams corresponding to the same colour flow? ➥

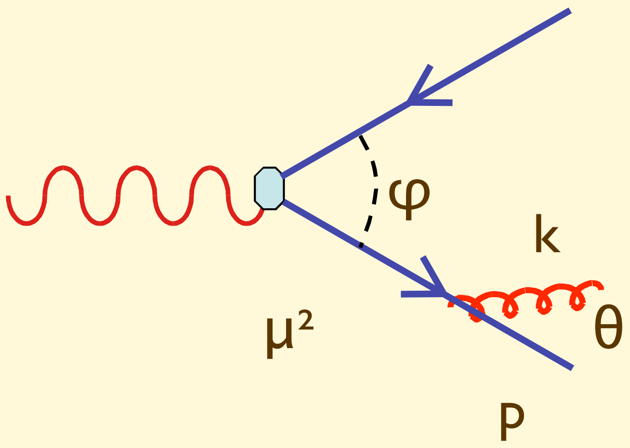
Angular ordering



Radiation inside the cones is allowed, and described by the eikonal probability, radiation outside the cones is suppressed and averages to 0 when integrated over the full azimuth



An intuitive explanation of angular ordering



Lifetime of the virtual intermediate state:

$\tau < \gamma/\mu = E/\mu^2 = 1 / (k_0\theta^2) = 1/(k_\perp\theta)$

$\mu^2 = (p+k)^2 = 2E\, k_0\, (1-\cos\theta)$
$\sim E\, k_0\, \theta^2 \sim E\, k_\perp\, \theta$

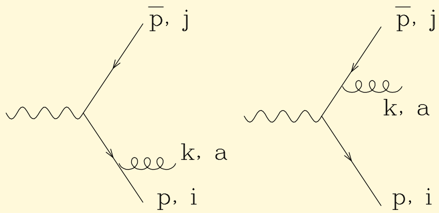
Distance between q and qbar after τ:

$d = \varphi\tau = (\varphi/\theta)\ 1/k_\perp$

If the transverse wavelength of the emitted gluon is longer than the separation between q and qbar, the gluon emission is suppressed, because the q qbar system will appear as colour neutral (=> dipole-like emission, suppressed)

Therefore $d > 1/k_\perp$, which implies $\theta < \varphi$

The formal proof of angular ordering



$$d\sigma_g = \sum |A_{soft}|^2 \frac{d^3k}{(2\pi)^3 2k^0} \sum |A_0|^2 \frac{-2p^\mu \bar{p}^\nu}{(pk)(\bar{p}k)} g^2 \sum \varepsilon_\mu \varepsilon_\nu^* \frac{d^3k}{(2\pi)^3 2k^0}$$

$$= d\sigma_0 \frac{\alpha_s C_F}{\pi} \frac{dk^0}{k^0} \frac{d\phi}{2\pi} \frac{1-\cos\theta_{ij}}{(1-\cos\theta_{ik})(1-\cos\theta_{jk})} d\cos\theta$$

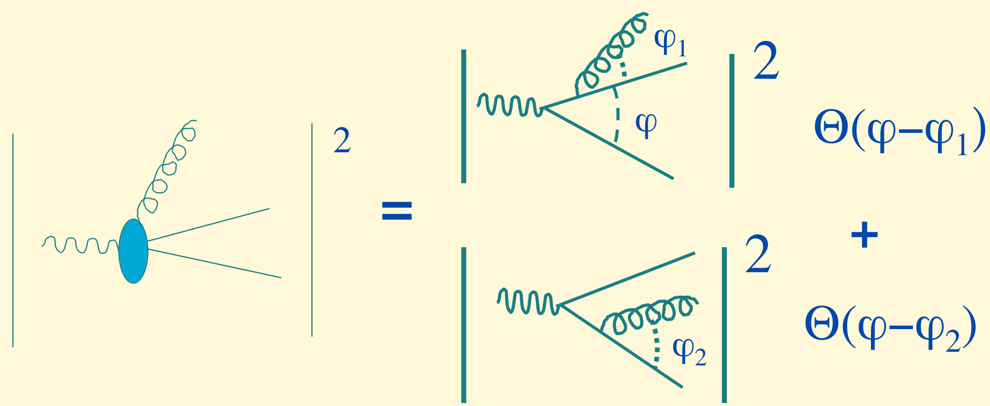
You can easily prove that:

$$\frac{1-\cos\theta_{ij}}{(1-\cos\theta_{ik})(1-\cos\theta_{jk})} = \frac{1}{2}\left[\frac{\cos\theta_{jk}-\cos\theta_{ij}}{(1-\cos\theta_{ik})(1-\cos\theta_{jk})} + \frac{1}{1-\cos\theta_{ik}}\right] + \frac{1}{2}[i \leftrightarrow j] \equiv W_{(i)} + W_{(j)}$$

where:

$$W_{(i)} \rightarrow finite\ if\ k \parallel j\ (\cos\theta_{jk} \rightarrow 1)$$
$$W_{(j)} \rightarrow finite\ if\ k \parallel i\ (\cos\theta_{ik} \rightarrow 1)$$

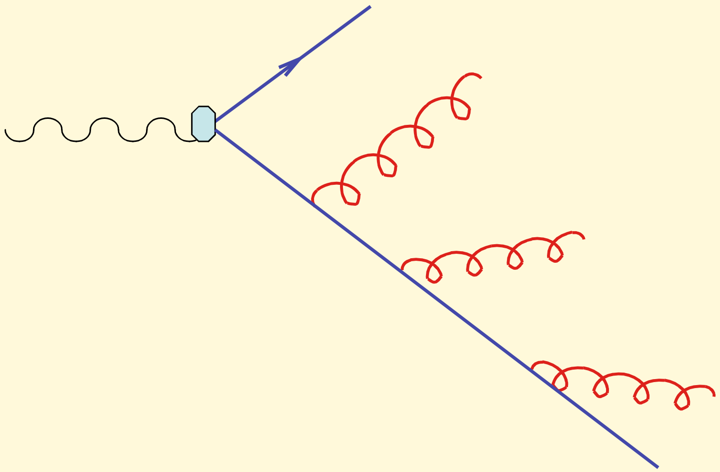
The probabilistic interpretation of W(i) and W(j) is a priori spoiled by their non-positivity. However, you can prove that after azimuthal averaging:



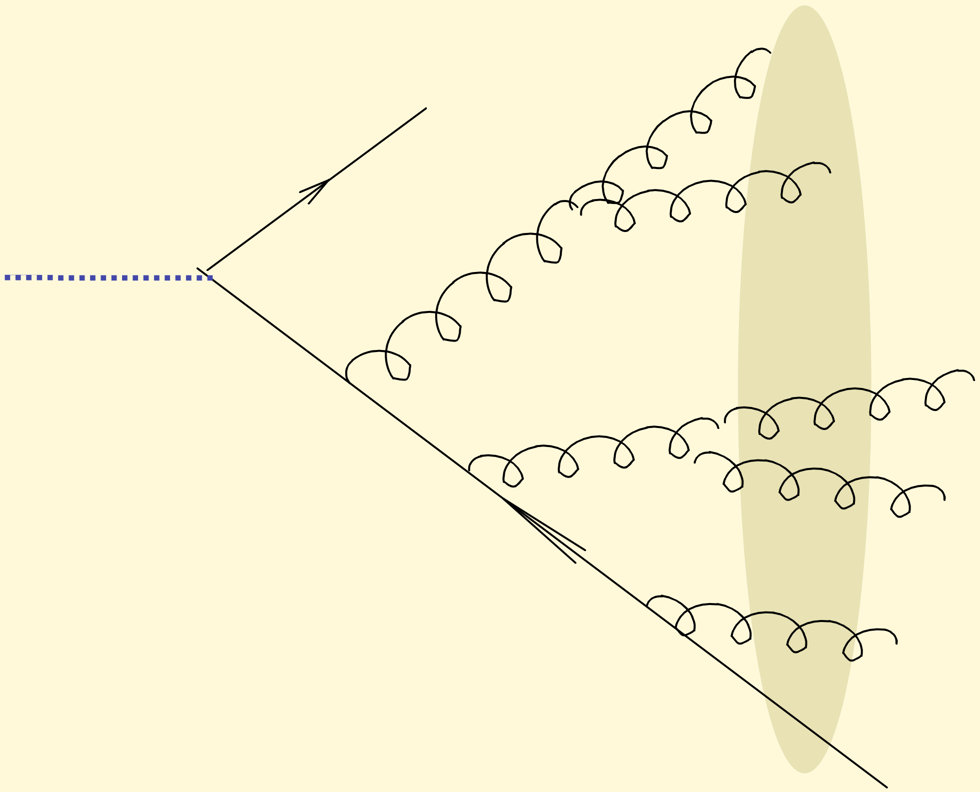
$$\int \frac{d\phi}{2\pi} W_{(i)} = \frac{1}{1-\cos\theta_{ik}} \text{ if } \theta_{ik} < \theta_{ij}\,, \quad 0 \text{ otherwise}$$

$$\int \frac{d\phi}{2\pi} W_{(j)} = \frac{1}{1-\cos\theta_{jk}} \text{ if } \theta_{jk} < \theta_{ij}\,, \quad 0 \text{ otherwise}$$

Further branchings will obey angular ordering relative to the new angles. As a result emission angles get smaller and smaller, squeezing the jet

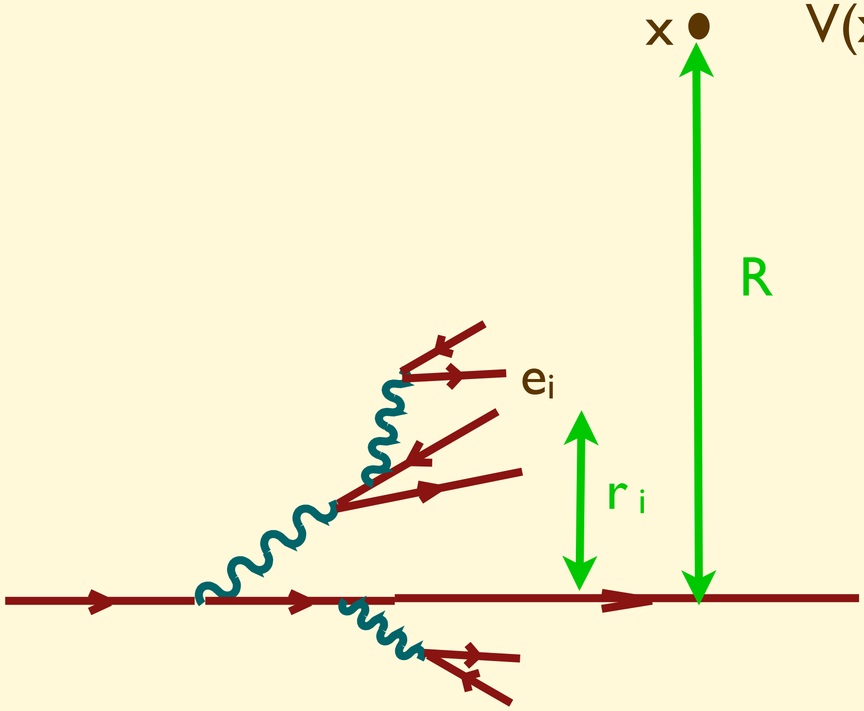


The construction can be iterated to the next emission, with the result that emission angles keep getting smaller and smaller => **jet structure**

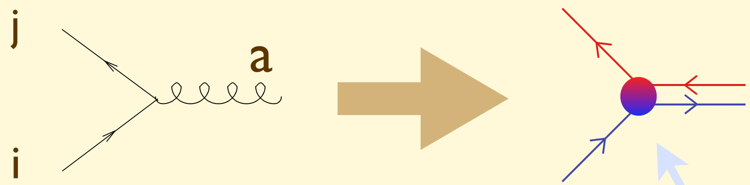


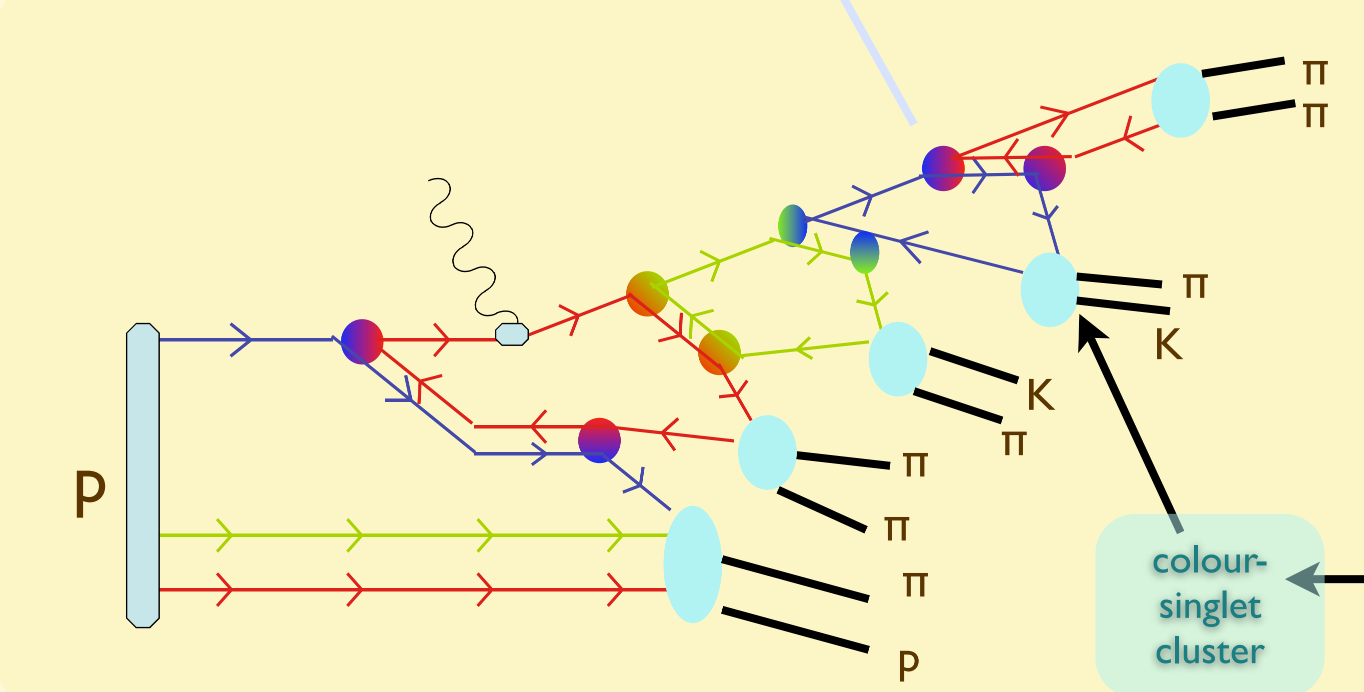
Total colour charge of the system is equal to the quark colour charge. Treating the system as the incoherent superposition of N gluons would lead to artificial growth of gluon multiplicity. Angular ordering enforces coherence, and leads to the proper evolution with energy of particle multiplicities.

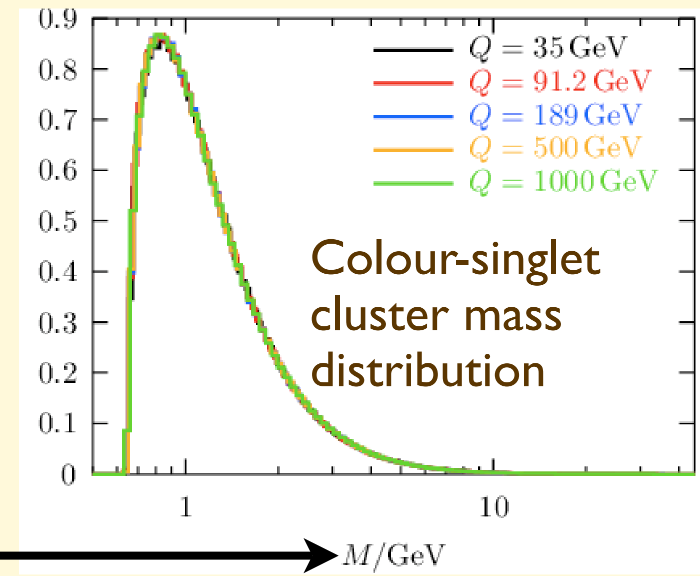
$$V(x) = \frac{e}{R} + \frac{1}{R^2} \left| \sum e_i \mathbf{r}_i \right|$$

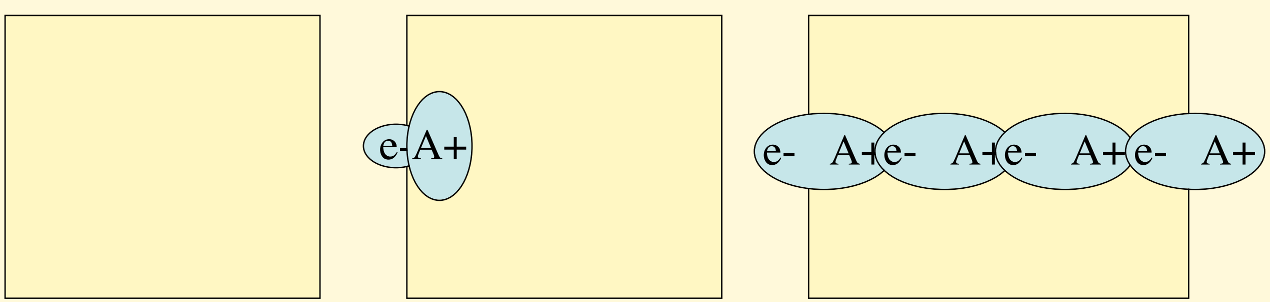


The structure of the perturbative evolution leads naturally to the clustering in phase-space of colour-singlet parton pairs (”preconfinement”). Long-range correlations are strongly suppressed. Hadronization will only act locally, on low-mass colour-singlet clusters.

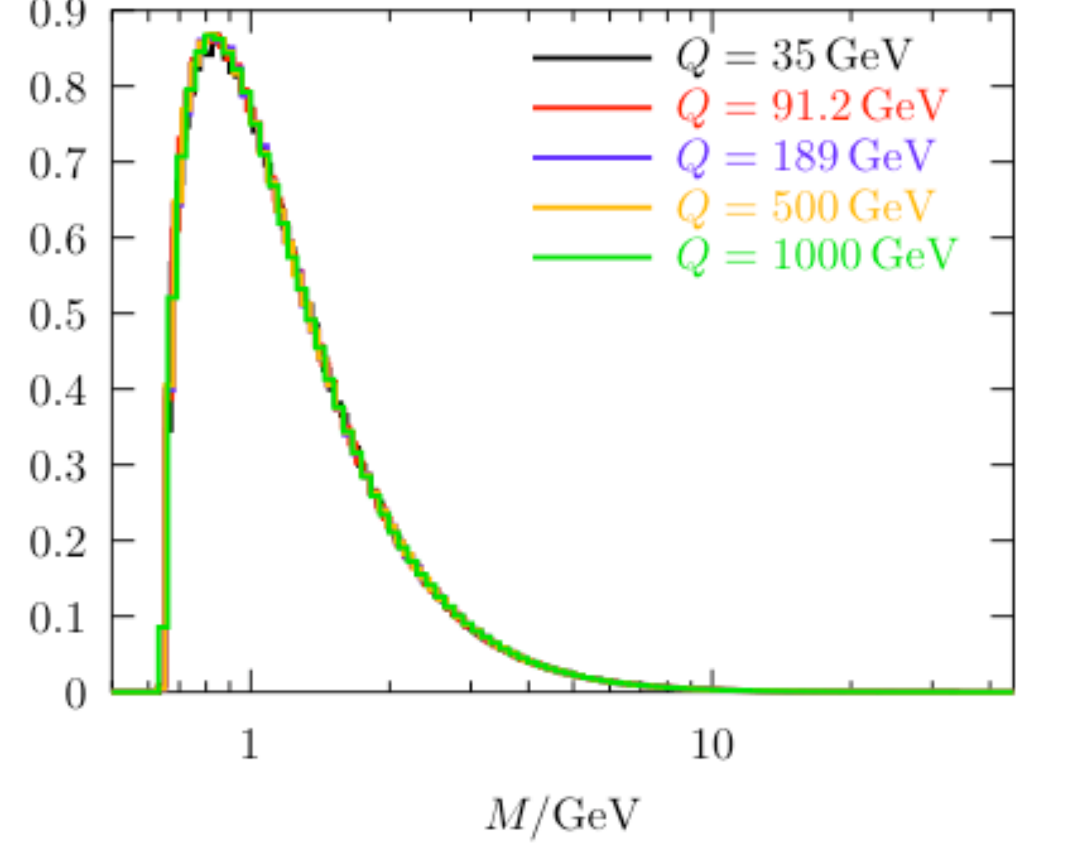








Colour is left “behind” by the struck quark. The first soft gluon emitted at large angle will connect to the beam fragments, ensuring that the beam fragments can recombine to form hadrons, and will allow the struck quark to evolve without having to worry about what happens to the proton fragments.

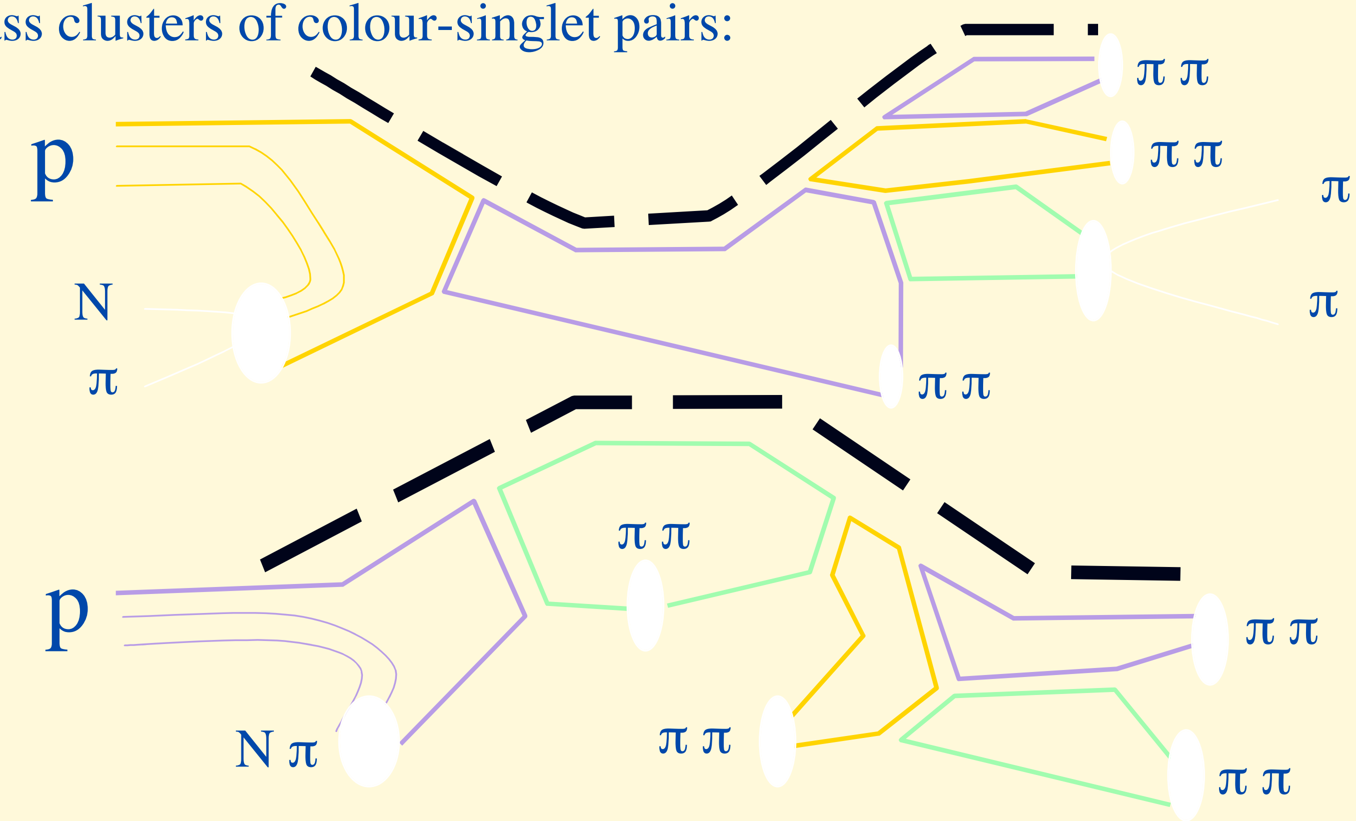


The existence of high-mass clusters, however rare, is unavoidable, due to IR cutoff which leads to a non-zero probability that no emission takes place. This is particularly true for evolution of massive quarks (as in, e.g. Z→bb or cc). Prescriptions have to be defined to deal with the "evolution" of these clusters. **This has an impact on the z→1 behaviour of fragmentation functions.**

Phenomenologically, this leads to uncertainties, for example, in the background rates for H→γγ (jet→γ).

Hadronization

At the end of the perturbative evolution, the final state consists of quarks and gluons, forming, as a result of angular-ordering, low-mass clusters of colour-singlet pairs:



Thanks to the cluster pre-confinement, hadronization is local and independent of the nature of the primary hard process, as well as of the details of how hadronization acts on different clusters. Among other things, one therefore expects:

N(pions) = C N(gluons),
C=constant~2