Dynamical Particle Mass Generation

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Outline of the talk

- Prologue & Motivations
- Summary
- Mass Generation in LQCD
- The quark mass term in Ward-Takahashi Identities
  - Perturbation Theory
  - Beyond Perturbation Theory
- Outlook
- Conclusions

Prologue & Motivations

Prologue & Motivations

What if the Higgs is not found/confirmed?

Summary and ... Conclusions - 1

Voilà an unconventional scenario for mass generation

- Elementary fermion masses can be (are?) dynamically generated by the mechanism of spontaneous chiral breaking
- The latter is triggered by explicit renormalizable χ-breaking terms (like the Wilson term in Lattice QCD) living at the UV cut-off, Λ_{UV}
- The overall mass scale is provided by the RGI scale, Λ_T, of the strongest (vector gauge) interaction exhibiting SχSB

- Actually visible features in Lattice QCD!

Summary and ... Conclusions - 2

Can we make use of this mechanism?

- Elementary fermion masses are “modulated" by powers of the coupling constants of the interactions they are subjected to
- A natural mass hierarchy emerges, as the stronger is the interaction a particle is subjected to, the larger is its mass
- To give the right mass to the “top quark" one must have $\Lambda_T \gg 300$ MeV $\to$ a new vector gauge interaction is required

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- It remains to discuss/see how to incorporate weak interactions

- The Wilson LQCD action

$$S^L_{QCD} = S^L_{YM} + S^L_F$$

$$S^L_F = a^4 \sum_x \bar{\psi}(x)\Big[\frac{1}{2}\sum_\mu \gamma_\mu(\nabla^\star_\mu + \nabla_\mu) - \frac{a}{2}\sum_\mu \nabla^\star_\mu \nabla_\mu + m_0\Big]\psi(x)$$

- m_{cr} is the value of m_0 at which chiral symmetry is restored

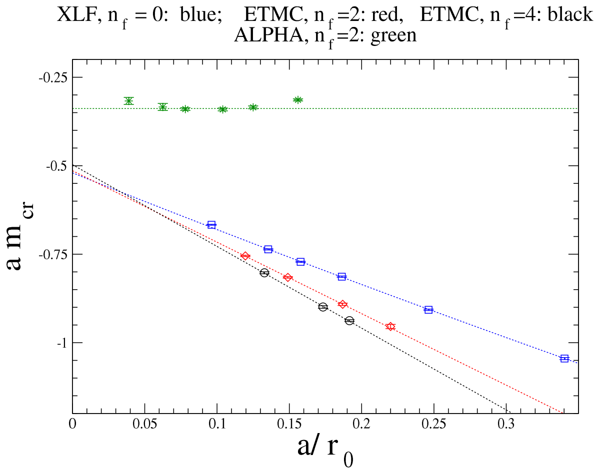
$$m_{cr} = \frac{c_0}{a} + c_1\Lambda_{QCD} + c_2 a\Lambda^2_{QCD} + \mathrm{O}(a^2)$$

- From actual simulation data (see figure), one finds

$$-c_1\Lambda_{QCD} \sim 800 \text{ MeV}$$

Lattice QCD - 2

$$am_{cr} = c_0 + c_1 a\Lambda_{QCD} + c_2 (a\Lambda_{QCD})^2 + O(a^3)\,, \qquad r_0^{-1} \sim \Lambda_{QCD}$$



Can one understand these features?

Chiral Ward–Takahashi Identites

- Lattice Ward–Takahashi Identity

$$\nabla_\mu \langle A^f_\mu(x)\hat{Q}(0)\rangle^L = \langle \Delta^f \hat{Q}(0)\rangle \delta(x) + 2m_0 \langle P^f(x)\hat{Q}(0)\rangle + \langle X^f(x)\hat{Q}(0)\rangle$$

- Renormalized Ward–Takahashi Identity

$$\nabla_\mu \langle \hat{A}^f_\mu(x)\hat{Q}(0)\rangle^L = \langle \Delta^f \hat{Q}(0)\rangle \delta(x) + 2(m_0 - \bar{M})\langle P^f(x)\hat{Q}(0)\rangle + \langle \bar{X}^f(x)\hat{Q}(0)\rangle$$

$\mathrm{O}(a)$

- $\Delta^f \hat{Q}$ → chiral variation of $\hat{Q}$
- $X^f = aO^f_5$ → chiral variation of the Wilson term in S^L_{QCD}
- $\hat{O}^f_5 = \left[O^f_5 + 2\frac{\bar{M}}{a}P^f + \frac{Z_A - 1}{a}\nabla_\mu A^f_\mu\right]$ → mult. renorm.
- $\bar{X}^f = a\hat{O}^f_5 = \mathrm{O}(a)$
- $\hat{A}^f_\mu = Z_A A^f_\mu$

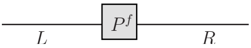
- What's $\bar{M}$ expression?
- How do we choose m_0?

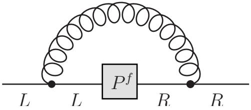
$\bar{M}$ in Perturbation Theory - 1

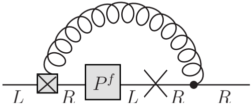
$\frac{\bar{M}}{a}$ is the quadratically divergent O_5^f - P^f mixing coefficient

- $\langle q_L(p) P^f(0) \bar{q}_R(p) \rangle \Big|_{1PI} =$
  $= \gamma_5 \tau^f \Big\{ \Big[1 + \delta_1 L_{\delta_1} g_s^2 + O(g_s^4) \Big] + a\, \epsilon_q \Big[\rho_1 L_{\rho_1} g_s^2 + O(g_s^4) \Big] + O(a^2) \Big\}$

- $\times$ = mass insertion, we set $m_0 = \epsilon_q$
- crossed box = Wilson vertex, aV_5, from S^L_{QCD}
- L's are logarithmic terms

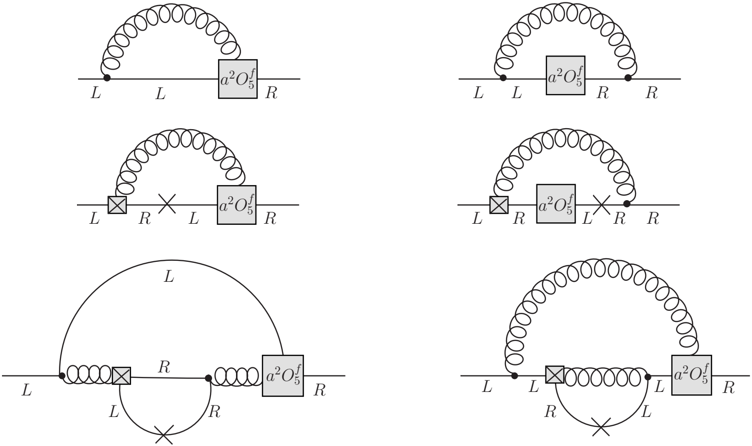






$\bar{M}$ in Perturbation Theory - 2

- $\langle q_L(p) a^2 O_5^f(0) \bar{q}_R(p) \rangle \Big|_{1PI} = \gamma_5 \tau^f \Big\{ a^2 p^2 \Big[1 + \zeta_1 L_{\zeta_1} g_s^2 + O(g_s^4) \Big] +$
  $+ \Big[\eta_1 g_s^2 + \eta_2 L_{\eta_2} g_s^4 + O(g_s^6) \Big] + a\,\epsilon_q \Big[\xi_1 g_s^2 + \xi_2 L_{\xi_2} g_s^4 + O(g_s^6) \Big] + O(a^2) \Big\}$



We impose O_5^f multiplicative renormalizability

- Setting

$$2a\bar{M} = [h_1 g_s^2 + h_2 g_s^4 + \mathrm{O}(g_s^6)] + a\,\epsilon_q [d_1 g_s^2 + d_2 g_s^4 + \mathrm{O}(g_s^6)]$$

- and imposing

$$\lim_{a\to 0} \langle q_L(p) \Big[O_5^f(0) + 2\frac{\bar{M}}{a} P^f(0) \Big] \bar{q}_R(p) \rangle \Big|^L = \text{finite}$$

- one can fix the $\bar{M}$-coefficients h's and d's

$$h_1 + \eta_1 = 0 \qquad d_1 + \xi_1 = 0$$
$$h_2 + h_1 \delta_1 L_{\delta_1} + \eta_2 L_{\eta_2} = 0 \qquad h_1 \rho_1 L_{\rho_1} + d_1 \delta_1 L_{\delta_1} + d_2 + \xi_2 L_{\xi_2} = 0$$

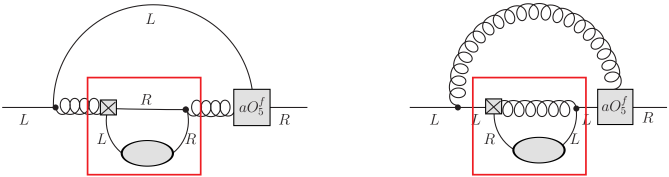
- getting (coefficients η's, δ's, ξ's are supposedly known)

$$2a\bar{M} = [-\eta_1 g_s^2 + (-\eta_2 L_{\eta_2} + \eta_1 \delta_1 L_{\delta_1}) g_s^4 + \mathrm{O}(g_s^6)] +$$
$$+ a\,\epsilon_q [-\xi_1 g_s^2 + (-\xi_2 L_{\xi_2} + \eta_1 \rho_1 L_{\rho_1} + \xi_1 \delta_1 L_{\delta_1}) g_s^4 + \mathrm{O}(g_s^6)] + \mathrm{O}(a^2)$$

Beyond Perturbation Theory

Here comes the crucial conjecture

- At $O(g_s^4)$ NP-soft contributions interfere with UV divergencies
  - effects coming from the dynamical breaking of χ-symmetry
  - triggered by the Wilson term in the UV regularized action
  - give in all $\langle q_L a O_5^f \bar{q}_R \rangle_{1PI} \to O(\frac{1}{a^2}\, a\, g_s^4\, a\, \Lambda_{QCD}) = O(g_s^4\, \Lambda_{QCD})$

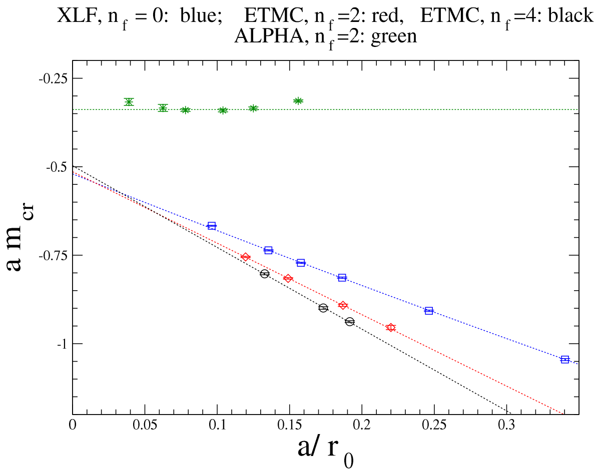


- One then gets (at $\epsilon_q = 0$)

$$2a\bar{M} = [-\eta_1 g_s^2 + (-\eta_2 L_{\eta_2} + \eta_1 \delta_1 L_{\delta_1}) g_s^4 + O(g_s^6)] +$$
$$+ a\Lambda_{QCD}[-\xi_2^{NP} g_s^4 + O(g_s^6)] + O(a^2\Lambda_{QCD}^2) \equiv$$
$$\equiv c_0 + a\, c_1 \Lambda_{QCD} + O(a^2\Lambda_{QCD}^2)$$

Recall

$$am_{cr} = c_0 + c_1 a\Lambda_{QCD} + c_2(a\Lambda_{QCD})^2 + O(a^3)\,, \qquad r_0^{-1} \sim \Lambda_{QCD}$$



Two possible m_0 choices

1. The lattice choice $\to$ for having an adjustable quark mass ϵ_q (the theory is renormalized in the broken phase)

$$m_0 = \epsilon_q + \bar{M} = \epsilon_q + \frac{c_0}{a} + c_1 \Lambda_{QCD} + \mathrm{O}(a)$$

$$\nabla_\mu \langle \hat{A}^f_\mu(x) \hat{Q}(0) \rangle^L = \langle \Delta^f \hat{Q}(0) \rangle \delta(x) + 2\epsilon_q \langle P^f(x) \hat{Q}(0) \rangle + \mathrm{O}(a)$$

- $\epsilon_q \neq 0 \to$ A theory of quarks with mass ϵ_q

2. The minimal choice $\to$ for a UV-finite theory it's enough to take (the theory is renormalized in the unbroken phase)

$$m_0 = \epsilon_q + \frac{c_0}{a}$$

$$\nabla_\mu \langle \hat{A}^f_\mu(x) \hat{Q}(0) \rangle^L = \langle \Delta^f \hat{Q}(0) \rangle \delta(x) + 2(\epsilon_q - c_1 \Lambda_{QCD}) \langle P^f(x) \hat{Q}(0) \rangle + \mathrm{O}(a)$$

- $\epsilon_q = 0 \to$ A theory of quarks with mass $-c_1 \Lambda_{QCD}$ ($c_1 < 0$)

A model with two coupled vector gauge interactions

- Assume an extra family of super-strongly interacting fermions, Q, exists that is also subjected to ordinary Yang-Mills interactions
- Arguments similar to the ones developed for pure QCD yield now the interesting result

$$m_q = \mathrm{O}(\alpha_s^2)\,\Lambda_T$$
$$m_Q = \mathrm{O}(\alpha_T^2)\,\Lambda_T$$

  - α_s is the strong coupling constant
  - α_T is the super-strong coupling constant
  - $\Lambda_T \gg \Lambda_{QCD}$ is the super-strong RGI scale
- A mass hierarchy has emerged!
- Reasonable top quark mass for $\Lambda_T \sim 1.5$ TeV

Outlook

- Question

Can we make use of the mechanism
of dynamical mass generation
when chiral gauge interactions of the weak type
are introduced?

- Answer

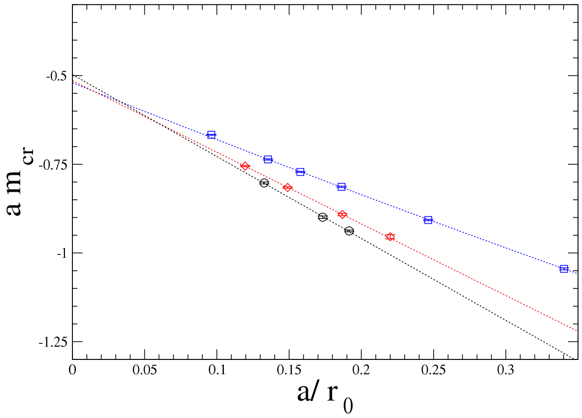
Yes, we think ... we can!

Conclusions

Happy Birthday, Pino!!

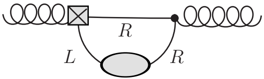
Recovery Slides - 1

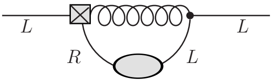
XLF, $n_f = 0$: blue; ETMC, $n_f = 2$: red, ETMC, $n_f = 4$: black



Recovery Slides - 2

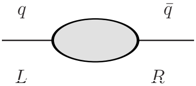
- The NP contribution to $\langle q_L a O_5^f \bar{q}_R \rangle_{1\text{PI}}$ comes from the subdiagram





$\alpha_s(\Lambda_{QCD})\, a\, \Lambda_{QCD}\, (k^2\delta_{\mu\nu} - k_\mu k_\nu)$ $\qquad$ $\alpha_s(\Lambda_{QCD})\, a\, \Lambda_{QCD}\, \not{k}$

- L-R transition is modeled assuming the formula (Banks-Casher)



$$\rightarrow \frac{\pi \rho_{BC}(0)}{\Lambda_{QCD}^2} \frac{\theta(-q^2 + \Lambda_{QCD}^2)}{q^2 + \Lambda_{QCD}^2} \sim \Lambda_{QCD} \frac{\theta(-q^2 + \Lambda_{QCD}^2)}{q^2 + \Lambda_{QCD}^2}$$

- q_L interacting with the $\bar{q}\, q$ pairs, filling the vacuum, becomes $\bar{q}_R$
- for the process to occur another χ-flip is needed, provided by aV_5
- O($a\Lambda_{QCD}$) terms “resurrected" by PT power divergences

Recovery Slides - 3

- Constructing “chiral" gauge invariant Wilson-like terms requires introducing scalar fields
- It is like we're having a scalar field with a v.e.v. = $O(\Lambda_{UV})$
- After renormalizing the theory in the unbroken phase, masses are dynamically generated
  - $\rightarrow$ no “unnaturalness" problem
- In the broken phase (i.e. including dynamical chiral symmetry breaking effects) the effective LE theory ($E \ll \Lambda_T$) looks like the SM with two differences
  - particle masses $\propto \Lambda_T$ are generated
  - no Higgs, only non-linearly interacting Goldstones's
- Full quark and lepton hierarchy, as well as family and isospin splitting, from an appropriate set of Wilson-like terms of increasing dimensions
- Unification of EW and strong couplings at $\Lambda_{GUT} \sim 10^{17}$ GeV

This model vs. Technicolor

- Elementary particle content is similar to Technicolor
- All fermion & weak boson masses are dynamically generated
- Wilson-like terms occur in the basic action, living at the UV-cutoff
  - invariant under a basic chiral invariance of the gauge type
  - induce O(Λ_T) scale invariance breaking terms coming from O(Λ_T/Λ_{UV}) effects enhanced by O(Λ_{UV}^{-1}) divergences
- Wilson-like terms of larger and larger dimensions associated to lighter and lighter fermions (no FCNC problem)
- EW and strong coupling unification at $\Lambda_{GUT} \sim 10^{17}$ GeV
- T-hadrons are expected with $M_T \propto \Lambda_T \sim$ a few TeV's
- In $W_L\, W_L \to W_L\, W_L$, T-mesons replace SM Higgs