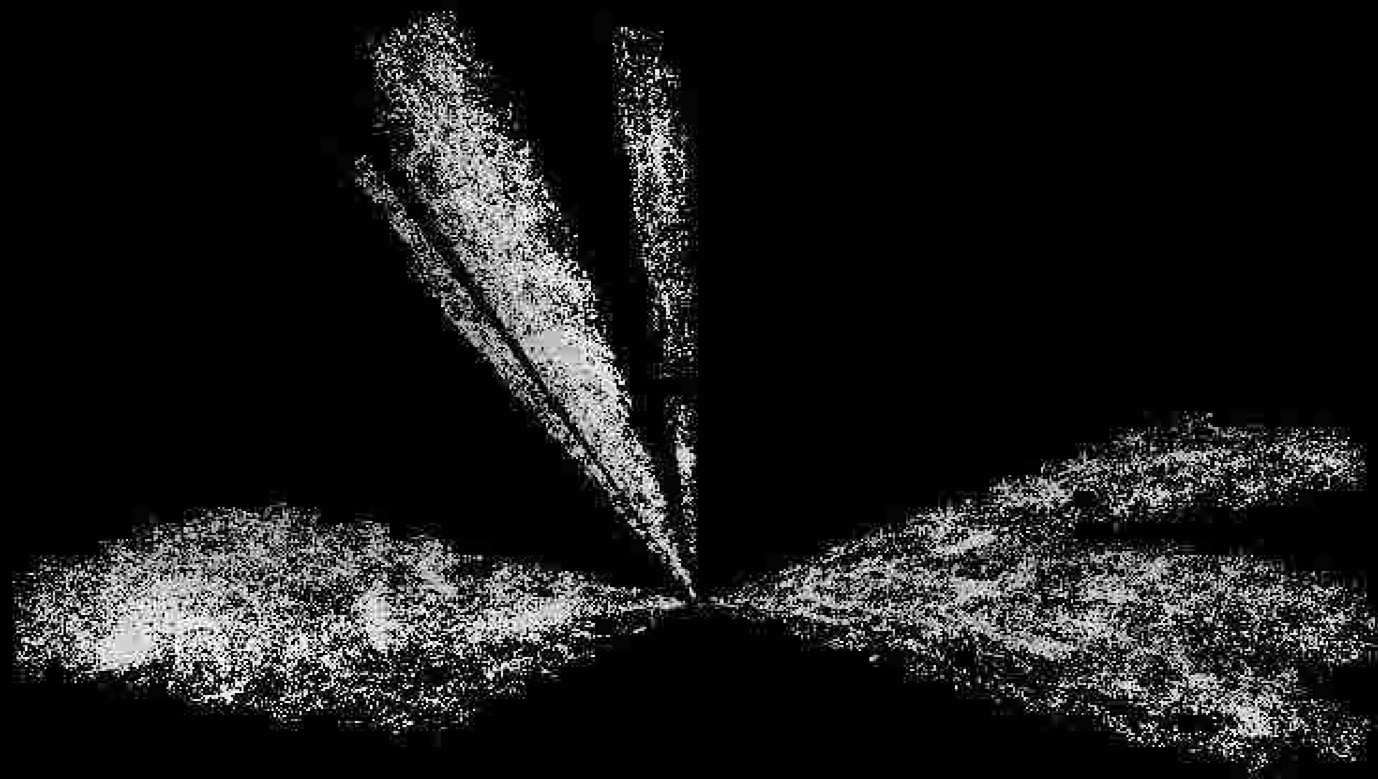
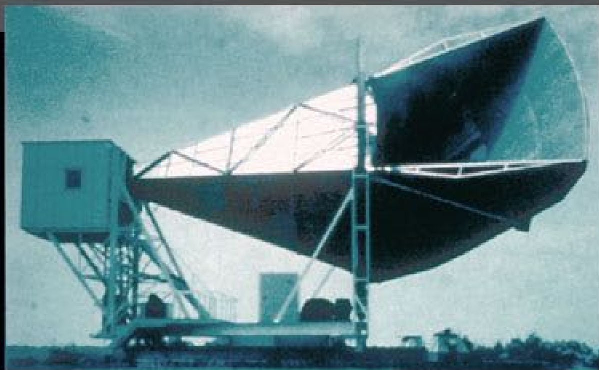
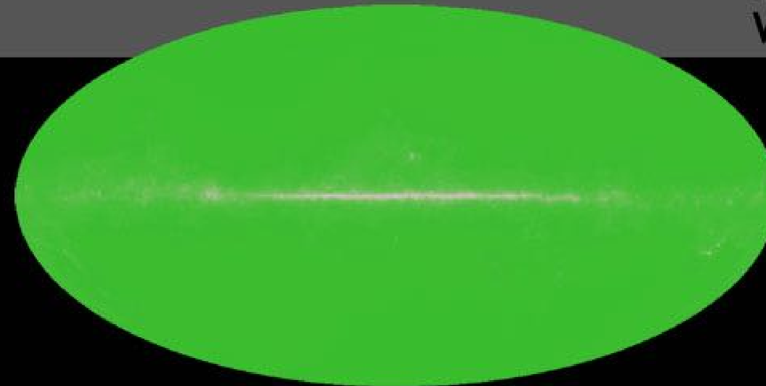


1965

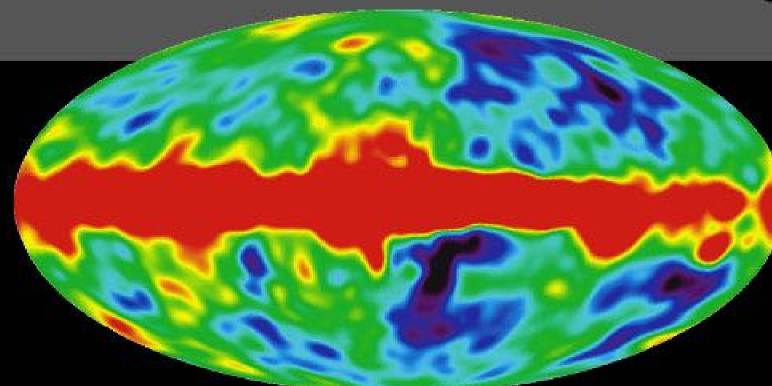
Penzias and Wilson





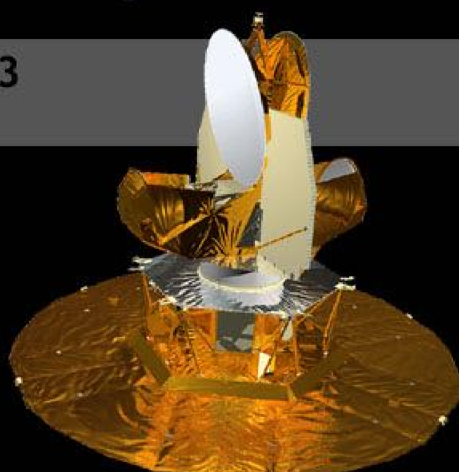
1992

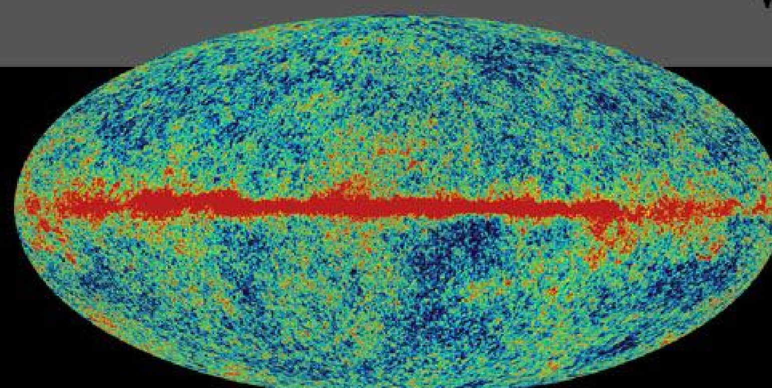
COBE



2003

WMAP






t=-13.42 Gyr
z=50.00

INFLATION:
Conjectures vs. Facts

V. Mukhanov

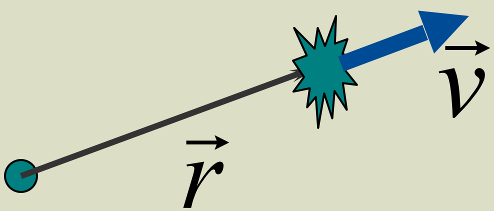
Sektion Physik, LMU, München

Expanding Universe: Facts

- Isotropy of Background Radiation

$$\frac{\delta\varepsilon}{\varepsilon} \le 10^{-5} \quad \text{in big scales up to} \quad ct \approx 10^{28}\, cm$$

- Hubble law



$$\vec{v} = H(t)\vec{r} = \dot{a}\vec{\chi}_{com}$$

rate of expansion

$$\vec{r} = a(t)\vec{\chi}_{com}$$

$$H = \dot{a}/a$$

- Age of the Universe

$$t_0 \approx \frac{r}{v} = \frac{1}{H_0} \approx 10^{17}\sec \quad \text{for} \quad H_0 \cong 50-80\ km/\sec Mpc$$

Initial Conditions

- ## Matter Distribution

The size l_i should be compared with the size of causally connected region $ct_i \approx 10^{-33} cm$

$$\frac{l_i}{ct_i} \approx \frac{\dot{a}_i}{\dot{a}_0}$$

initial rate of expansion

current rate of expansion

In 10^{90} causally disconnected regions $\delta\varepsilon / \varepsilon \leq 10^{-5}$!!!

Homogeneity, isotropy problem

Initial velocities

- Today $(t_0 \approx 10^{17}\ \mathrm{sec})$:

$$\left|E_0^{kin} + E_0^{pot}\right| \sim \left|E_0^{kin}\right| \sim \left|E_0^{pot}\right|$$

- At "initial moment" $(t_i \approx 10^{-43}\ \mathrm{sec})$

$$\boxed{\frac{\left|E_i^{kin} + E_i^{pot}\right|}{E_i^{kin}}} \le \ldots \left(\frac{\dot{a}_0}{\dot{a}_i}\right)^2 \boxed{\le \ldots 10^{-60}}$$

Flatness (≡ initial velocities) problem

Initial conditions were VERY SPECIAL (nongeneric)!?

- Root of the problem: Gravity is attractive force ⟶ $\frac{\dot{a}_i}{\dot{a}_0} \gg 1$
- *Conjecture* (Guth, 81): Gravity was REPULSIVE during some period of the Universe evolution ⟶ $\dot{a}_i / \dot{a}_0 \leq 1$? $\Rightarrow$ *no* problems?
- How gravity can become "repulsive"?

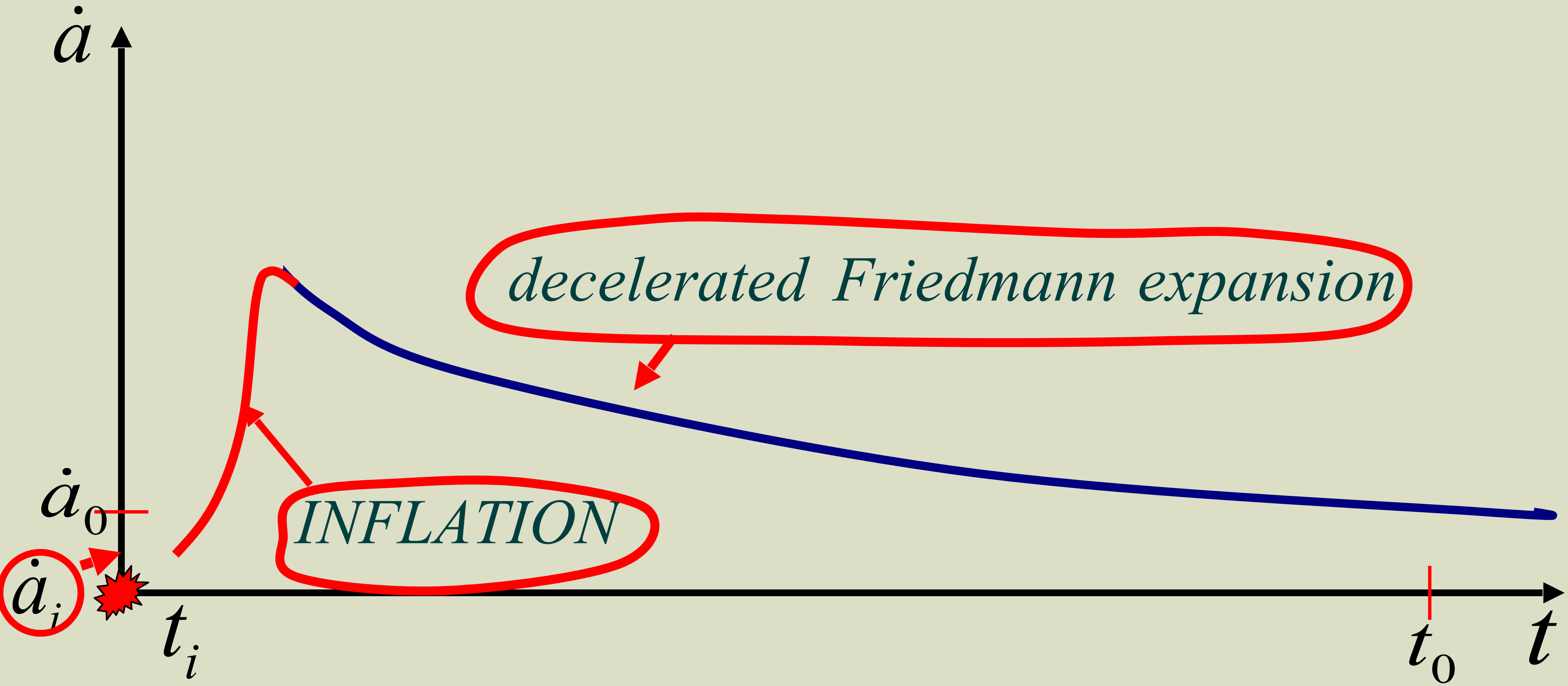
$$\ddot{a} = -\frac{4\pi G}{3}(\varepsilon + 3p)a$$

acceleration energy density pressure

If $\varepsilon + 3p > 0$ (energy dominance condition) $\Rightarrow$ ä<0

Only if $\varepsilon + 3p < 0$ $\Rightarrow$ $\ddot{a} > 0$ $\equiv$ "antigravity"

INFLATION is the stage of accelerated expansion of the Universe when the energy dominance condition is broken



Necessary conditions for successful inflation:

- $\dot{a}_i \ll \dot{a}_0 \Rightarrow \boxed{\Omega_0} \equiv \dfrac{|E_0^{pot}|}{E_0^{kin}} = \boxed{1}$ Prediction of inflation!

- Transition from inflation to Friedmann era should be "smooth"

Scenarios

$$S=\int\left(-\frac{R}{16\pi G}+\ldots R^2+\boxed{f\left(\frac{1}{2}\varphi_{,\alpha}\varphi^{,\alpha}\right)-V(\varphi)}+\ldots\right)\sqrt{-g}\,d^4x$$

Higher derivative
"new", chaotic,....
scenarios

k – inflation
New, chaotic...

"Old", "New"
Chaotic (Linde,83)
Extended
Natural, Supernatural
Hybrid, Power law
Fast oscillating,.......

Which scenario was realized in nature???

Quantum fluctuations and CMB:

V. Mukhanov

Sektion Physik, LMU, München

- Main bonus from inflation-generation of primordial spectrum of inhomogeneities (Mukhanov, Chibisov 1981)
- Inflation "washes away" all pre-inflationary inhomogeneities
  However, in all scales there always remain
  ineveitable quantum fluctuations

Example: Quantum metric fluctuations in Minkowskii space

$$h_\lambda \approx \frac{l_{Pl}}{\lambda} \approx \frac{10^{-33}cm}{\lambda}$$

Today in galactic scales $h \sim 10^{-58}$

Can quantum fluctuations be amplified up to "needed" value 10^{-5} in expandng Universe???

- $L = -\frac{1}{6}R + p\left(X, \varphi, \ldots\right), \quad X = \frac{1}{2} g^{\alpha\beta} \varphi_{,\alpha} \varphi_{,\beta}$

$$T^{\mu}{}_{\nu} = \left(\varepsilon + p\right) u^{\mu} u_{\nu} - p \delta^{\mu}{}_{\nu}$$

where

$$\varepsilon = 2 X p_{,X} - p$$

- If $2Xp_{,X} \ll p$ then $p \approx -\varepsilon$ $\Longrightarrow$ *Inflation*

Perturbations

$$\varphi(x,t) = \varphi_0(t) + \delta\varphi(x,t)$$

$$ds^2 = (1+2\Phi)dt^2 - a^2(t)((1-2\Phi)\delta_{ik} + h_{ik}^{TT})dx^i dx^k$$

gravitational potential

gravity waves

Gravitational waves

$$h_{ik}^{TT} = \frac{u}{a} e_{ik}$$

$$u'' - \Delta u - \frac{a''}{a} u = 0$$

where $(') = a\dfrac{\partial}{\partial t}$

Scalar perturbations

$$\delta\varphi + \sqrt{\frac{3}{2}\left(1 + \frac{p}{\varepsilon}\right)}\Phi = \frac{v}{a}$$

$$v'' - c_s^2 \Delta v - \frac{z''}{z} v = 0$$

where $z = a\left(1 + p/\varepsilon\right)^{1/2}$

$$c_s^2 = p_{,X}/\varepsilon_{,X}$$

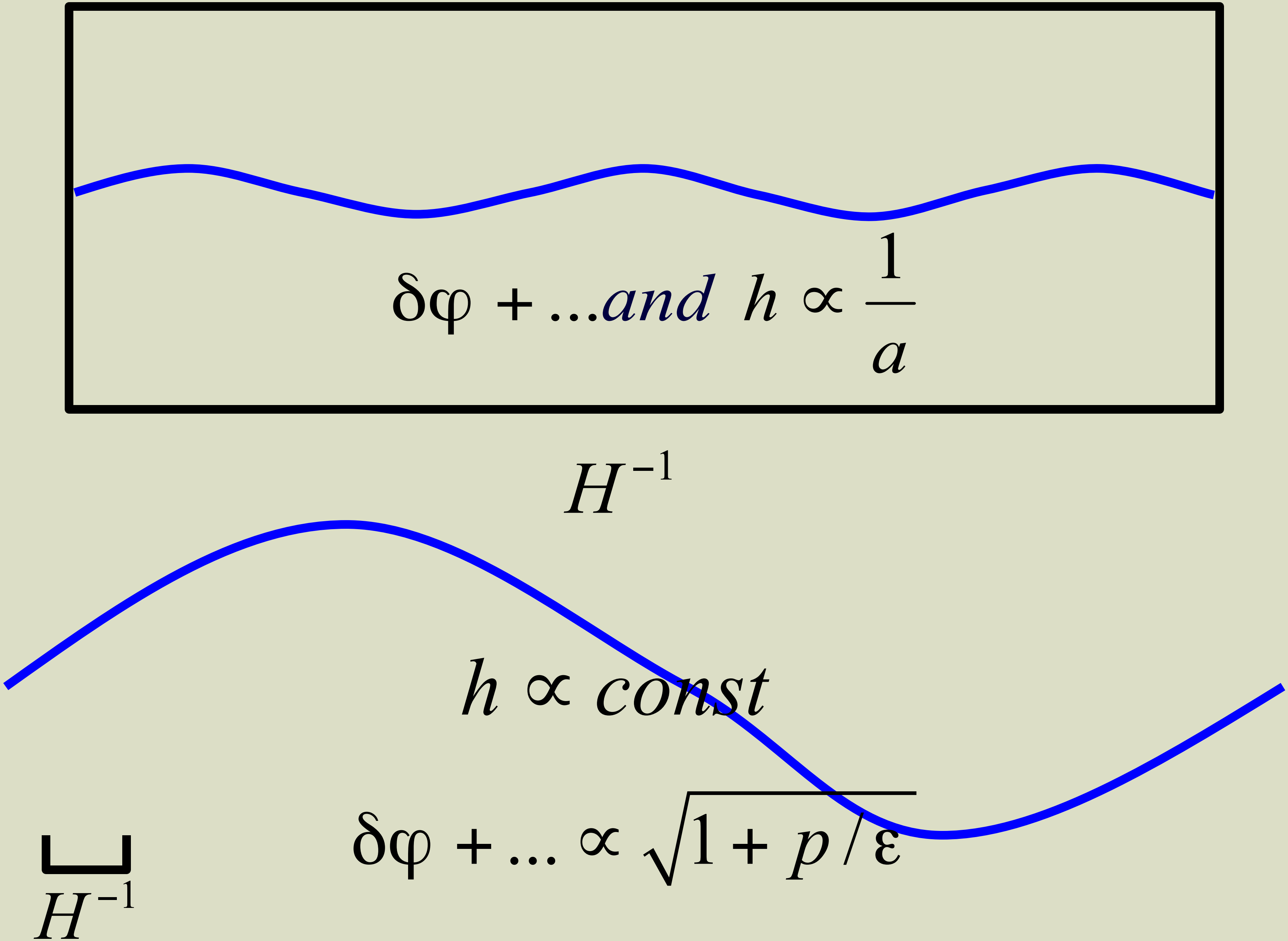
$$a''/a \approx z''/z \approx a^2 H^2$$

Quantum metric fluctuations are big enough (10^{-5}) only in the scales close to the Planckian scale ($10^{-33}cm$)

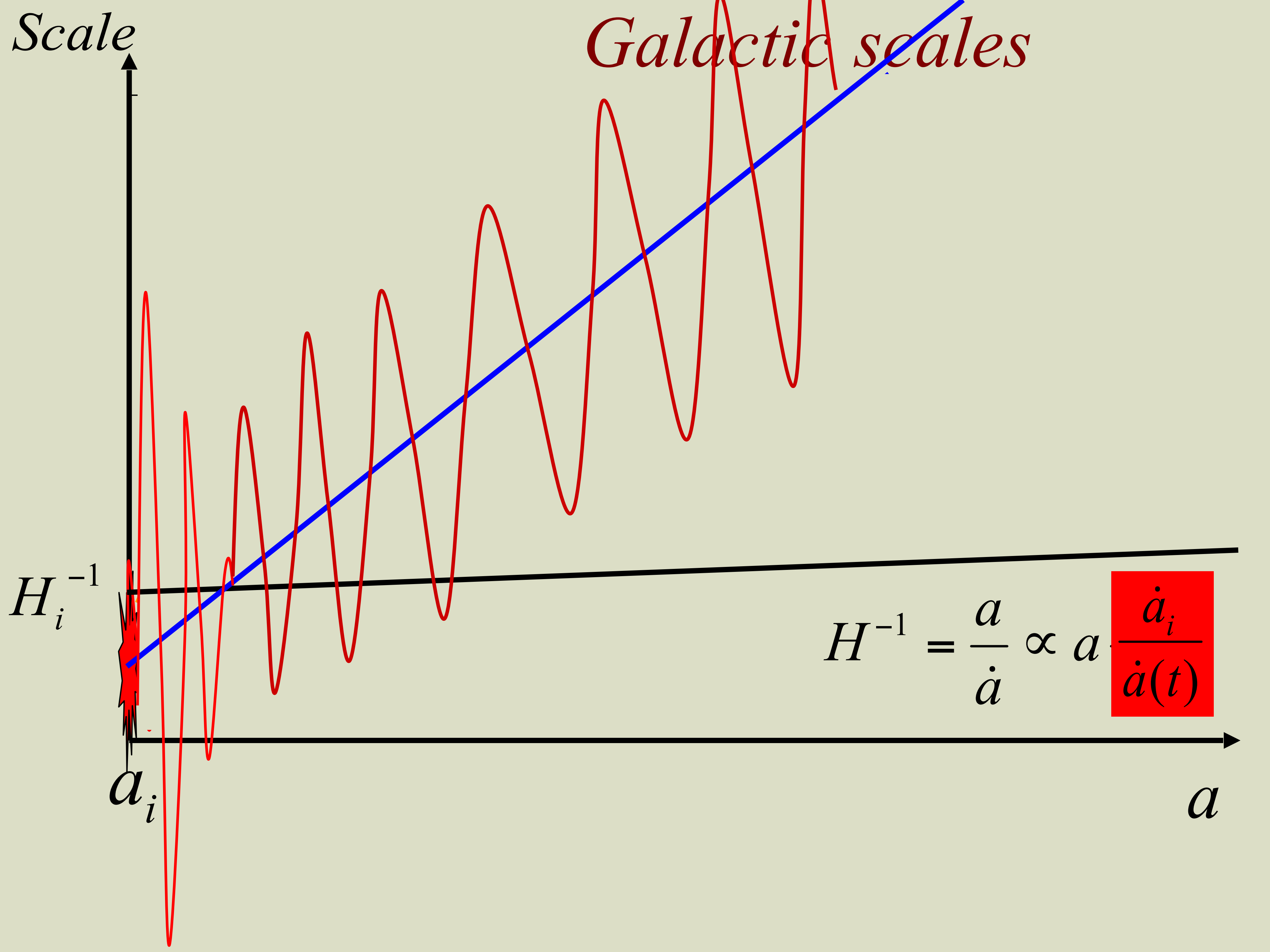
Purpose : Transfer these fluctuations to galactic scales ($10^{28}cm$)

- Consider plane wave perturbation: $\delta\varphi, \Phi \propto \exp\left(i\vec{k}_{com}\vec{x}\right)$

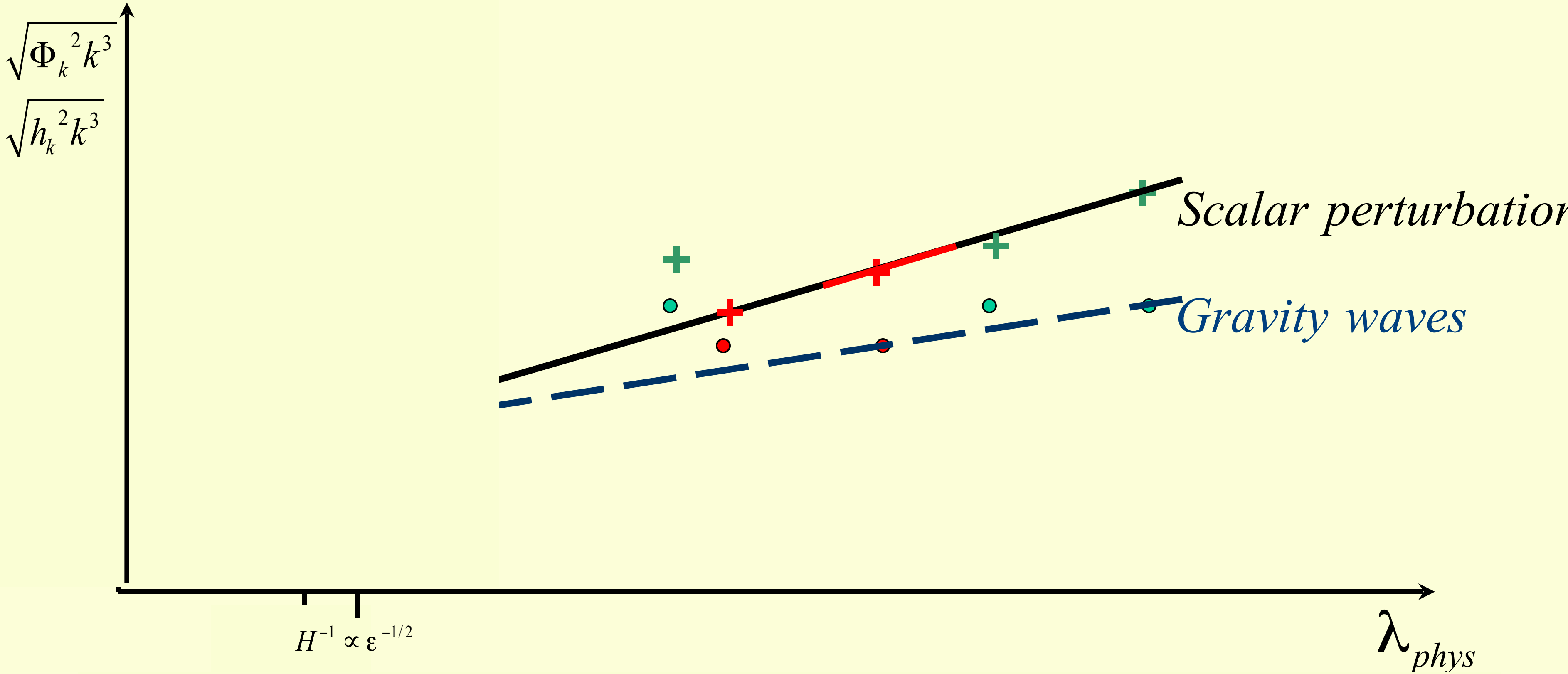
For given k_{com}, $\lambda_{ph}(cm) \propto a / k_{com} \propto a(t)$ and the change of the amplitude with time depends on how big is λ_{phys} compared to the curvature scale (size of Einstein lift) $H^{-1} = a / \dot{a}$



$\delta\varphi + \ldots and\ h \propto \frac{1}{a}$
H^{-1}
$h \propto const$
$\delta\varphi + \ldots \propto \sqrt{1 + p/\varepsilon}$
H^{-1}



Scale
Galactic scales
H_i^{-1}
$H^{-1} = \frac{a}{\dot{a}} \propto a \frac{\dot{a}_i}{\dot{a}(t)}$
a_i
a



$$0.92 < n_S < 0.97$$

$$n_T = -3\left(1 + \frac{p}{\varepsilon}\right)_{k\approx Ha} \qquad \frac{T}{S} = O(1)\, c_S^{\,1/2}\left(1 + \frac{p}{\varepsilon}\right)^{1/2}_{k\approx Ha}$$

Summary

Input from HEP

???

Idea and basic properties of inflation are established:

Inflation is the stage of accelerated expansion of the universe with graceful exit to Friedmann stage

SCENARIO ???

???

- Homogeneity, isotropy and flatness of the Universe mechanism for the origin of perturbations plus solutions of numerios "man made" problems (monopoles,...)
- Robust predictions:
  - $Spatially\ flat\ Universe:\ \Omega_{total} = 1 \pm 10^{-5}$
  - *Nearly* $scale-invariant\ spectrum\ (n_S \le 0{,}96)$

  Perturbations are Gaussian
  - *Gravity waves*

- Energy scale of inflation → prediction of the perturbations amplitude, concrete n_s
- Transition from inflation to Friedmann, reheating mechanism
- The origin of small number 10^{-5} characterizing perturbations

"What really interests me is whether God had any choice when he created the World"

A. Einstein

Inflation was inevitable!!! (it is unique opportunity to create World starting from generic initial conditions with minimal efforts)

Q & A

Q : Does inflation have a serious competitor?

A : No

PBB and *TD* vs. *INFLATION*

	Inflation	PBB	TD
Causality	+	+	n/a
"No-hair"	+	-	n/a
Graceful exit	+	-?	n/a
Perturbations	+	-?	-

Q : Could we get from observations something useful for "M-theory" ?

A : Unfortunately not too much...

Q : Can we get in inflationary models $\Omega_0 < 1$, large deviations from scale invarience, nongaussian perturbations?

A : In principle YES adding extra fine-tuned parameters in the model, BUT

$$\frac{\text{price}}{\text{perfomance}} > 10^{70}$$ compared to 1/3 for standard inflation

Q : Does inflation completely solve the problem of initial conditions?

A : ???

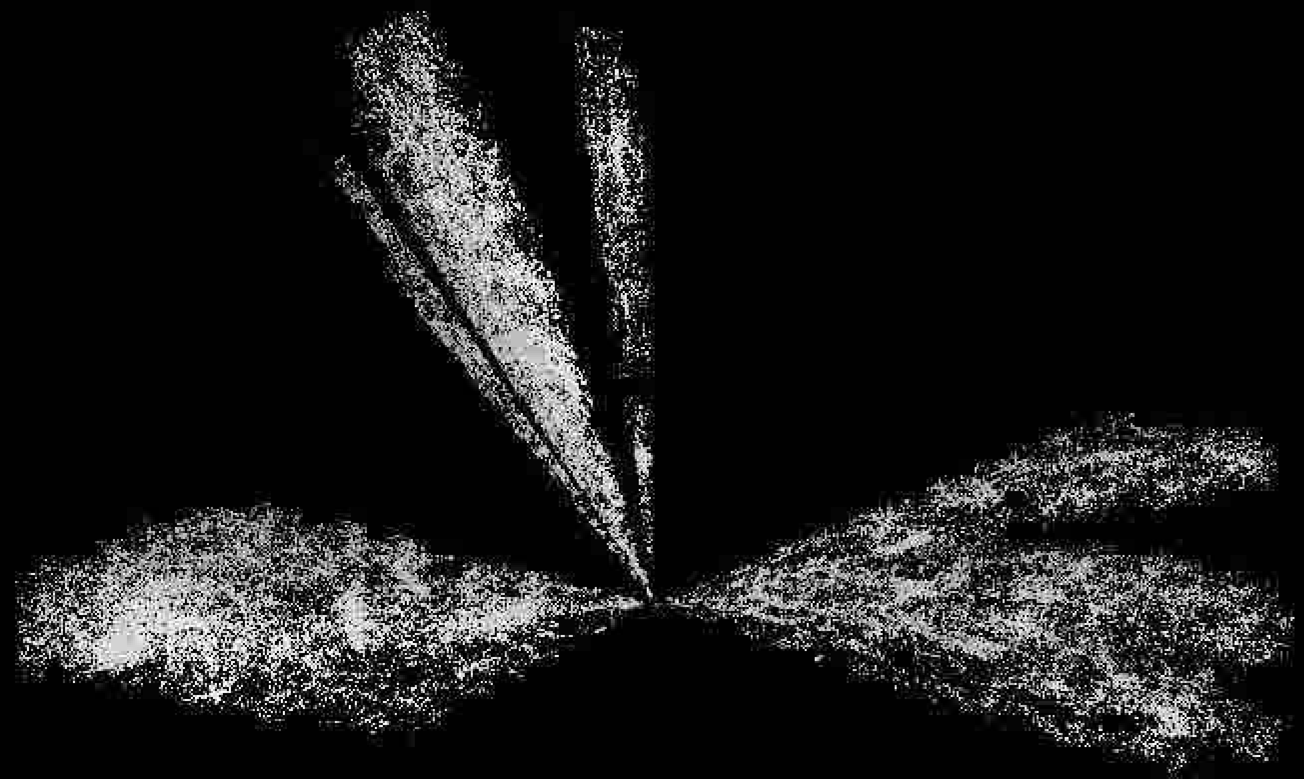
Q : What was before inflation?

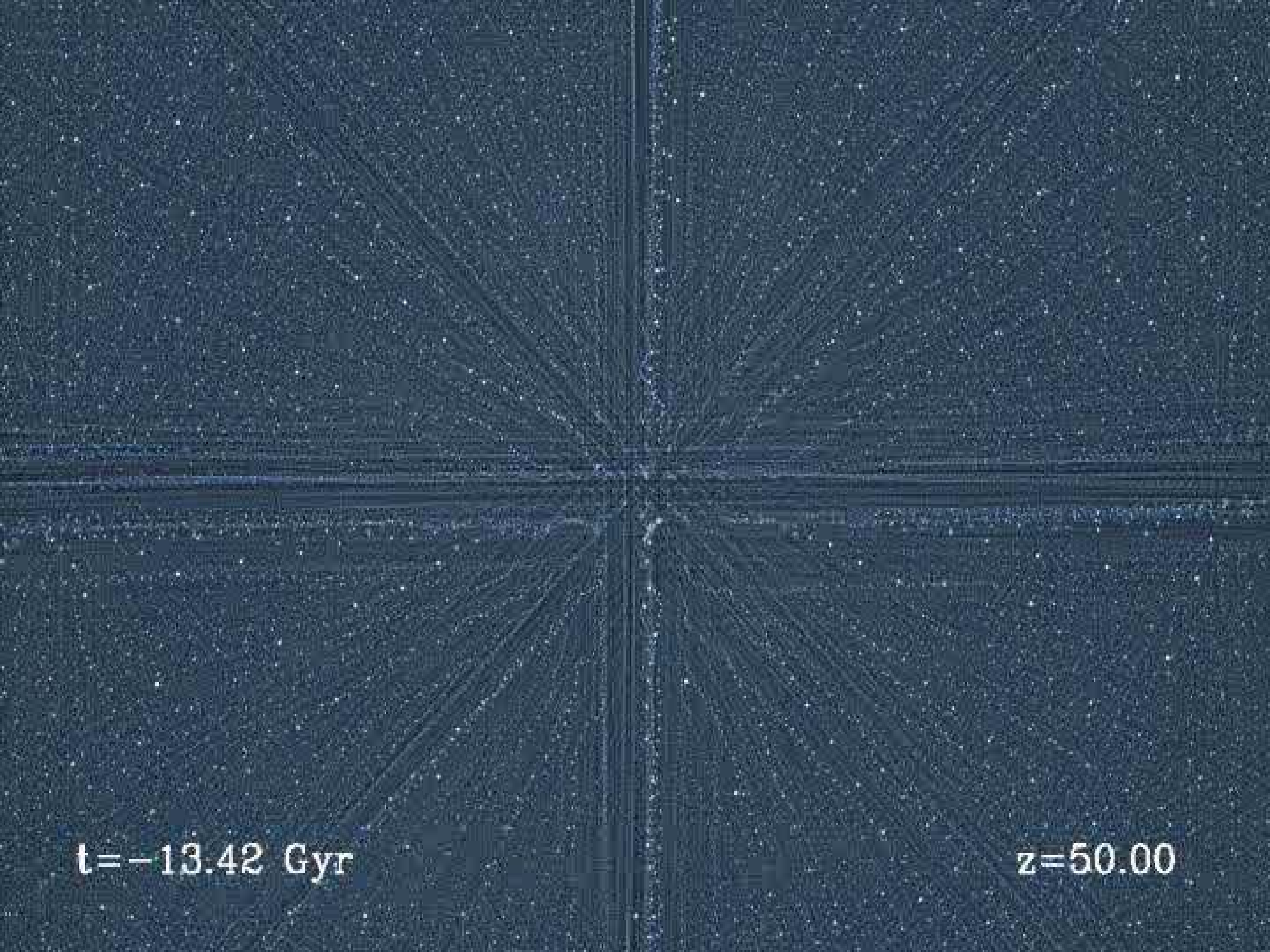
A : ???Instanton??? $\Leftrightarrow$ "The world and time had both one beginning. The world was made not in time, but simulteneously with time "

Saint Augustine of Hippo, 354-430

Q : Can preinflationary stage have observationally varifiable consequences?

A : Probably not, but who knows...?




t=-13.42 Gyr
z=50.00

CMB-slow:
How to estimate cosmological parameters by hand

V. Mukhanov

Sektion Physik, LMU, München

Parameters

- Metric

$$ds^2 = (1+2\Phi)dt^2 - a^2(t)((1-2\Phi)\delta_{ik} + h_{ik}^{TT})dx^i dx^k$$

gravitational potential

gravity waves

After inflation

$$\left|k^3\Phi_k^2\right| = Ak^{n_s-1}$$

$$\left|k^3 h_k^2\right| = Bk^{n_T}$$

where

$$n_s \approx 0.92 \ to \ 0.97$$

$$n_t \approx n_s - 1 + \ldots$$

- Matter

$$1 = \Omega_{tot}^0 = \Omega_\gamma^0 + \Omega_\nu^0 + \Omega_b^0 + \Omega_{CDM}^0 + \Omega_{\Lambda,Q}^0$$

Prediction of inflation!

- Main unknown parameters determining the spectrum

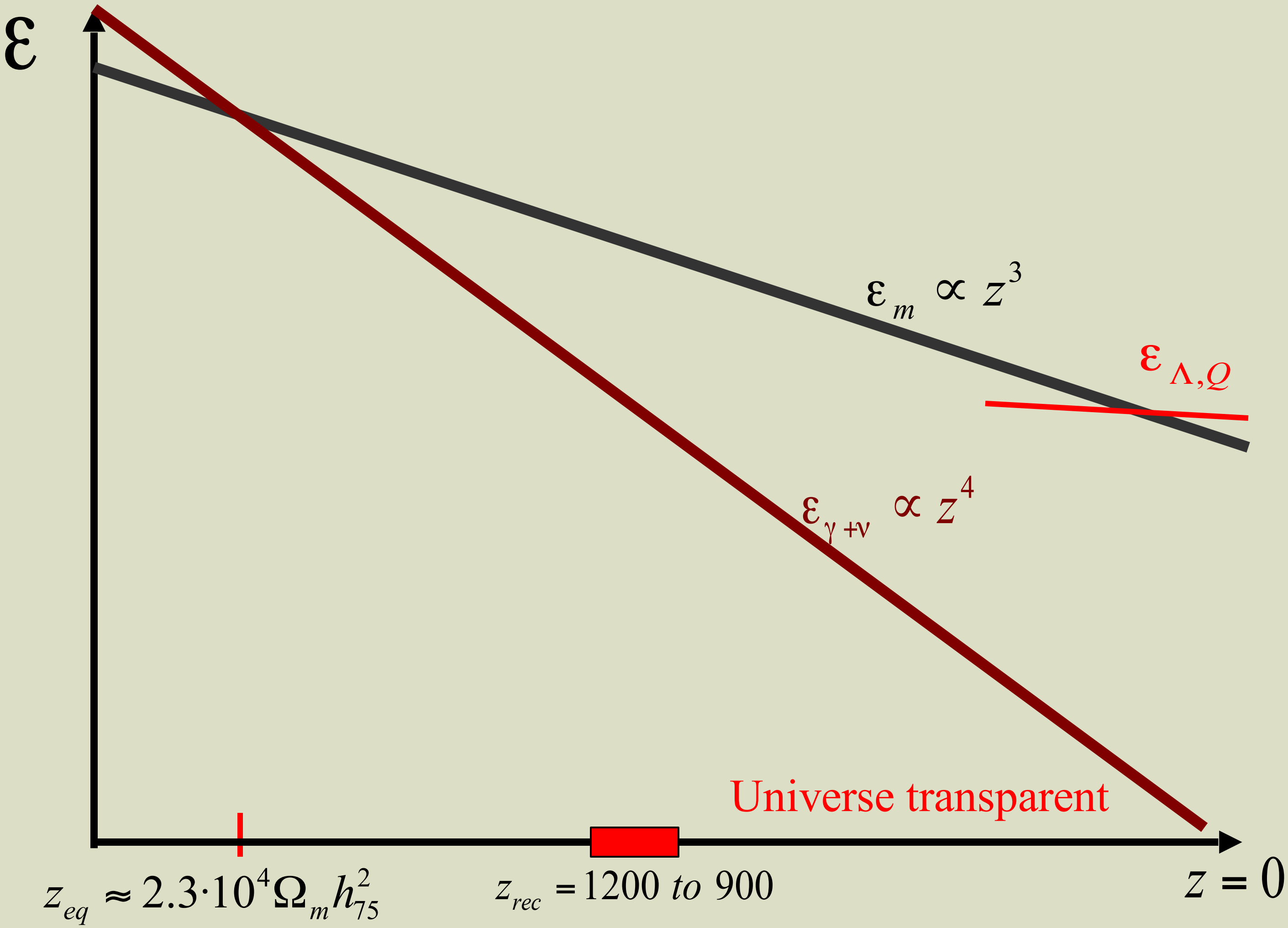
1) $h_{75} = \dfrac{H_0}{75\,\dfrac{km}{s \cdot Mpc}}$ 2) $\Omega_m^0 = \Omega_b^0 + \Omega_{CDM}^0$ 3) Ω_b^0

4) $\Omega_{\Lambda,Q}^0$ 5) amplitude A 6) spectral index n_s

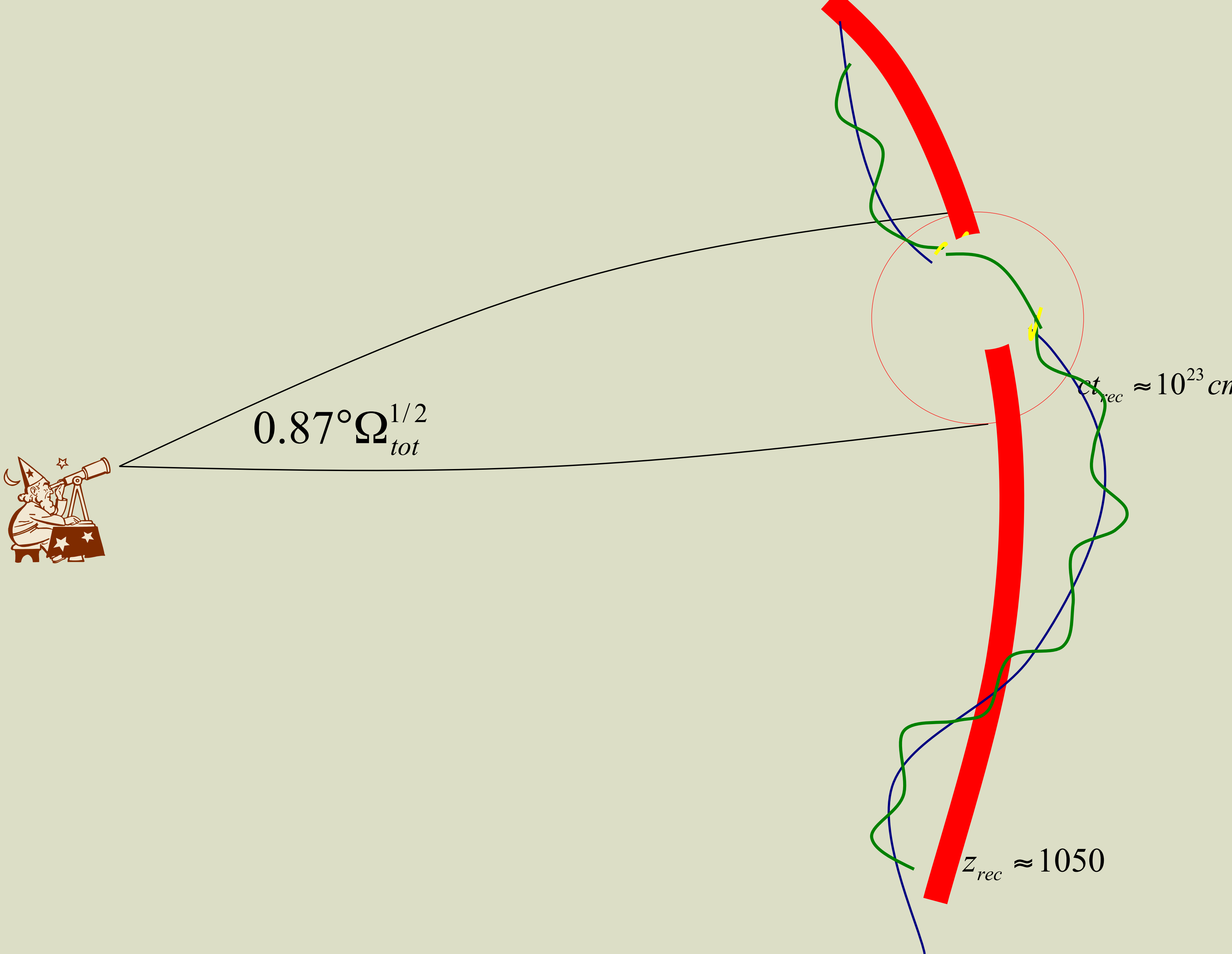
If $\Omega_{tot}^0 = 1$ then $\Omega_{\Lambda,Q}^0 = 1 - \Omega_m^0$

- Extra parameters influencing the spectrum

number of light neutrinos, gravity waves, reionization



ε
$\varepsilon_m \propto z^3$
$\varepsilon_{\Lambda,Q}$
$\varepsilon_{\gamma+\nu} \propto z^4$
Universe transparent
$z_{eq} \approx 2.3 \cdot 10^4 \Omega_m h_{75}^2$
$z_{rec} = 1200$ *to* 900
$z = 0$



$0.87° \Omega_{tot}^{1/2}$
$ct_{rec} \approx 10^{23}$ cm
$z_{rec} \approx 1050$

Before recombination

-(Im)perfect fluid: coupled baryon-radiation plasma

$$T_\alpha^\beta = (\varepsilon_\gamma + \varepsilon_b + p_\gamma) u_\alpha u^\beta - p_\gamma \delta_\alpha^\beta + \text{viscosity terms}$$

-CDM particles which interact with plasma only gravitationally

The fractional density perturbations in radiation $\delta = \delta\varepsilon_\gamma / \varepsilon_\gamma$ satisfy the equation:

$a\dfrac{\partial}{\partial t}$

viscosity

$$\left(\frac{\delta'}{c_S^2}\right)' - \frac{3\zeta}{\varepsilon_\gamma a}\Delta\delta' - \Delta\delta = \frac{4}{3c_S^2}\Delta\Phi + \left(\frac{4\Phi'}{c_S^2}\right)' - \frac{12\zeta}{\varepsilon_\gamma a}\Delta\Phi'$$

speed of sound

Gravitational potential is generated mainly by radiation before equality and by cold matter after that

After recombination

$$\left(\frac{\partial}{\partial\eta}+n^i\frac{\partial}{\partial x^i}\right)\left(\frac{\delta T}{T}+\Phi\right)=0$$

$$\left(\frac{\delta T}{T}+\Phi\right)=const$$

along photon's geodesic

$$x^i(\eta)=x_0^i+n^i(\eta-\eta_0)$$

$$\frac{\delta T}{T}(\eta_0,x_0^j,n^i)=\frac{\delta T}{T}(\eta_r,x^j(\eta_r),n^i)+\Phi(\eta_r,x^j(\eta_r))$$

n^i

rec

n^i

$$\frac{\delta T}{T}(\eta_r,x^j(\eta_r),n^i)-?$$

Instanteneous recombination

- Matching conditions for T_{α}^{β}

$$\frac{\delta T}{T}(\eta_0, \mathbf{x_0}, \mathbf{n}) = \int \left(\Phi_k + \frac{\delta_k}{4} - \frac{3\delta_k^{/}}{4k^2} \frac{\partial}{\partial \eta_0} \right)_{\eta_r} e^{i\mathbf{k}(\mathbf{x_0} + \mathbf{n}(\eta_r - \eta_0))} \frac{d^3 k}{(2\pi)^{3/2}}$$

- Correlation function

$$C(\theta) = \left\langle \frac{\delta T}{T}(\mathbf{n_1}) \frac{\delta T}{T}(\mathbf{n_2}) \right\rangle = \frac{1}{4\pi} \sum_{l=2}^{\infty} (2l+1) C_l P_l(\cos\theta)$$

$\boxed{\cos\theta = \mathbf{n_1} \bullet \mathbf{n_2}}$

Recombination is non-instantaneous

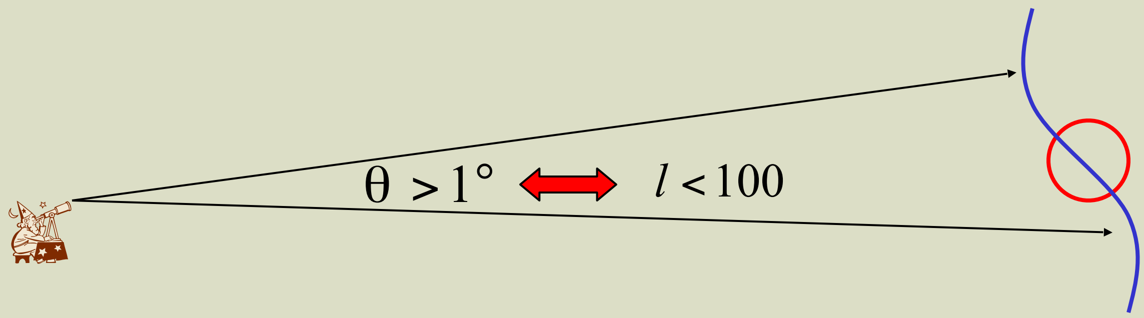
where

$$C_l = \frac{2}{\pi} \int \left| \left(\Phi_k + \frac{\delta_k}{4} \right)_{\eta_r} j_l(k\eta_0) - \frac{3\delta_k^{/}(\eta_r)}{4k} \frac{dj_l(k\eta_0)}{d(k\eta_0)} \right|^2 e^{-2(\sigma k \eta_r)^2} k^2 dk$$

$\sigma \approx 2 \bullet 10^{-2}$

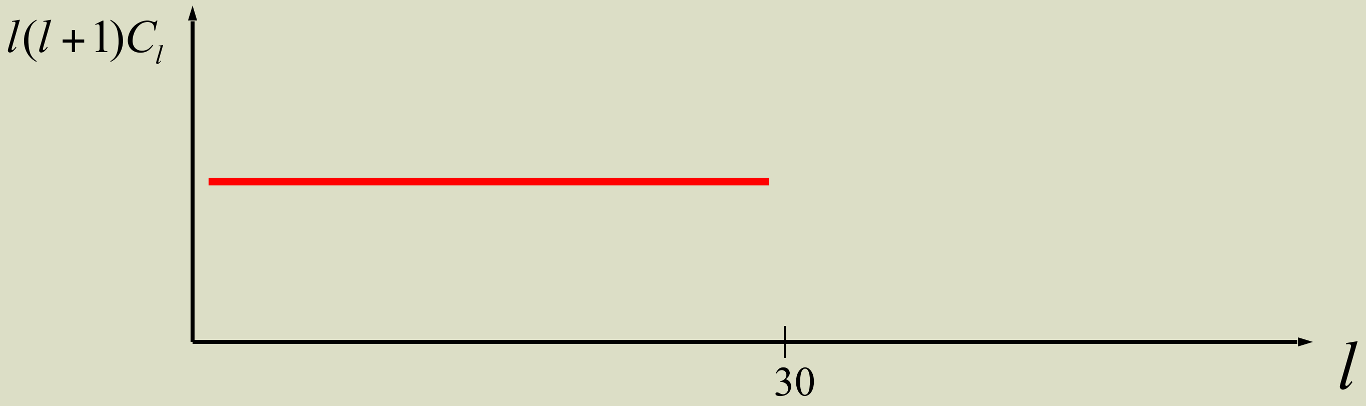
$l(l+1)C_l$ characterizes fluctuations on angular scales $\theta \approx \dfrac{\pi}{l}$

- Longwave inhomogeneities ($k\eta_r << 1$)



$$\left| k^3 \Phi_k^2 \right|_{\eta_r} = A^{/} k^{n_S - 1}, \quad \delta(\eta_r) = \frac{\delta\varepsilon_r}{\varepsilon_r} = \frac{4\delta\varepsilon_m}{3\varepsilon_m} = -\frac{8}{3}\Phi, \quad \delta^{/}(\eta_r) = 0$$

$$l(l+1)C_l \approx \frac{9\,A^{/}}{100\pi} \quad \text{for} \quad l < 30 \quad \text{if} \quad n_S \approx 1$$



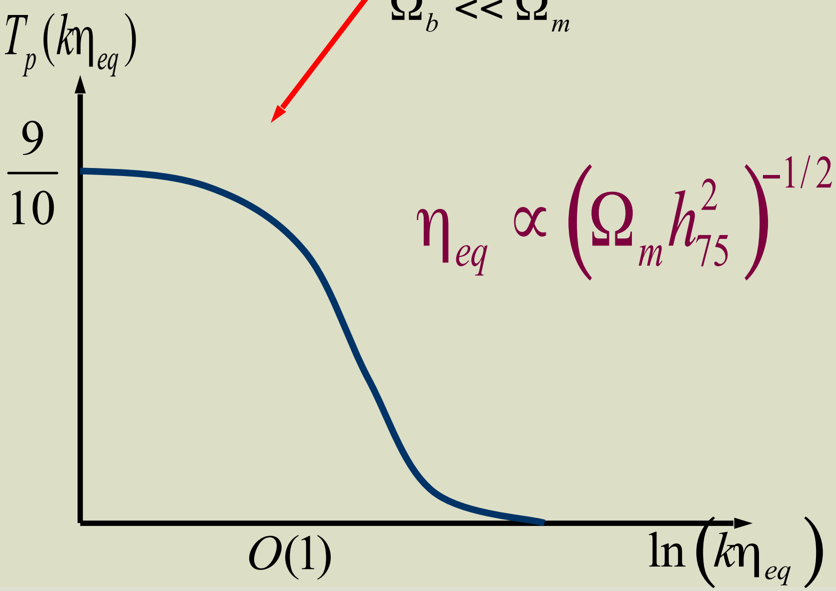
Shortwave inhomogeneities $(k\eta_r \gg 1)$

$$C_l \approx \frac{1}{16\pi}\int_{l\eta_0^{-1}}^{\infty}\left(\frac{\left|\Phi+4\delta\right|^2 k^2}{(k\eta_0)\sqrt{(k\eta_0)^2-l^2}}+\frac{9\sqrt{(k\eta_0)^2-l^2}}{(k\eta_0)^3}\left|\delta\right|^2\right)e^{-2(\sigma k\eta_r)^2}dk \quad \text{for } l \gg 1$$

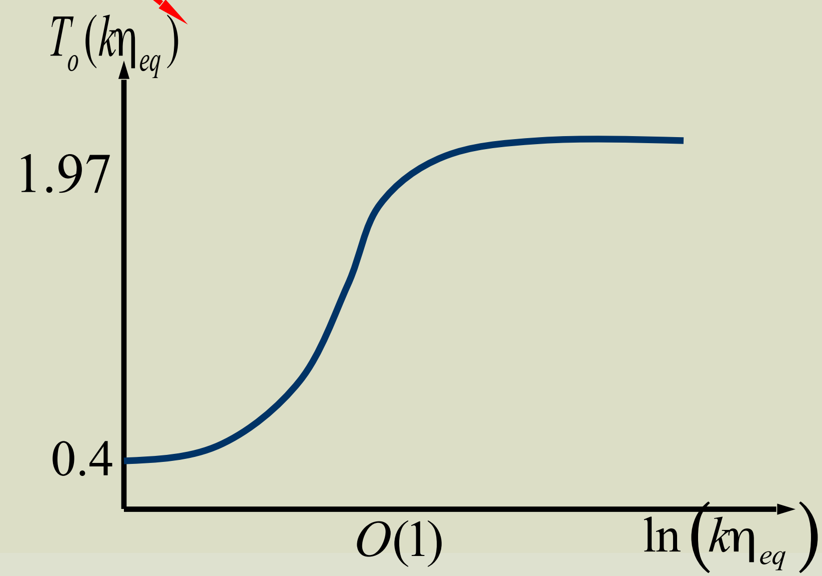
$\propto \Omega_b h_{75}^2$

$$\left(\Phi_k+\frac{\delta_k}{4}\right)_{\eta_r} \approx \left[T_p\left(1-\frac{1}{3c_S^2}\right)+T_o\sqrt{c_S}\cos\left(k\int_0^{\eta_r}c_S\,d\eta\right)e^{-(k/k_D)^2}\right]\Phi_k^0$$

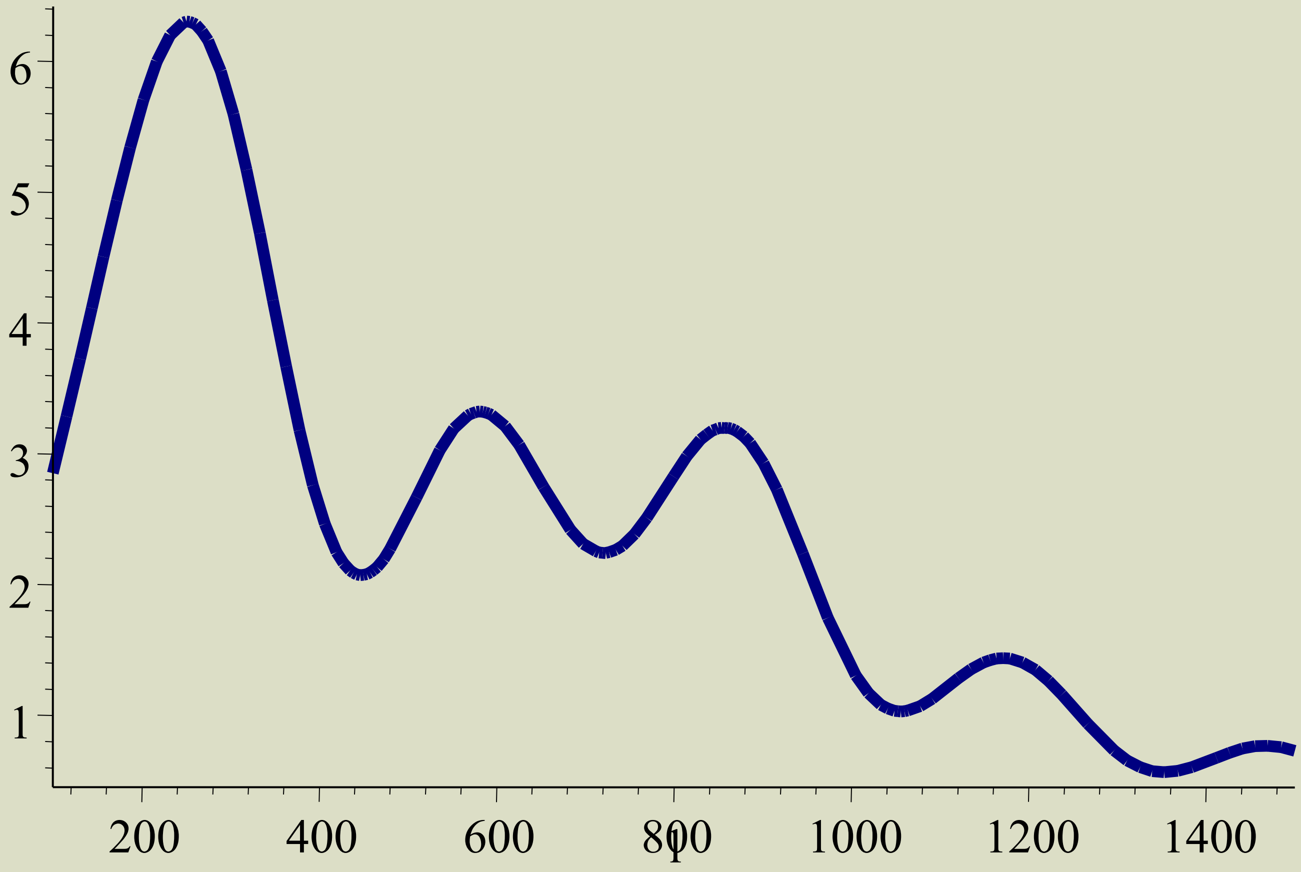
$\Omega_b \ll \Omega_m$



$$\eta_{eq} \propto \left(\Omega_m h_{75}^2\right)^{-1/2}$$



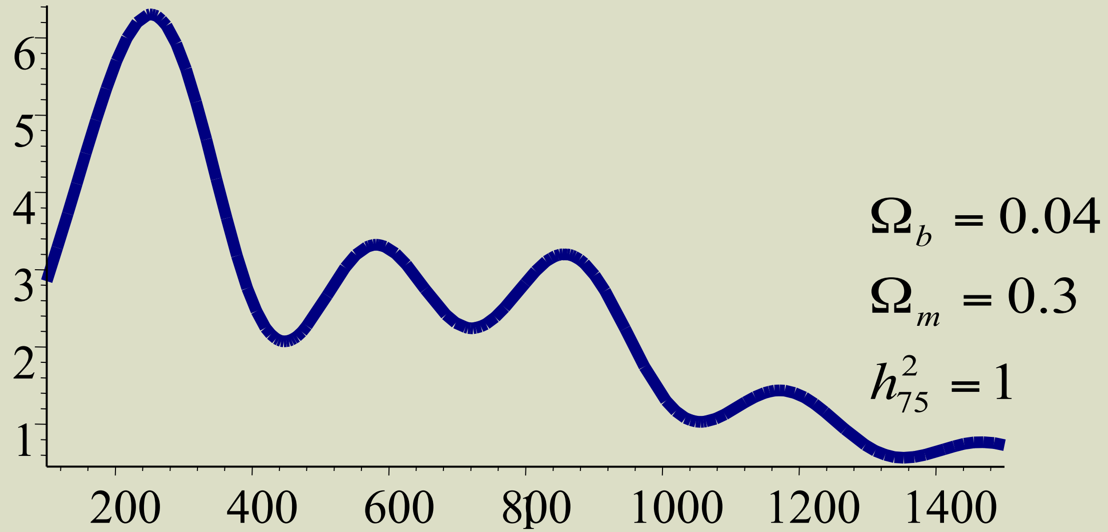
Spectrum



- Determining the cosmological parameters

$$h_{75},\ \Omega_m,\ \Omega_b,\ \boxed{\Omega_{tot}} = \Omega_m + \Omega_{\Lambda,Q},\ A,\ n_s$$

$$\longmapsto \propto \frac{1}{\sqrt{\Omega_{tot}}} \quad (\Lambda=0)$$



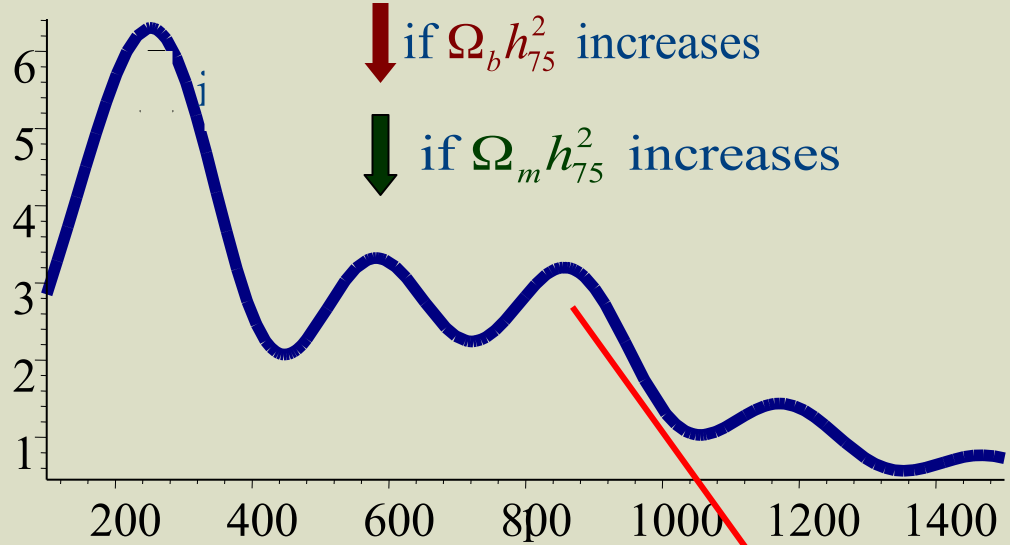
– The location of the peaks $(\Omega_{tot} = 1)$

$$l_n \approx \frac{\pi}{\rho} n \propto \frac{\left(1+2.2\Omega_b h_{75}^2\right)}{\left(\Omega_m h_{75}^{3.1}\right)^{0.16}} \Rightarrow$$

$\Delta l_1 = +40$ when $\Omega_b h_{75}^2$ increases twice

$\Delta l_1 = -40$ when $\Omega_m h_{75}^2$ increases twice

–The heights of the peaks



↓ if $\Omega_b h_{75}^2$ increases

↓ if $\Omega_m h_{75}^2$ increases

The second peak exists ⟹ $\Omega_b h_{75}^2 < 0.07$

⇓

$\Omega_m h_{75}^2 < 0.3$ if $\Omega_b h_{75}^2 > 0.03$

⇓

$\Omega_{\Lambda,Q} \neq 0$!!! because $\Omega_{tot} = 1$

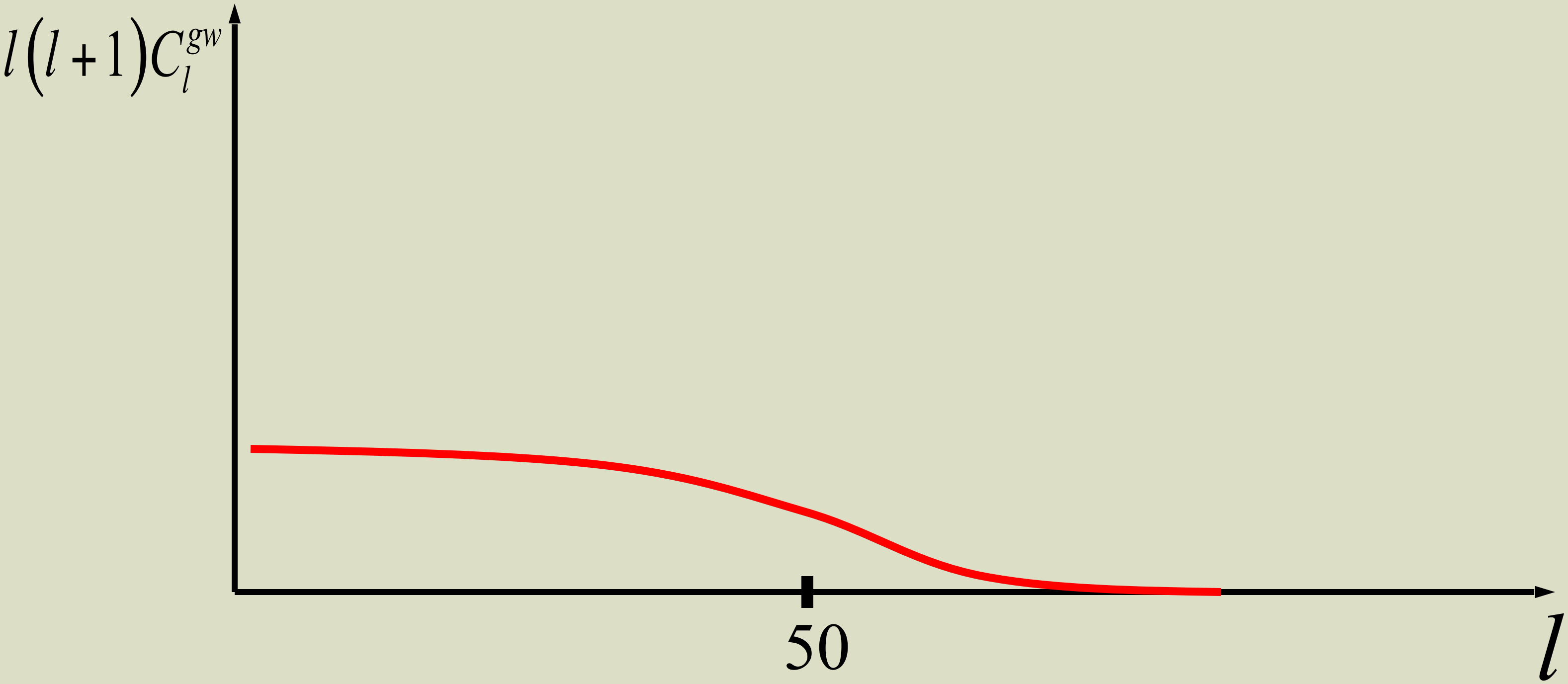
$n_S - ?$ for given $\Omega_b h_{75}^2$ and $\Omega_m h_{75}^2$

$h_{75}^2 - ?$ peaks location depends on $\Omega_m h_{75}^2$

peaks heights depends on $\left(\Omega_m h_{75}^{3.1}\right)^{0.16}$

⟹ 1% in location ⟺ 7% in h_{75}

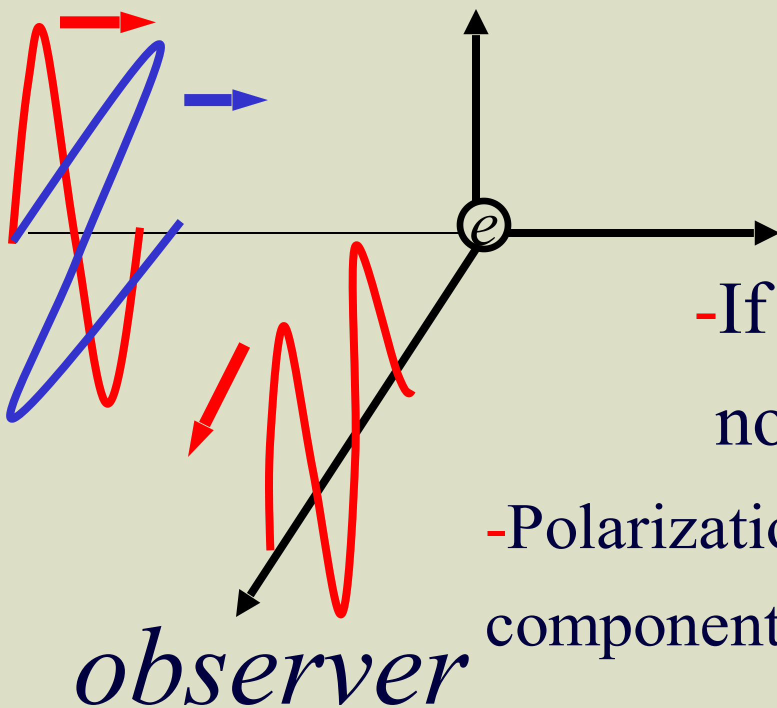
Gravitational waves



$$\frac{T}{S} \approx 7(1-n_S)+\ldots$$

CMB polarization-way to detect primordial gravitational waves

Thomson scattering ➡ linear polarization



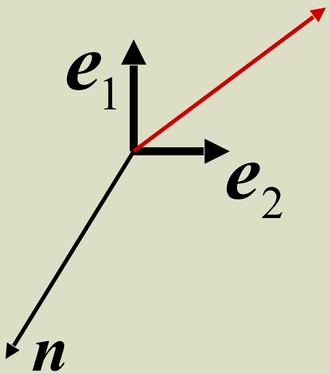
100% polarized

-If incident radiation is isotropic-
no polarization is generated

-Polarization is proportional to the quadrupole component of incoming radiation

-Quadrupole anisortopy is generated by both, scalar pertirbations and gravity waves, only after the recombination begins ➡ $polarization \propto duration\ of\ recombination$

Polarization tensor



$\boldsymbol{E} = E^a \boldsymbol{e}_a$

$$P_{ab}(\boldsymbol{n}) = \frac{1}{I}\left(\langle E_a E_b \rangle - \frac{1}{2} g_{ab} \langle E_c E^c \rangle\right)$$

$2d$ second rank traceless tensor

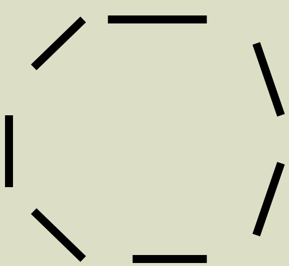
E – mode: $E(\boldsymbol{n}) = P^{b;a}_{a;b}$

B – mode: $B(\boldsymbol{n}) = P^{b;ac}_{a} \varepsilon_{bc}$

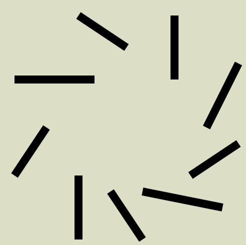
Only gravitational waves contribute to B!!!

$$P_{ab} = p_a p_b - \frac{1}{2} g_{ab} p^2$$

$p_a(\boldsymbol{n})$ – ?

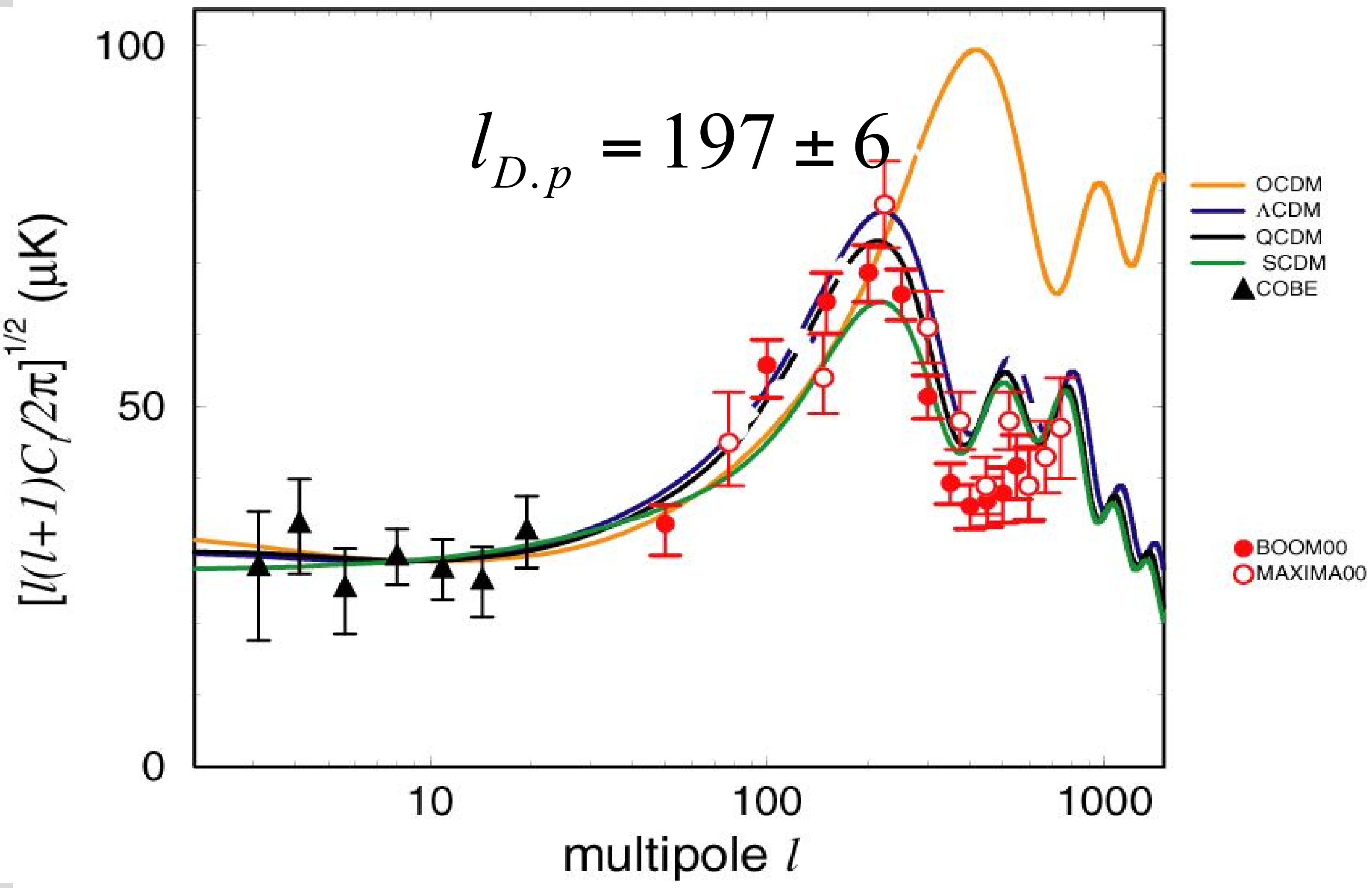


Scalar perturbations

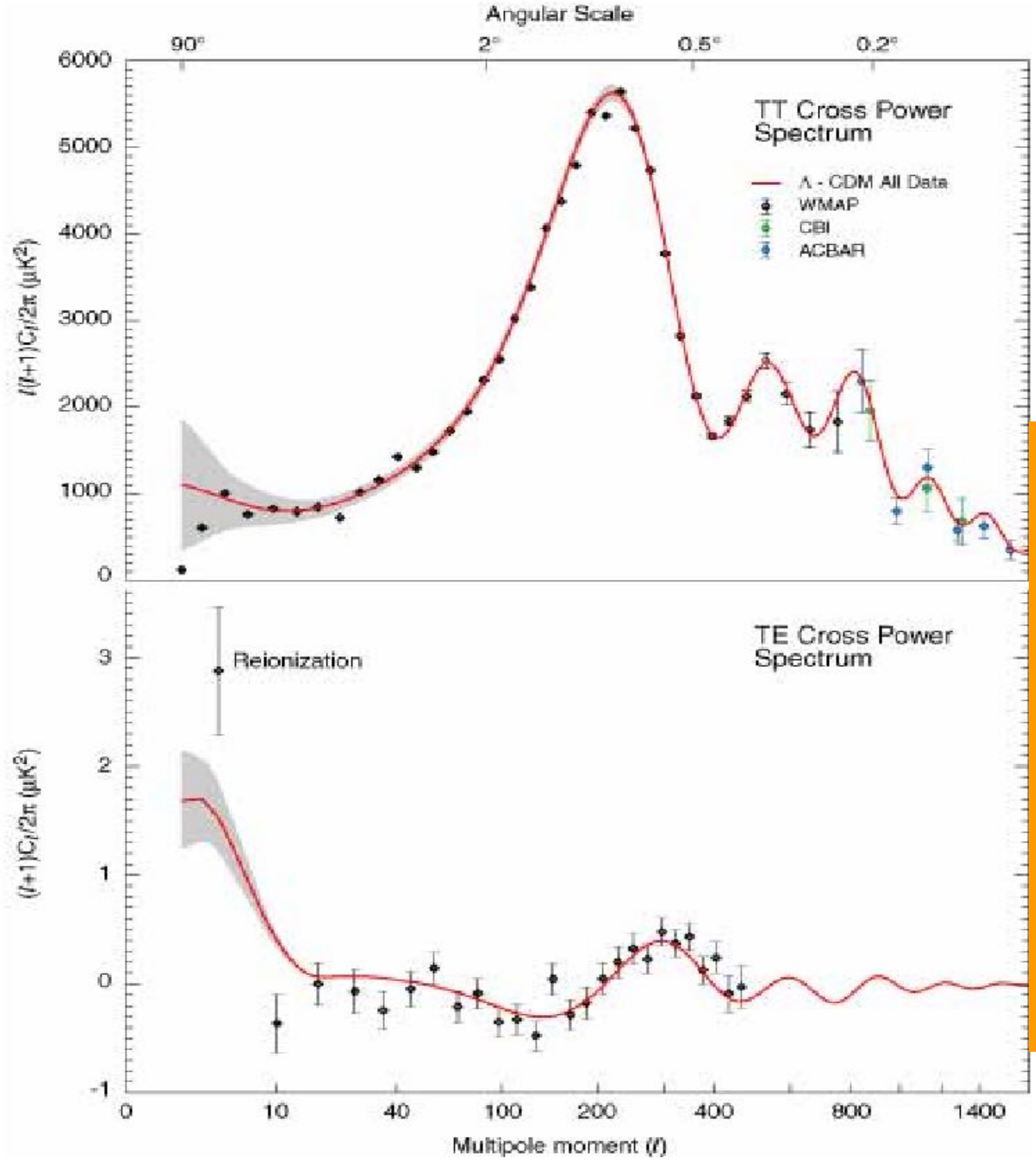


Gravity waves

$\langle BB \rangle$ gets maximum at $l \approx 100$



$l_{D.p} = 197 \pm 6$
100
50
0
10
100
1000
multipole l
$[l(l+1)C_l/2\pi]^{1/2}$ (μK)
OCDM
ΛCDM
QCDM
SCDM
COBE
BOOM00
MAXIMA00



$$\Omega_{tot} = 1,02 \pm 0,02 \quad n_S$$

$$H_0 = 71 \ km/\sec Mpc$$

$$\Omega_b h^2 = 0,024 \pm 0,001$$

$$\Omega_m h^2 = 0,14 \pm 0,02$$