P. Nason

QCD at the LHC

- Hadron machine: QCD everywhere;
- Previous experience up to 2 TeV;
- Increase in complexity: more open channels, more jets;
- Need better tools for calculations and simulations.

Outline

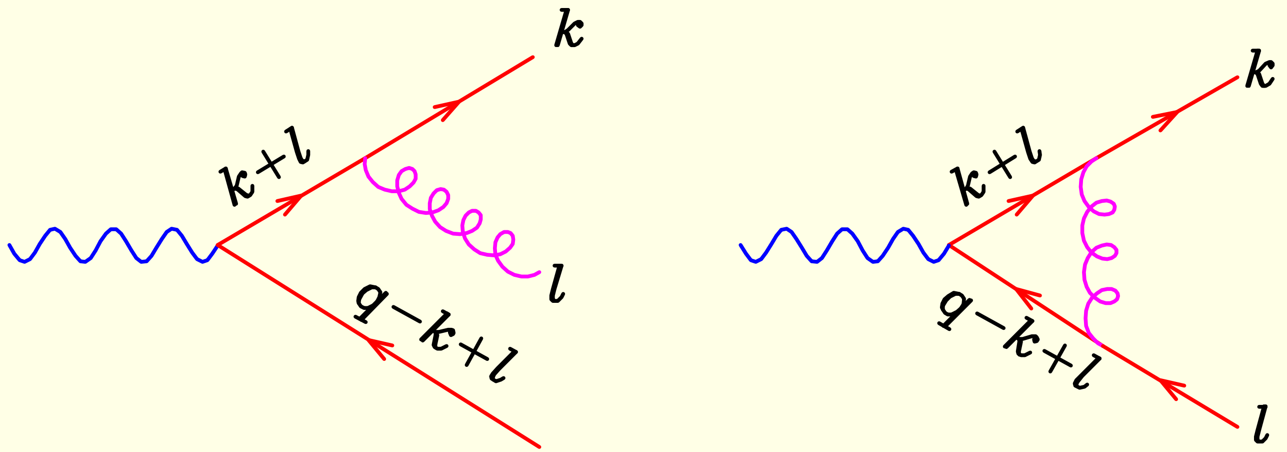
- Basics of QCD calculations;
- Fixed order calculations, “parton generators”
- Exclusive final states: shower Monte Carlo
- MC@NLO: the new frontier

Basics

$$R = \frac{\sigma(e^+e^- \to \text{hadrons})}{\sigma(e^+e^- \to \mu^+\mu^-)} = 3 \times \sum_i q_i^2 + \mathcal{O}(\alpha_S)$$

angular distributions of jets and muons equal at high energy.

How do radiative corrections enter?



$\mathcal{O}(\alpha_S)$ corrections from square of real emission and interference of virtual correction with Born term. Both InfraRed divergent; IR singularity cancels in the total.

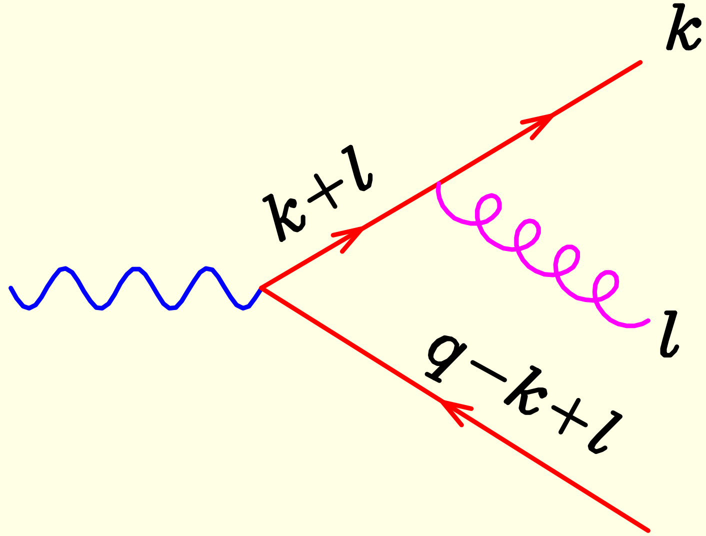
Anatomy of IR singularity

In the soft limit (small gluon energy):

$$\mathcal{M} = \overline{u}(k)\mathcal{N}, \quad \mathcal{N} = \epsilon^{\mu}\gamma_{\mu}v(k')$$

$$\mathcal{M}_1 = \overline{u}(k)(-i)\gamma_{\alpha}i\frac{\not{k}+\not{l}}{(k+l)^2}\mathcal{N}.$$

$$\mathcal{M}_1 = \overline{u}(k)\frac{\gamma_{\alpha}\not{k}+\not{k}\gamma_{\alpha}}{(k+l)^2}\mathcal{N} = \overline{u}(k)\frac{2k_{\alpha}}{2k\cdot l}\mathcal{N} = \frac{k_{\alpha}}{k\cdot l}\mathcal{M}.$$



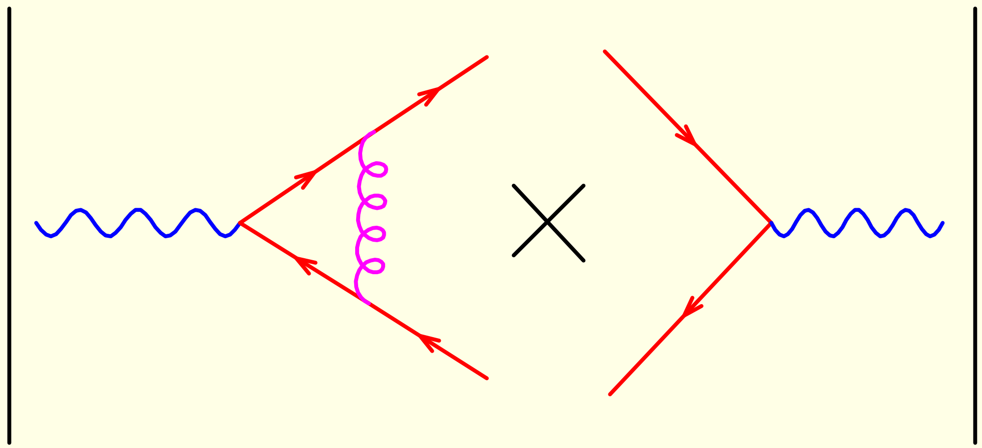
Including antiquark emission:

$$\mathcal{M}_{q\bar{q}g} = \mathcal{M}_1 + \mathcal{M}_2 = \left(\frac{k_{\alpha}}{k\cdot l} - \frac{k'_{\alpha}}{k'\cdot l}\right)\mathcal{M}, \quad |\mathcal{M}|^2_{q\bar{q}g} = 2\frac{k\cdot k'}{(k\cdot l)(k'\cdot l)}|\mathcal{M}|^2.$$

Plus phase space and color factors:

$$\sigma_{q\bar{q}g} = C_F\frac{\alpha_{\mathrm{S}}}{2\pi}\sigma^{\mathrm{Born}}_{q\bar{q}}\int d\cos\theta\frac{dl^0}{l^0}\frac{4}{(1-\cos\theta)(1+\cos\theta)}.$$

We know that the $\mathcal{O}(\alpha_S)$ correction to the $e^+e^-\to$hadrons cross section is equal to α_S/π times the Born term. Thus, the virtual correction must also be singular, and cancel (up to finite terms) the singularity of the real one.

2  $= -C_F \dfrac{\alpha_S}{2\pi} \sigma^{\text{Born}}_{q\bar{q}} \displaystyle\int d\cos\theta \frac{dl^0}{l^0} \frac{4}{(1-\cos^2\theta)}.$

Summary

- $q\bar{q}g$ cross section divergent
- $q\bar{q}$ cross section divergent (virtual corrections must cancel real part)
- Only "calorimetric" quantities become calculable

Only inclusive final states
are calculable at fixed order

Implementation: Parton Level Generators,
either with IR cutoff (slicing method) or by direct cancellation:

subtraction method

- Origin: Ellis, Ross and Terrano (1981), $\mathcal{O}(\alpha_S^2)$ corrections to 3-jet production in e^+e^-.
- First implementation as PLG in Kunszt, P.N. (1990).
- First implementation in hadron collisions:
  Mele, Ridolfi, P.N. in $h_1 h_2 \rightarrow ZZ + X$ (1991).
- General methods proposed by Catani Seymour, Frixione.

PLG example

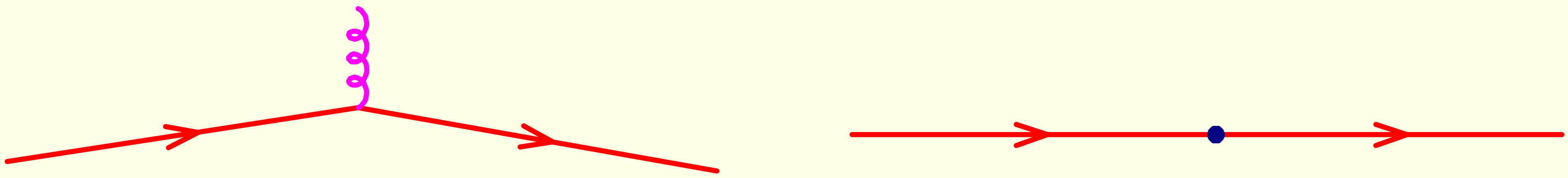
In $e^+e^- \to$hadrons, for unoriented events.
use Dallitz variables $x_{q/\bar{q}} = 2E_{q/\bar{q}}/E_{\text{CM}}$;

- generate one 2-body ($x_{q/\bar{q}} = 1$) event with weight 1.
- generate one 2-body ($x_{q/\bar{q}} = 1$) event with weight α_{S}/π.
- Generate N random pairs $x_k, \bar{x}_k$ ($k = 1, \ldots, N$), uniform in the triangle $x, \bar{x} > 0$, $x + \bar{x} > 1$.
- For each k generate an event, with $x_q = x_k$, $x_{\bar{q}} = \bar{x}_k$ and weight

$$w = \frac{1}{2N} \frac{\alpha_{\text{S}} C_F}{2\pi} \frac{x_k^2 + \bar{x}_k^2}{(1 - x_k)(1 - \bar{x}_k)}$$

  and a 2-body event ($x_{q/\bar{q}} = 1$) with weight $-w$.
- For each generated event, build up observables, and histogram them with the event weight.

IR singularities cancel in the region $x_q \to 1$ or $x_{\bar{q}} \to 1$ in Collinear and Soft insensitive observables.



$x_q \to 1$ $x_{\bar{q}} \to 1$; counterterm $x_q = 1$ $x_{\bar{q}} = 1$. Cancellation operational for IR insensitive observables (insensitive to the emission of an extra soft gluon), like the calorimetric quantities.

Example of not IR safe variable: multiplicity.

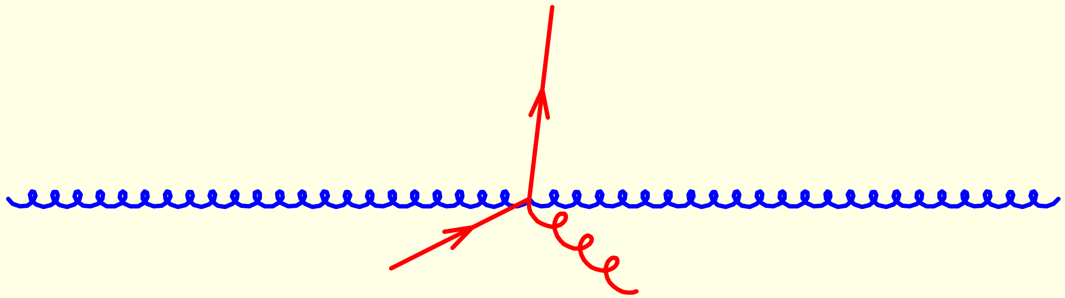
only $x_q \to 1$; antiquark and gluon become collinear (total energy $\to E_q$). Counterterm: $x_q = 1$ $x_{\bar{q}} = 1$. Cancellation for Collinear insensitive observables, like calorimetric quantities.

IR safe but not Collinear safe: $\sum_i \vec{k}_i^2$, equal to $4E^2(x_q^2 + x_{\bar{q}}^2 + x_g^2)$ in our case ($4E^2 \times 2$ for counterterm).

Event $\rightarrow$ Counterterm: *projection*

Straightforward in our example. In general complex and not unique.

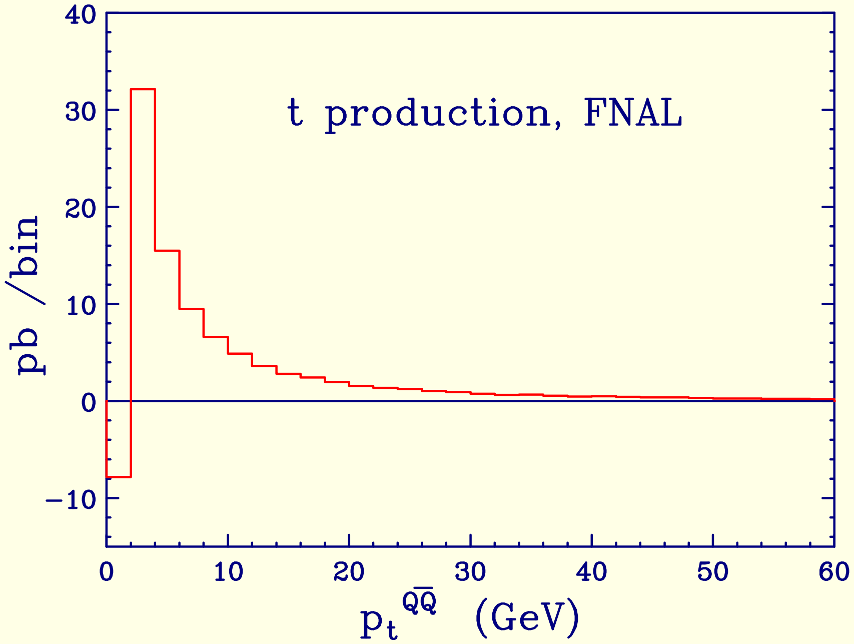
Example: Heavy Flavour Production (Mangano, Ridolfi, P.N.).



Projection:

- Longitudinal boost B_1, to frame where $y_q = y_{\bar{q}}$;
- Transverse boost B_2 to frame where $p_T^{q\bar{q}} = 0$;
- Longitudinal boost B_1^{-1}.

Notice: projection does nothing to collinear or soft events ($B_2 = 1$).
This guarantees the cancellation for Soft/Collinear insensitive observables.



In practical calculations, too small bins in sensitive regions can produce negative results. Very difficult to understand how big bins to use...

This problem is always present with FO calculations. Also at tree level: cannot trust prediction with soft or near collinear final state partons.

Exclusive Final State: Shower Monte Carlo

- Alternative approach to FO calculations (but limited precision).
- Most popular tool for HEP
- Heavily used by experimentalists

Ingredients

- Sophisticated theoretical input:
  infinite set of dominant QCD Feynman graphs.
- Modeling of hadronization phenomena.
- PDG encyclopedia:
  all known (and unknown!) particle branching ratios and decays.

History

PQCD:

- First SMC's: Fox+Wolfram, Odorico, (1980) for $e^+e^- \to$ hadrons
- Soft gluon interference Marchesini+Webber (1983)
- Backward evolution for initial state radiation Sjöstrand (1985)

Hadronization

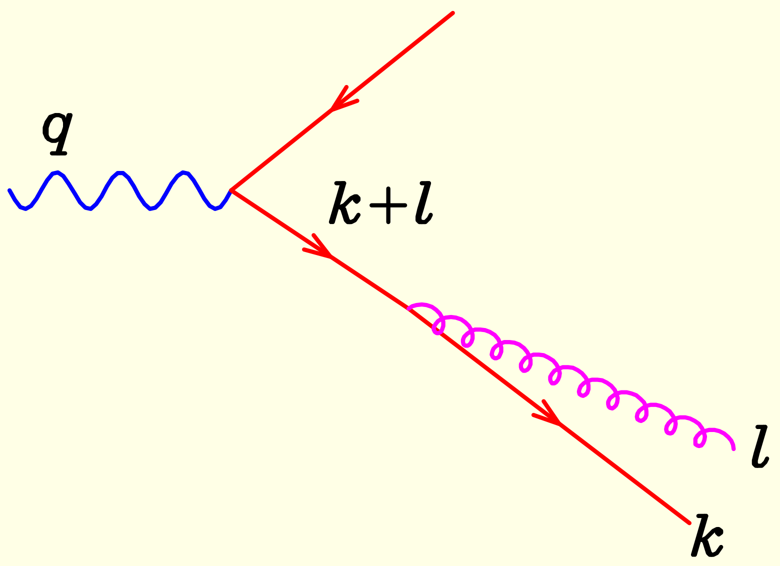
- Independent fragmentation Field+Feynman, (1977)
- String model Artru+Mennessier (1974), Bowler (1981), Andersson+Gustafson+Söderberg (1983), Andersson+Gustafson+Ingelman+Sjöstrand (1983), Sjöstrand (1984)
- Cluster model Field+Wolfram (1983), Webber (1984)

Models

- COJETS Odorico (1984)
- ISAJET Page+Protopopescu (1986)
- FIELDAJET Field (1986)
- JETSET Sjöstrand (1986)
- PYTHIA Bengtsson+Sjöstrand (1987), Sjöstrand (1994)
- ARIADNE Lönnblad (1991)
- HERWIG Marchesini+Webber (1988),
  Marchesini+Webber+Abbiendi+Knowles+Seymour+Stanco (1992)

Basics

Dominant QCD emissions: collinear singularities



When $l \parallel k$: $(l+k)^2 \to 0$. Cross section equals

$$\text{Born} \times \frac{\alpha_S}{2\pi} \frac{dt}{t} P_{qq}(z)\, dz \frac{d\Phi}{2\pi}$$

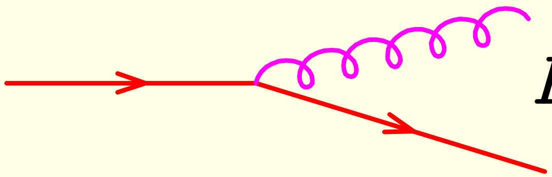
where $t \propto (l+k)^2$, z is the energy (or longitudinal momentum, or light cone momentum) fraction of the quark, and

$$P_{qq}(z) = C_F \frac{1+z^2}{1-z}$$

is the Altarelli-Parisi splitting function.

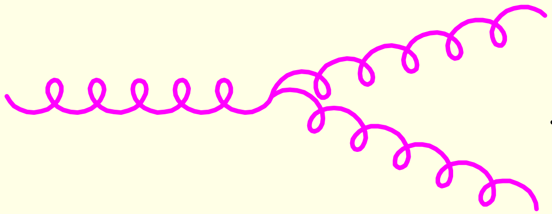
Diverges as $t \to 0$. (Ignore for now the infrared divergence at $z = 1$).

Present in all splitting processes



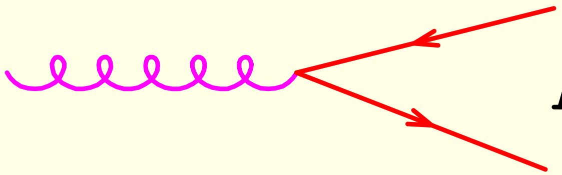
$P_{qq}(z)$

Same structure for all processes (after azimuthal average). Only differences in the form of $P_{ij}(z)$



$P_{gg}(z)$

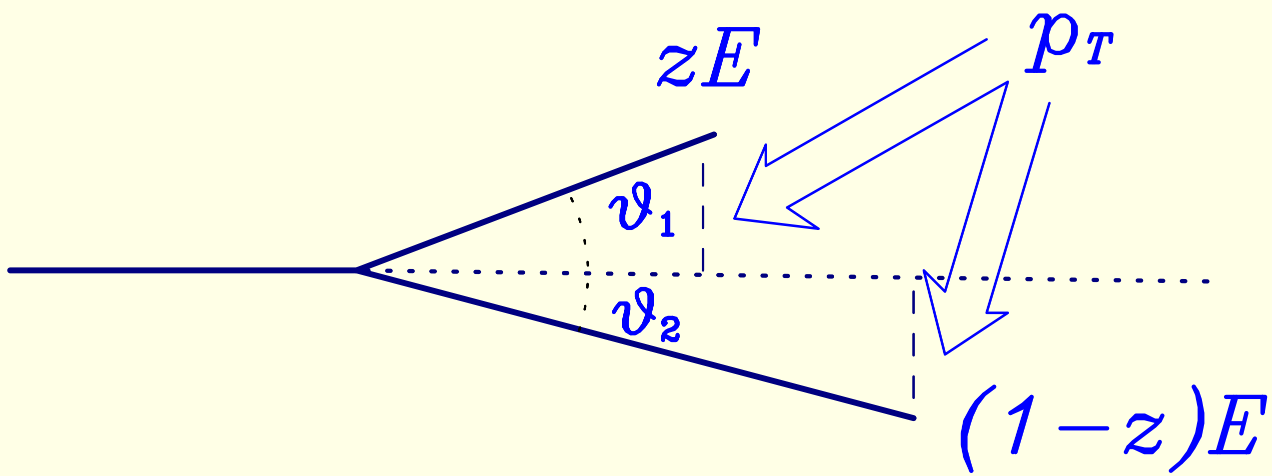
$$
\begin{aligned}
P_{gg} &= C_A\,(z/(1-z)+(1-z)/z+z(1-z)) \\
P_{gq} &= T_F\left(z^2+(1-z)^2\right)\ .
\end{aligned}
$$



$P_{gq}(z)$

Notice: dt/t singularity, not dt/t^2. All splitting processes violate angular momentum in collinear limit.

Kinematics:

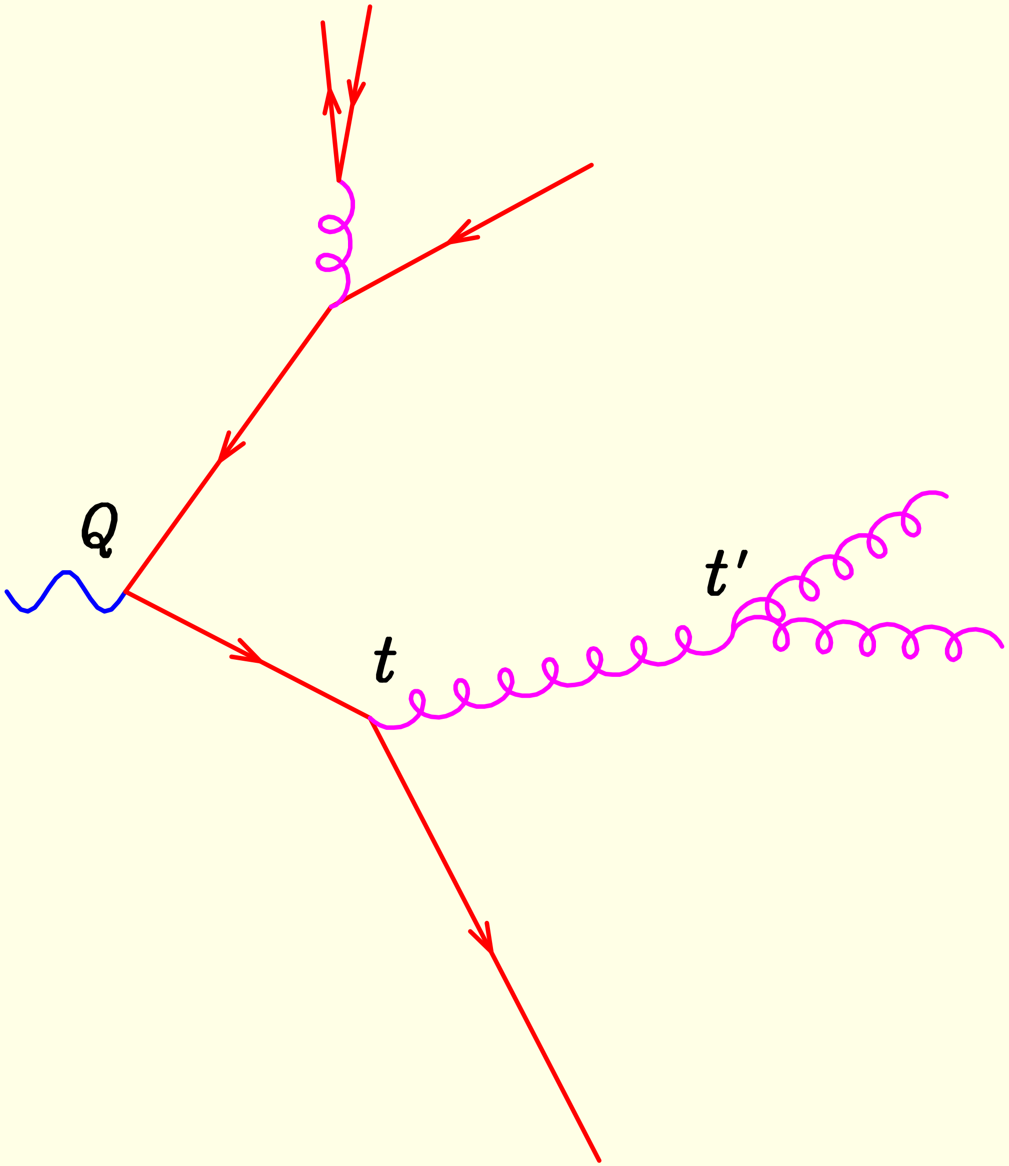


$$\theta = \theta_1 + \theta_2\,, \quad z\theta_1 = (1-z)\theta_2 \quad \Longrightarrow \quad \theta_1 = (1-z)\theta\,, \quad \theta_2 = z\theta$$

$$\begin{array}{llcll} \text{virtuality:} & t & = & z(1-z)E^2(1-\cos\theta) & = \frac{1}{2}z(1-z)E^2\,\theta^2 \\ p_T^2: & t & = & (Ez\theta_1)^2 & = E^2\,z^2(1-z)^2\,\theta^2 \\ \text{angle:} & t & & & = E^2\,\theta^2 \end{array}$$

Which t makes a difference only if soft ($z \to 0$, 1) region is singular.

Multiple emission



Dominant configurations for multiple emissions: strongly ordered region

$$Q^2 > t > t' > t'' \ldots$$

so that logarithmic integral builds into a

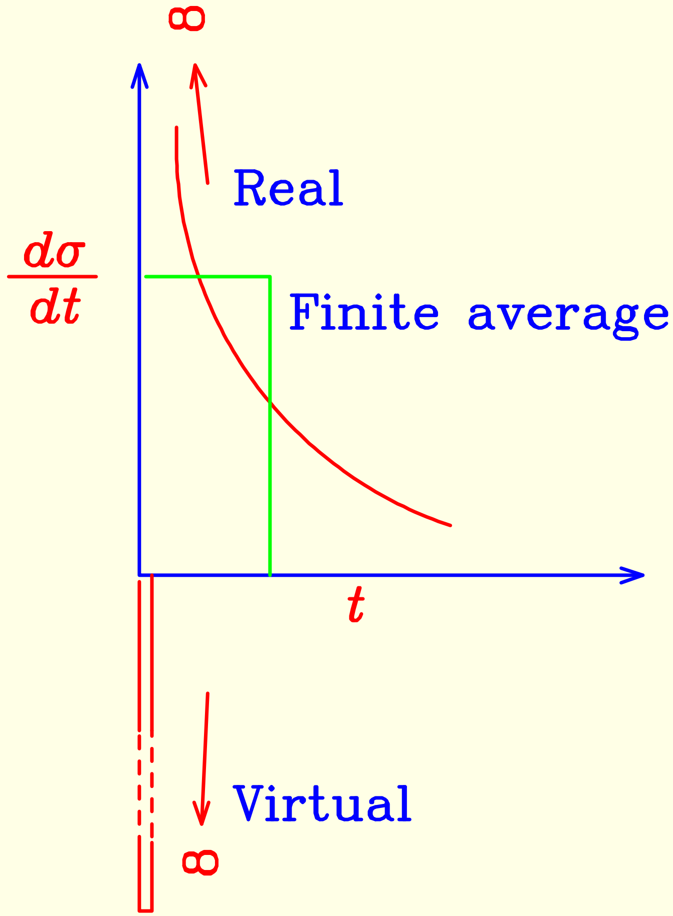
$$\frac{1}{n!} \log^n \frac{Q}{\lambda}$$

(leading log approximation)

Divergences cancellation

Virtual graphs must cancel the collinear divergence in the total cross section, by the Kinoshita-Lee-Nauenberg theorem. So

$$\text{Virtual term} = -\text{Born} \times \int \frac{\alpha_S}{2\pi} \frac{dt}{t} P_{qq}(z)\, dz \frac{d\Phi}{2\pi} + \text{non-singular terms.}$$



The cancellation takes place at all finite orders.

Can compute inclusive quantities at finite orders.
But how does the exclusive final state looks like?
Finite order unphysical for exclusive quantities.
Cancellation is among different final states!

Exclusive final state

Need to sum up all virtual corrections for a given final state! Recipe:

- Consider all tree graphs from the initial parton to all final states
- Include a factor
$$\frac{dt}{t} dz \, \frac{\alpha_S(t)}{2\pi} \, P_{ij}(z)$$
at each splitting vertex; use renormalized coupling at scale t.
- Order the splittings in t: later splittings have smaller t.
- Include the factor $\Delta_i(t_1)/\Delta_i(t_2)$ on each internal line going from a splitting at the scale t_1 to a splitting at t_2, with (t_0: IR cutoff)

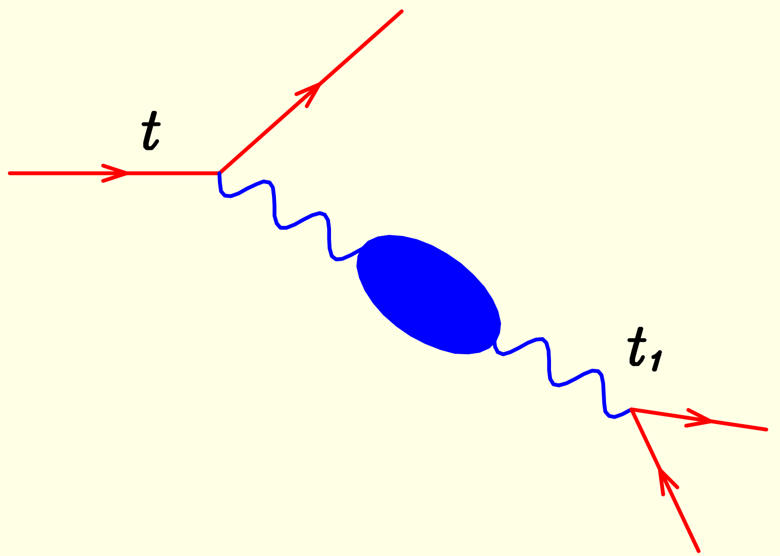
$$\Delta_i(t) = \exp\left[-\sum_j \int_{t_0}^{t} \frac{dt'}{t'} \int dz \, \frac{\alpha_S(t')}{2\pi} \, P_{ij}(z) \right]$$

Easy to check that $\mathcal{O}(\alpha_S)$ expansion of Δ yields correct virtual term.

How do we prove our recipe?

Original papers on MC derive these results from AP equations.

Simplest example (in QED)



Here the virtual corrections amount to

$$\alpha^2 \Longrightarrow \alpha^2(t_1)$$

Our recipe wants

$$\alpha(t) \times \alpha(t_1) \times \frac{\Delta(t)}{\Delta(t_1)}$$

so

$$\frac{\Delta(t)}{\Delta(t_1)} = \frac{\alpha(t_1)}{\alpha(t)} = \frac{\log\frac{\Lambda}{t}}{\log\frac{\Lambda}{t_1}}$$

and indeed using

$$\alpha(t) = \frac{1}{b_0 \log\frac{\Lambda}{t}}, \quad b_0 = \frac{4n_\mathrm{f}}{12\pi}$$

we get

$$\Delta(t) = \exp\left[-n_\mathrm{f}\int_{t_0}^{t}\frac{dt'}{t'}\frac{\alpha(t')}{2\pi}\left(z^2+(1-z)^2\right)\,dz\right] = \frac{\log\frac{\Lambda}{t}}{\log\frac{\Lambda}{t_0}}$$

Notation

Introduce the notation:

$$|k_1, m_1; ...; k_l, m_l\rangle \, ;$$

k_i and m_i are the momenta and quantum numbers of the particles; normalization:

$$\langle k_1, m_1; ...; k_l, m_l | k'_1, m'_1; ...; k'_{l'}, m'_{l'}\rangle = \delta_{l,l'} \prod_{i=1}^{l} \delta^3(k_i - k'_i)\delta_{m_i, m'_i} \, .$$

A shower is defined as

$$\mathbb{S} = \sum_{l=1}^{\infty} \sum_{m_1 ... m_l} \int d^3k_1 \ldots d^3k_l \, C(k_1, m_1; \ldots ; k_l, m_l) \, \langle k_1, m_1; ...; k_l, m_l|$$

given the cell

$$d\Psi = |k'_1, m'_1; ...; k'_{l'}, m'_{l'}\rangle \, d^3k'_1 \ldots d^3k'_{l'}$$

the product

$$S \cdot d\Psi$$

is the probability to generate a state in the cell $d\Psi$.

A final state observable $g(k_1, m_1; \ldots; k_l, m_l)$ is

$$\mathbb{G} = \sum_{l=1}^{\infty} \sum_{m_1 \ldots m_l} \int d^3k_1 \ldots d^3k_l \; g(k_1, m_1; \ldots; k_l, m_l) \; |k_1, m_1; k_2, m_2; ...; k_l, m_l\rangle$$

Its average is $\mathbb{S} \cdot \mathbb{G}$.

One can describe the final state using the shower variables themselves:

t, z, ϕ at each vertex

(the whole final state can be reconstructed from them).

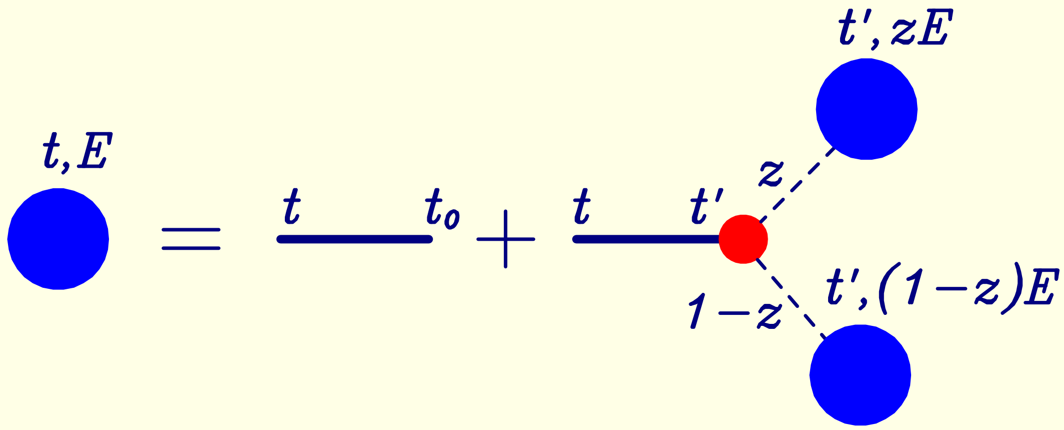
For ease of notation ϕ will be ignored.

Monte Carlo equation

The whole shower is defined recursively by the equation

$$\mathbb{S}(t,E) = \Delta(t)\,\langle\mathbb{I}| + \int_{t_0}^{t} \frac{\Delta(t)}{\Delta(t')}\,\frac{dt'}{t'}\,\frac{\alpha_S(t')}{2\pi}\,P(z)\,\mathbb{S}(t',Ez)\,\mathbb{S}(t',E(1-z))\,dz$$

or graphically



The integral over all final state configurations must give 1 by KLN; so

$$1 = \Delta(t) + \int_{t_0}^{t} \frac{\Delta(t)}{\Delta(t')}\,\frac{dt'}{t'}\,\frac{\alpha_S(t')}{2\pi}\,P(z)\,dz \Rightarrow \frac{d\Delta^{-1}(t)}{d\log t} = \Delta^{-1}(t)\frac{\alpha_S(t)}{2\pi}\int P(z)\,dz$$

Simple probabilistic interpretation:

$\Delta(t)/\Delta(t')$ is the probability for having no branching from t to t'.
The probability to have a branching in the interval $t', t' + dt'$ is

$$P(t')dt' = \frac{\Delta(t)}{\Delta(t'+dt')} - \frac{\Delta(t)}{\Delta(t')} = dt' \frac{d}{dt'} \frac{\Delta(t)}{\Delta(t')} = \frac{\Delta(t)}{\Delta(t')} \frac{dt'}{t'} \frac{\alpha_S(t')}{2\pi} \int P(z)\, dz$$

$\Delta(t)/\Delta(t')$ has uniform distribution!

Shower algorithm:

- Generate a random number $0 < r < 1$;
- Solve the equation $\Delta(t)/\Delta(t') = r$ for t';
- If $t' < t_0$ stop there (unresolvable emission);
- generate a z distributed according to $P(z)$;
- restart for each branch, at an initial value t'.

Elementary example

Simulate a source with a probability p for emission per unit time.
Probability distributions for first emission:

$$P(t)\,dt = \lim_{n \longrightarrow \infty} \left(1 - p\frac{t}{n}\right)^n p\,dt = e^{-pt}\,p\,dt = -de^{-pt}$$

so $\int P(t)dt$ is distributed uniformly between 0 and 1.
Monte Carlo implementation for emissions between $t = t_0$ and $t = t_f$

- generate a random number $0 < r < 1$
- solve the equation $e^{-p(t-t_0)} = r$ for t
- if $t > t_f$ stop.
- continue starting from t

Evolution equation for fragmentation functions

inclusive cross section for the production of a parton of energy Ez starting from a parton of energy E and "virtuality" t.

$$D(x,t) = \mathbb{S}(t,E) \cdot \sum |k_1 \dots k_l\rangle \sum_{i=1}^{l} \delta(E_i/E - x)$$

from the shower equation

$$\mathbb{S}(t,E) = \Delta(t)\,\langle\mathbb{I}| + \int_{t_0}^{t} \frac{\Delta(t)}{\Delta(t')}\,\frac{dt'}{t'}\,\frac{\alpha_S(t')}{2\pi}\,P(z)\,\mathbb{S}(t',Ez)\,\mathbb{S}(t',E(1-z))\,dz$$

we get (assuming $z \rightarrow 1-z$ symmetry)

$$D(x,t) = \Delta(t)\delta(1-x) + \int_{t_0}^{t} \frac{\Delta(t)}{\Delta(t')}\,\frac{dt'}{t'}\,\frac{\alpha_S(t')}{2\pi}\,2P(z)\,D(t',x/z)\frac{dz}{z}$$

Applying

$$\Delta(t) t \frac{\partial}{\partial t} \frac{1}{\Delta(t)}$$

we obtain

$$-D(x,t)\, t \frac{\partial \log \Delta(t)}{\partial t} + t \frac{\partial D(x,t)}{\partial t} = \int \frac{\alpha_{\mathsf{S}}(t)}{2\pi}\, 2P(z)\, D(t', x/z) \frac{dz}{z}$$

or

$$t \frac{\partial D(x,t)}{\partial t} = \int \frac{\alpha_{\mathsf{S}}(t)}{2\pi} \underbrace{\left[2P(z)\, \frac{1}{z} D(t', x/z) - 2P(z) \theta(z - 1/2) D(t', x) \right]}_{\widehat{P}(z)} dz$$

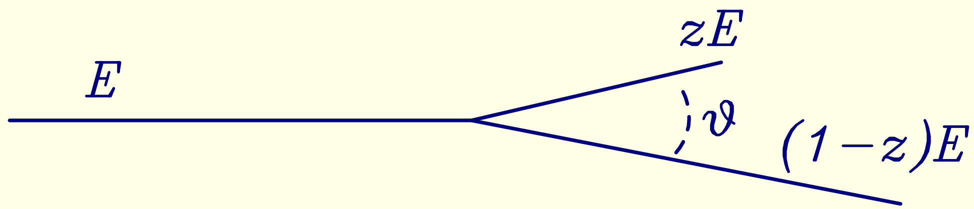
which corresponds to the regularized splitting vertex of the AP equation (check that $\int_{1/2}^{1} \widehat{P}_{gg}(z) dz = 0$ for pure glue)

Soft divergences

$z \to 1$ ($z \to 0$) region problematic: for $z \to 1$: $P_{qq}, P_{gg} \propto \dfrac{1}{1-z}$

Choice of shower variables makes a difference

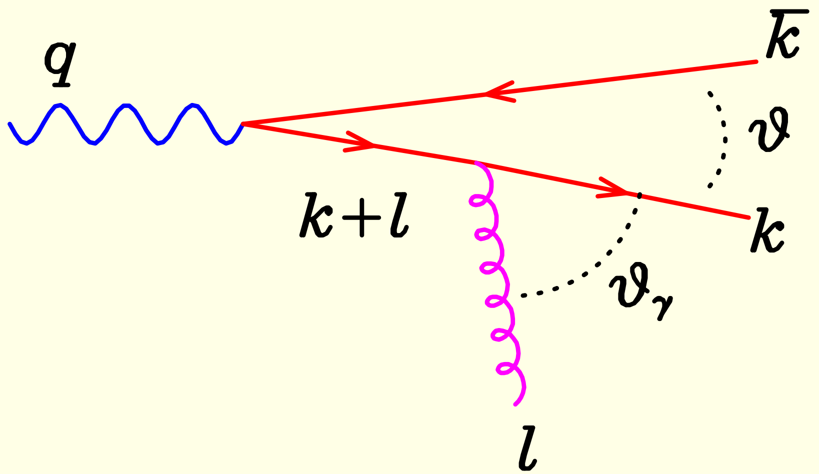
$$
\begin{aligned}
\text{virtuality:} \quad t &= E^2\, z(1-z)\, \theta^2 \\
p_T: \quad t &= E^2\, z^2(1-z)^2\, \theta^2 \\
\text{angle:} \quad t &= E^2\, \theta^2
\end{aligned}
$$



$$
\int \frac{dt}{t} \int_{t/E^2}^{1-t/E^2} \frac{dz}{1-z} \Leftrightarrow \int \frac{dt}{t} \int_{\sqrt{t}/E}^{1-\sqrt{t}/E} \frac{dz}{1-z}
$$

for example: factor of 2 difference in double log!

Soft Emission



Soft emission factor:

$$\frac{k^\mu}{k \cdot l} - \frac{\bar{k}^\mu}{\bar{k} \cdot l}$$

vanishes for $\theta_\gamma \gg \theta$.

Alternatively: energy unbalance

$$\delta E \propto \frac{(k+l)^2}{k^0} = \frac{2k \cdot l}{k^0} \propto \theta_\gamma^2 l^0$$

time for emission: $\delta t = 1/\delta E = \lambda/\theta_\gamma^2$.

In order for the photon to resolve the emitter we must have $b \propto \theta \delta t > \lambda/\theta_\gamma$, but $\theta \delta t \propto (\lambda/\theta_\gamma) \times (\theta/\theta_\gamma)$ so this is possible only if $\theta_\gamma < \theta$.

In summary: soft emission from splitted pair is incoherent for the emission angle below the splitting angle, coherent in the other case.

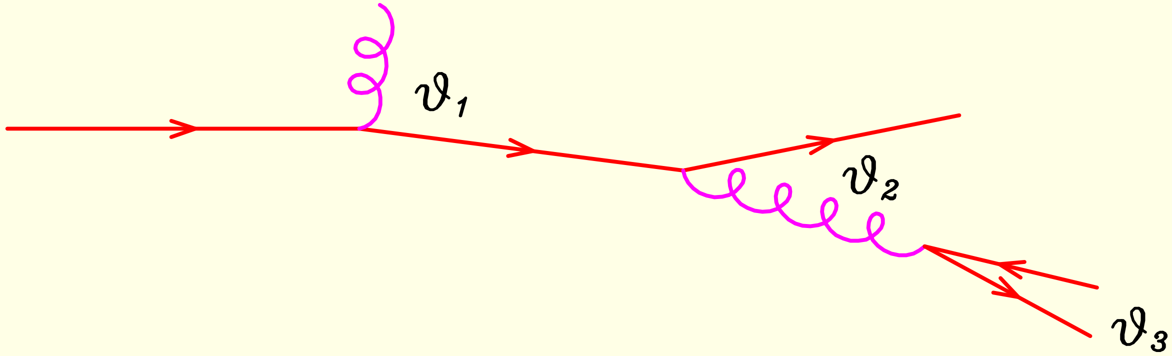
Angular ordering

is the correct choice (Mueller 1981)

$$\frac{d\theta}{\theta}\,\frac{\alpha_S(p_T^2)}{2\pi}\,P(z)\,dz$$

$$\theta_1 > \theta_2 > \theta_3 \ldots$$

$$p_T^2 = E^2\,z^2\,(1-z)^2\,\theta^2$$



Collinear and soft divergence for emission off an off-shell line!

(in fact coherent emission from remaining final state particles)

Important reduction in soft parton multiplicity

(because of the angular constraint)

Notice: $\alpha_S(p_T^2)$ also needed for correct treatment of soft region.

HERWIG's equation

$$\mathbb{S}(t_I, E) = \Delta(t_I)\,\langle\mathbb{I}| + \int_{t_0}^{t_I} \Delta(t_I, t)\, F(z,t)\mathbb{S}(tz^2)\,\mathbb{S}(t(1-z)^2)\; dt\, dz$$

where

$$t = E^2\theta^2\,, \quad \Delta(t_1, t_2) = \frac{\Delta(t_1)}{\Delta(t_2)}\,, \quad F(z,t) = \frac{\alpha_S(tz^2(1-z)^2)}{2\pi}\,\frac{1}{t}\,P(z)$$

and

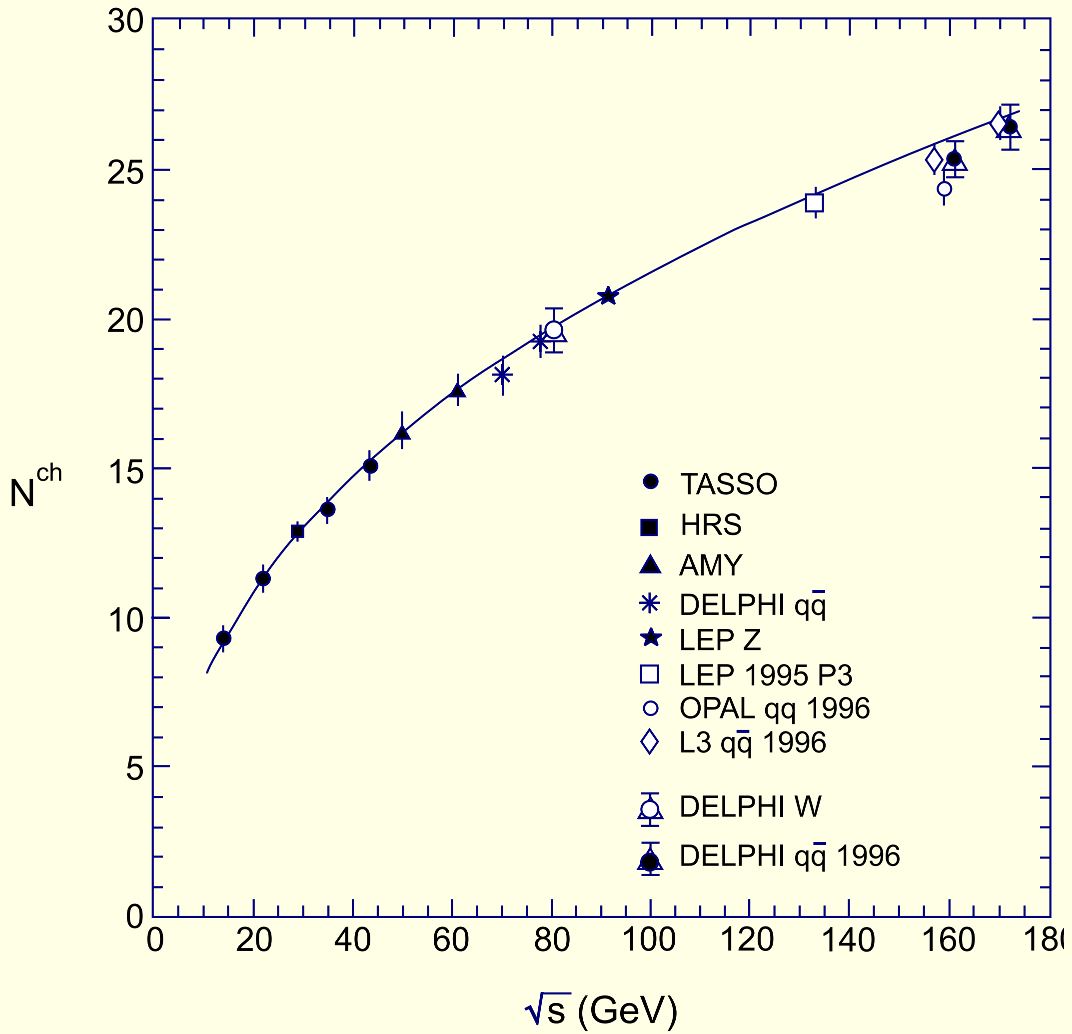
$$\Delta(t) = \exp\left[-\int_0^t \frac{dt'}{t'}\int_0^1 dz\,\frac{\alpha_S(t'z^2(1-z)^2)}{2\pi}\,P(z)\right]$$

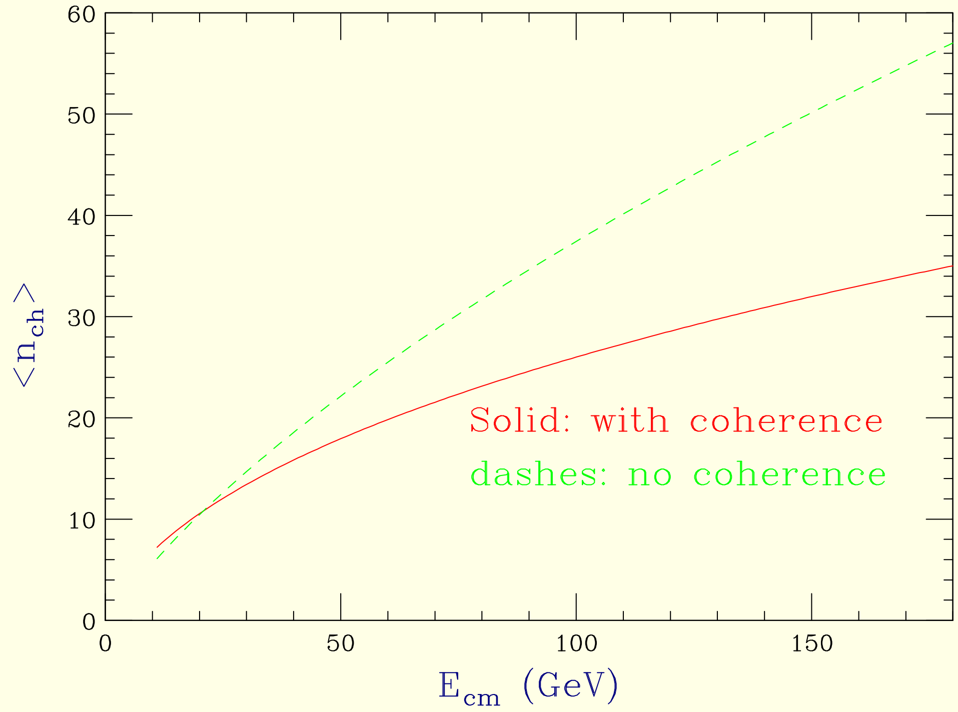
Lower cutoff: implicit $\theta(tz^2(1-z)^2 - t_0)$ sets lower limits for $z, 1-z, t$.

Notice: $\Delta(t) \propto (\alpha_S(t)/\alpha_S(t_0))^{c\log t/\Lambda^2}$ (faster than any power) instead of typical log power behaviour of anomalous dimensions.

Commonly called Sudakov form factors:

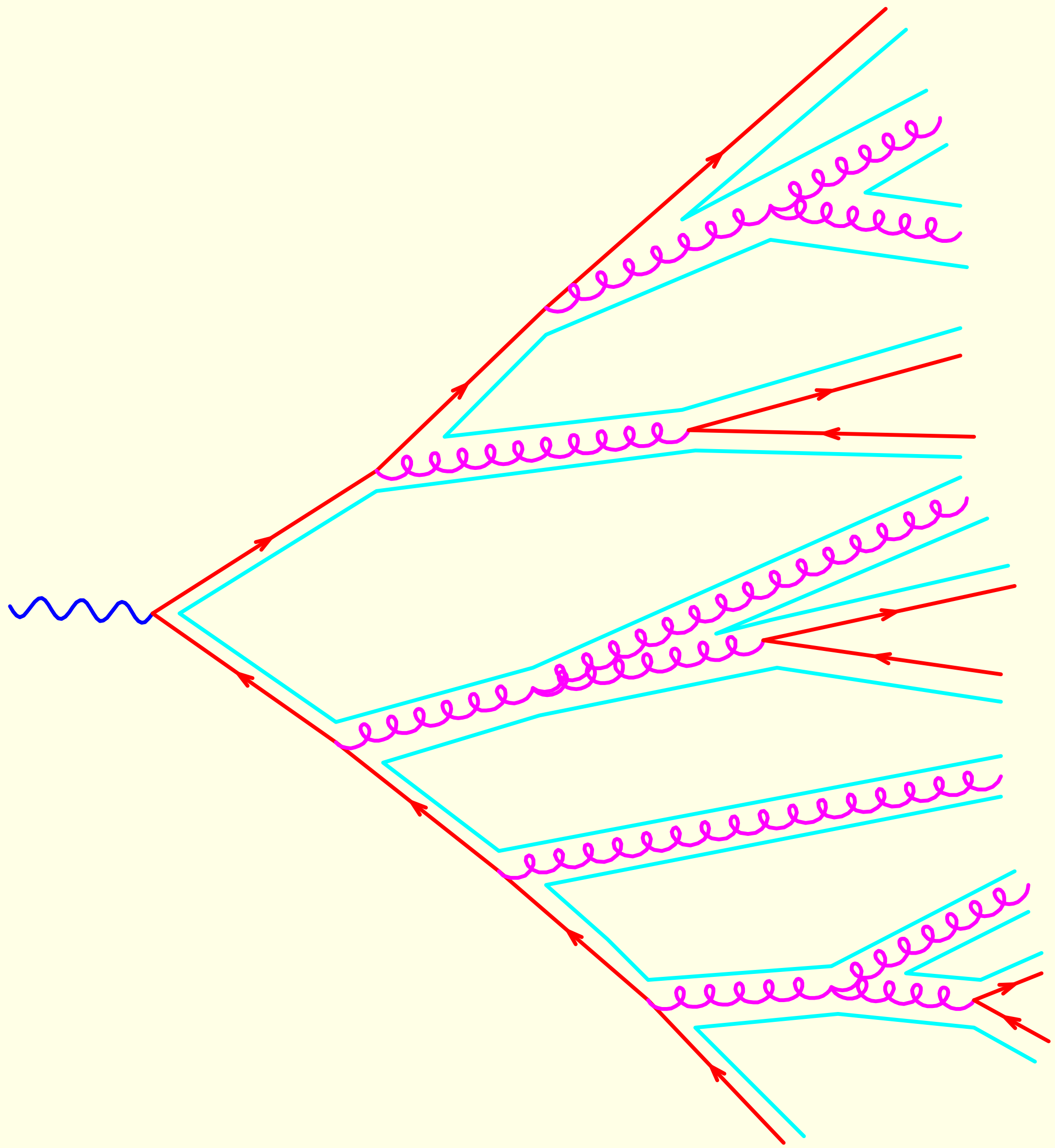
crucial in perturbative preconfinement of colour.





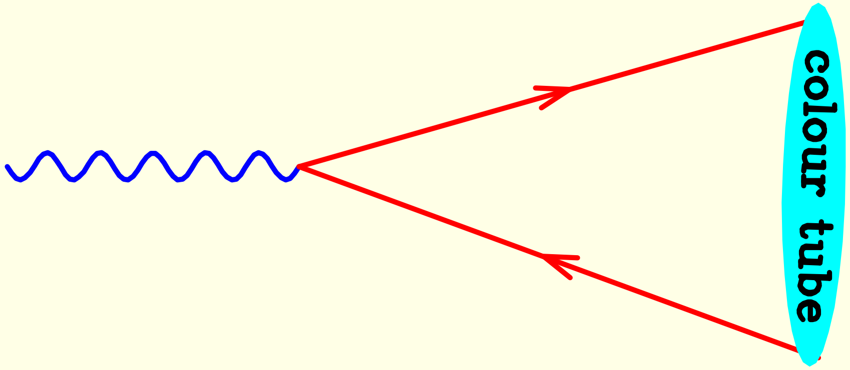
Slope of multiplicity versus energy in $e^+e^-\rightarrow$hadrons
strongly influenced by coherence effects

Soft emission and Hadronization



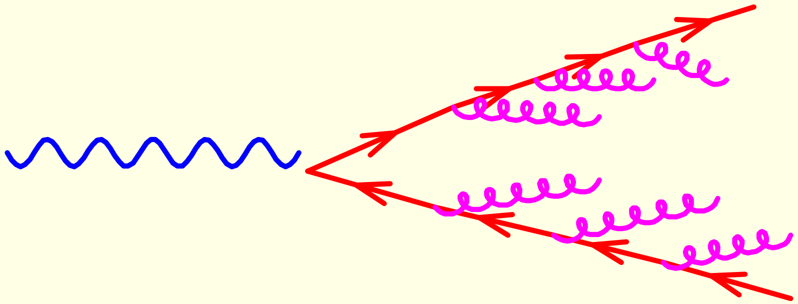
Hadronization according to color connections of final state particles.
In some approximation (large N_c limit) colour flow is described by single colour lines for quarks and double lines for gluons.
Large amount of soft radiation guarantees that colour connected pairs have small mass.
So, perturbation theory justifies a rearrangement of colour connections which is compatible with colour confinement:
(*preconfinement*).

Angular ordering and "String effect"

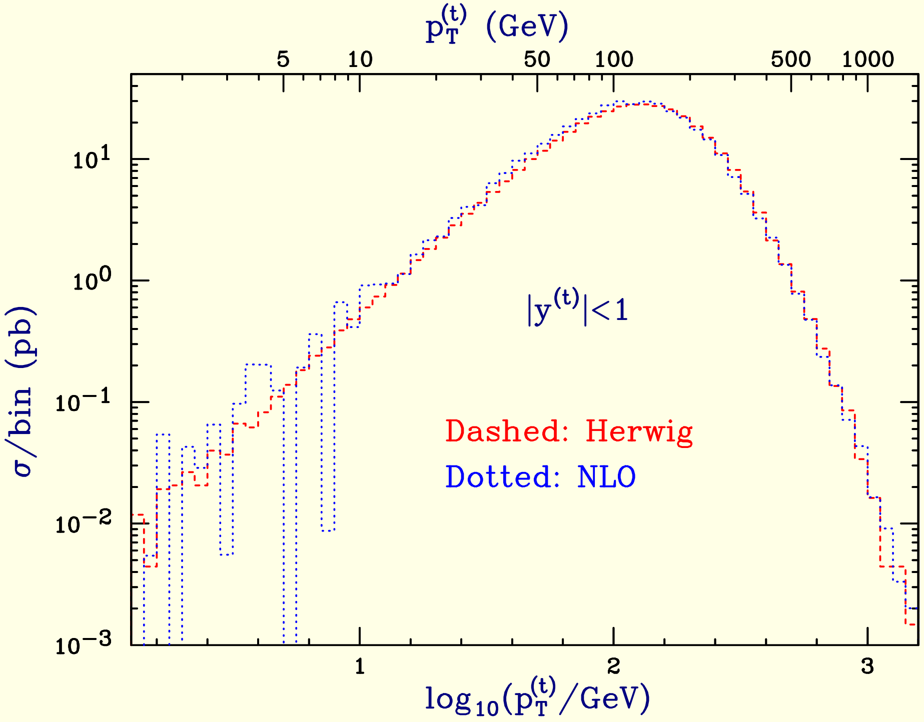


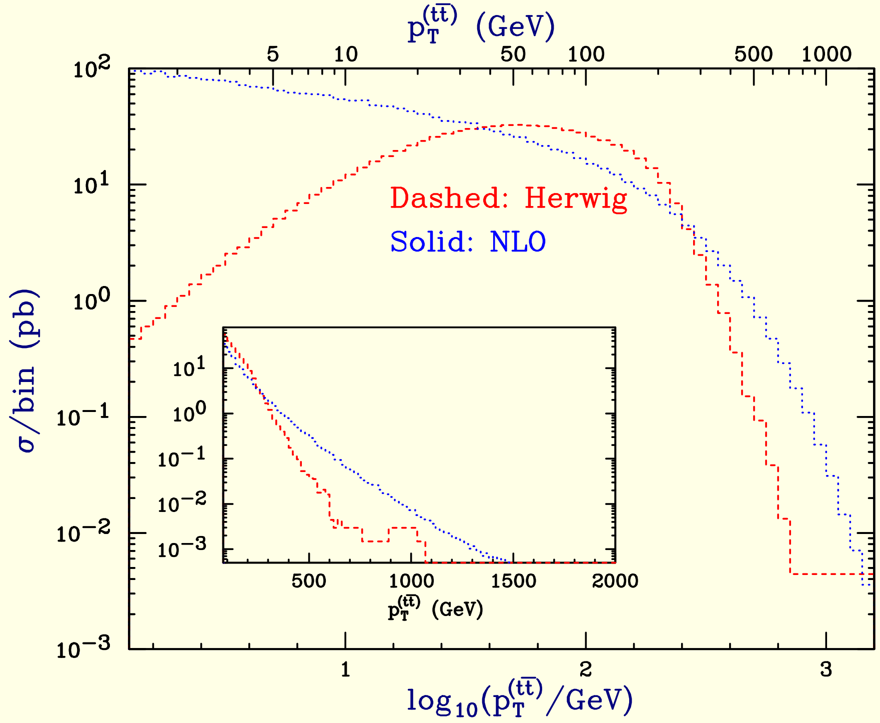
Hadronization may be described by the decay of a coloured string among colour connected pairs.

Angular ordered soft radiation produces a similar effect.



SMC versus NLO





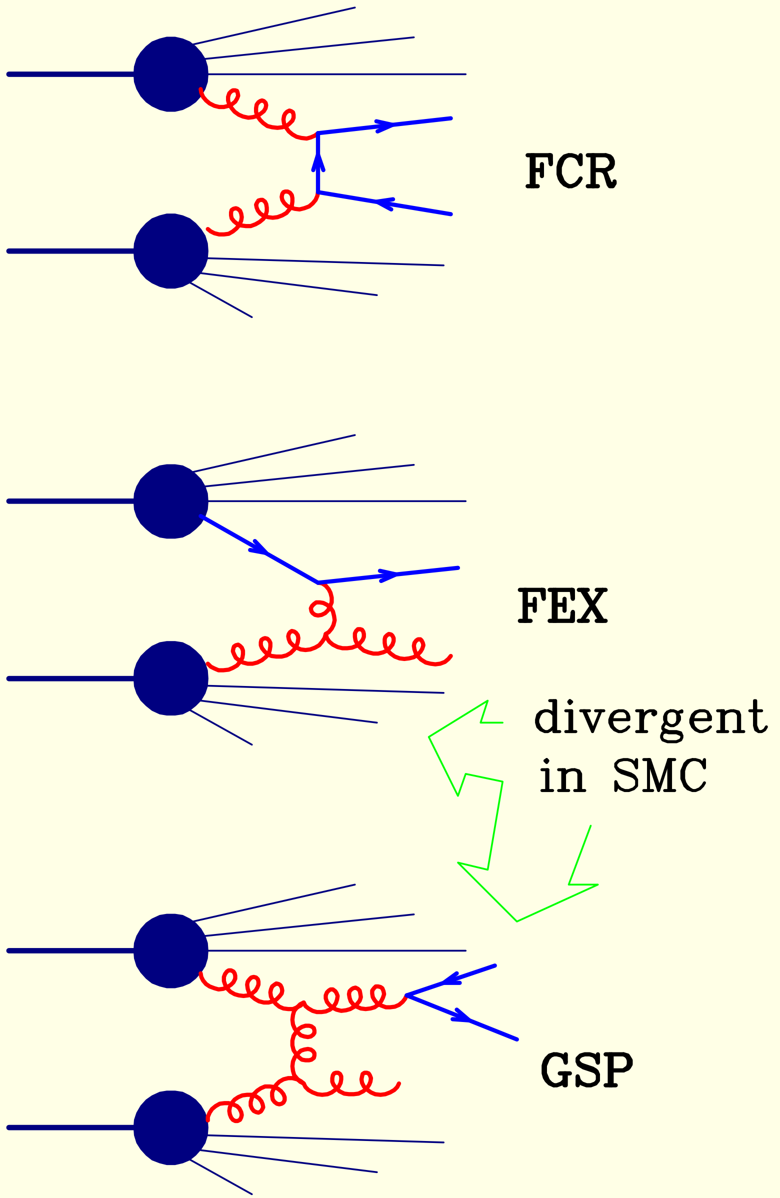
Observables integrated over the singular region are reasonably described by both calculations.

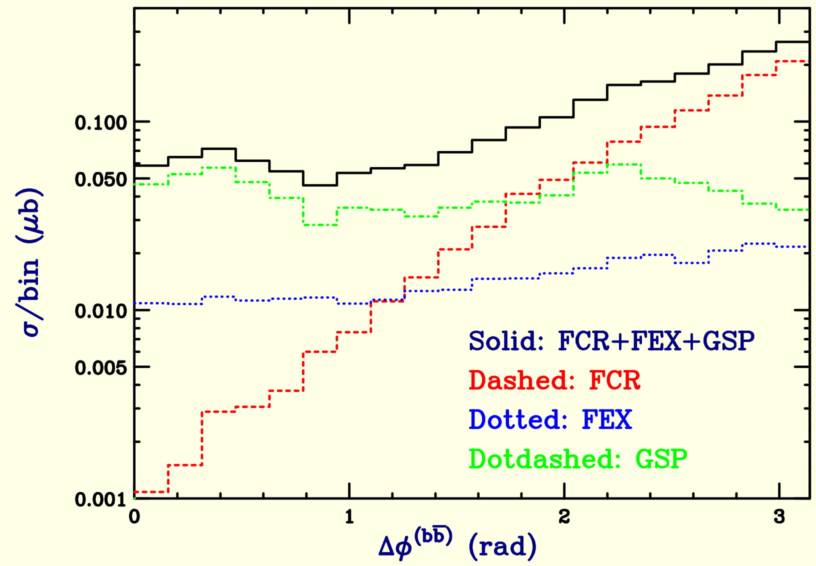
Observables sensitive to the singular region are reasonably described by the SMC near the singular region. NLO fails there.

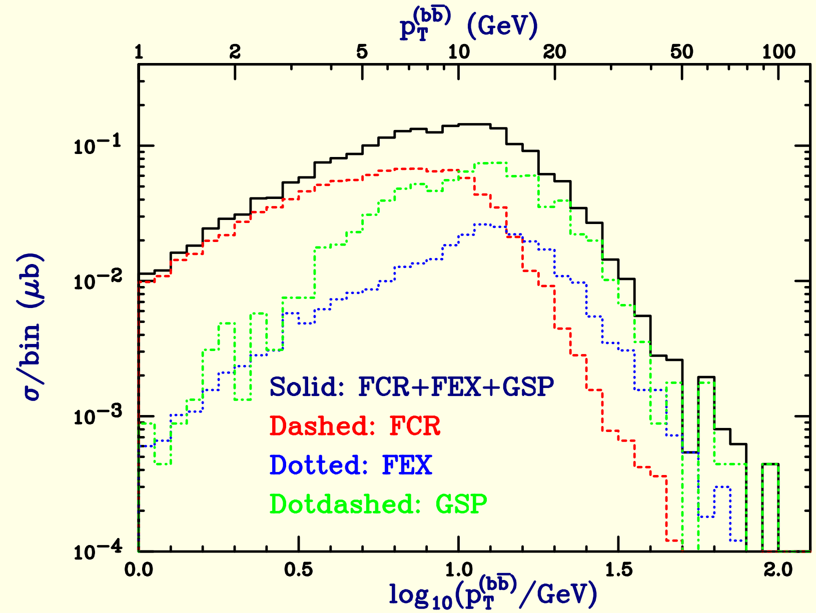
At large $p_T^{t\bar{t}}$ NLO is more reliable.

Limitations

Lack of NLO effects. Example: bottom production.







Cumbersome procedures to generate realistic events. Used since the 80's.

Improvements

Several: angular correlations, small-x resummation, etc.

Improve hardest emission description

Problem: hardest emission not necessarily the first one...

- Exact n body matrix elements (Catani+Krauss+Kuhn+Webber)
- Matrix elements corrections (Corcella+Seymour).
- MC@NLO: hardest emission exact at NLO (Frixione+Webber, 2003).
  MC implements an approximation to NLO corrections.
  Add difference between exact expression and MC approximation
  (If the MC is good the difference is not singular).
  Applications: Drell-Yan, W pairs, Higgs, Heavy flavour (with P.N.)

In FW approach, negative weighted events may occur.
In the following, I also discuss ideas to (possibly) avoid these problems

Strategy:

- Single out the hardest emission (largest p_T) in the shower.
  Formulate the shower in such a way
  that hardest emission is generated first.
- Correct the hardest emission, so that it is accurate at the NLO level.

This is all what you need to get calorimetric quantities correct at the NLO level. The shower developement is still accurate only at the Leading Logaritmic level.

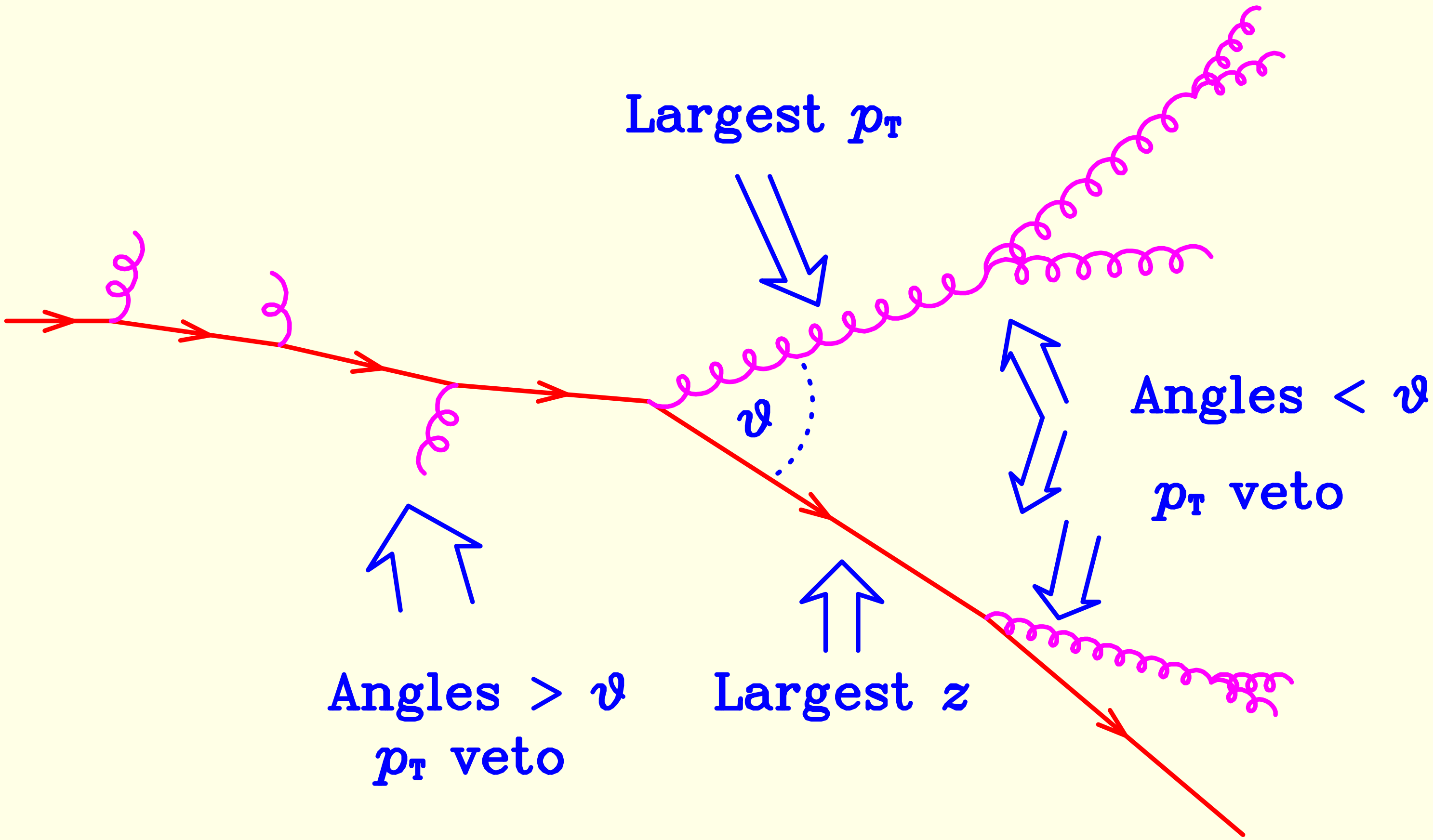
Recipe for hardest emission

- Generate hardest emission using the Sudakov factor

$$\Delta_R(t_I, p_\mathsf{T}) = e^{-\int_0^{t_I} dt \int dz F(t,z)\theta(z(1-z)\sqrt{t}-p_\mathsf{T})}$$

- Along the line from the initial t_I down to the t of the hardest event, generated an angular ordered, p_T limited shower. This means that the Sudakov form factor and the splitting vertices are modified by a $\theta(p_\mathsf{T} - z(1-z)\sqrt{t})$. This shower stops when you reach the angular variable of the hardest emission (truncated shower).
- Along the lines following the hardest emission, continue with an angular ordered, p_T limited shower, down to t_0.

A picture:



Important: no collinear log in truncated shower unless emissions are soft!
Because of ordering and p_T veto

$$\theta < \theta_i\,, \quad E\,\theta_i z_i(1-z_i) < (z_i E)\,\theta\, z(1-z) < \frac{z_i\, E\theta}{4} \Longrightarrow \theta < \theta_i < \frac{\theta}{4(1-z_i)}$$

already have $z > 3/4$, must have $z \to 1$ to have large θ_i range.

NLO correction

Typically implemented as

$$
\begin{aligned}
d\sigma &= B(p_1 \dots p_m) d\Phi_m + V(p_1 \dots p_m) d\Phi_m \\
&+ \left[R(p_1 \dots p_{m+1}) d\Phi_{m+1} - C(p_1 \dots p_{m+1}) d\Phi_{m+1} \mathbb{P}\right]
\end{aligned} \tag{1}
$$

where $\mathbb{P}$ defines a (soft-collinear insensitive) *projection* of the $m+1$ body final state to an m body final state

MC (approximate) NLO corrections

Primary event is generated according to its Born cross section.

MC hardest emission:

$$d\sigma = B(p_1 \dots p_m) d\Phi_m \left[\Delta_R(0) + \Delta_R(p_\mathrm{T}) \sum_{i=1,m} F_i(z,t)\, dz\, dt\, \frac{d\phi}{2\pi} \right]$$

with $\Delta_R(p_\mathrm{T}) = \prod_{i=1,m} \Delta_R^i(p_\mathrm{T})$.

$\mathcal{O}(\alpha_\mathrm{S})$ expansion:

$$d\sigma = B(p_1 \dots p_m) d\Phi_m \left[\left(1 - \int \sum_{i=1,m} F_i(z,t)\, dz\, dt \right) + \sum_{i=1,m} F_i(z,t)\, dz\, dt \right]$$

$$= B(p_1 \dots p_m) d\Phi_m + \left(\sum_{i=1,m} F_i(z,t) - \sum_{i=1,m} F_i(z,t) \mathbb{P}_i \right) d\Phi_m\, dz\, dt$$

where $\mathbb{P}$ applied to $p_1 \dots p_m$ with p_i splitting yields again $p_1 \dots p_m$.

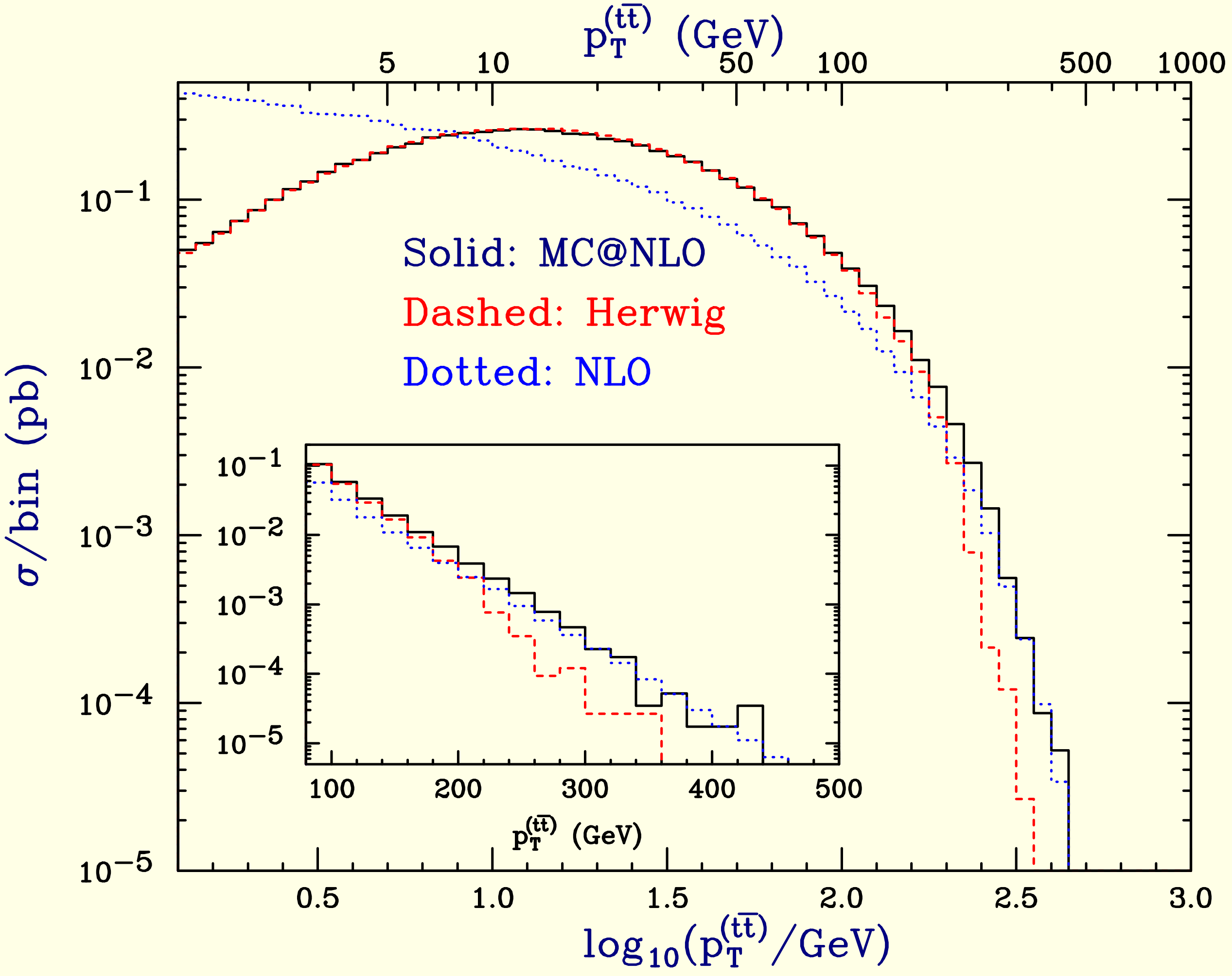
MC@NLO

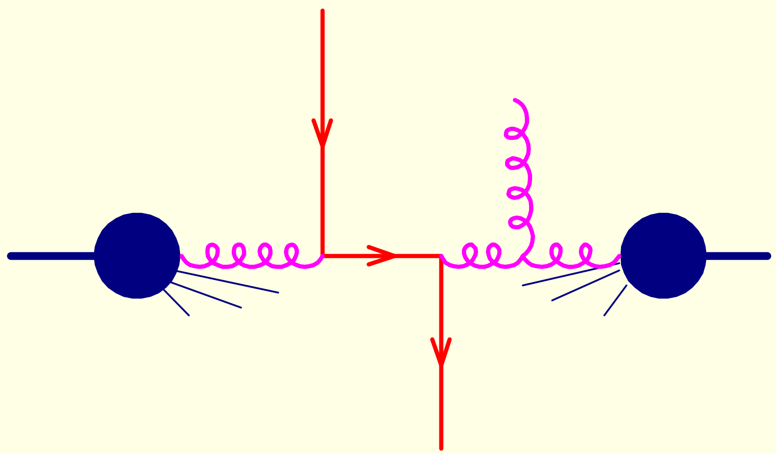
In the FW approach:

- Rewrite the NLO correction using the projection of the MC
- Add to the events generated with the standard MC, also events initiated by the Born term with one emission, weighted with the difference between the exact NLO and its MC approximation.

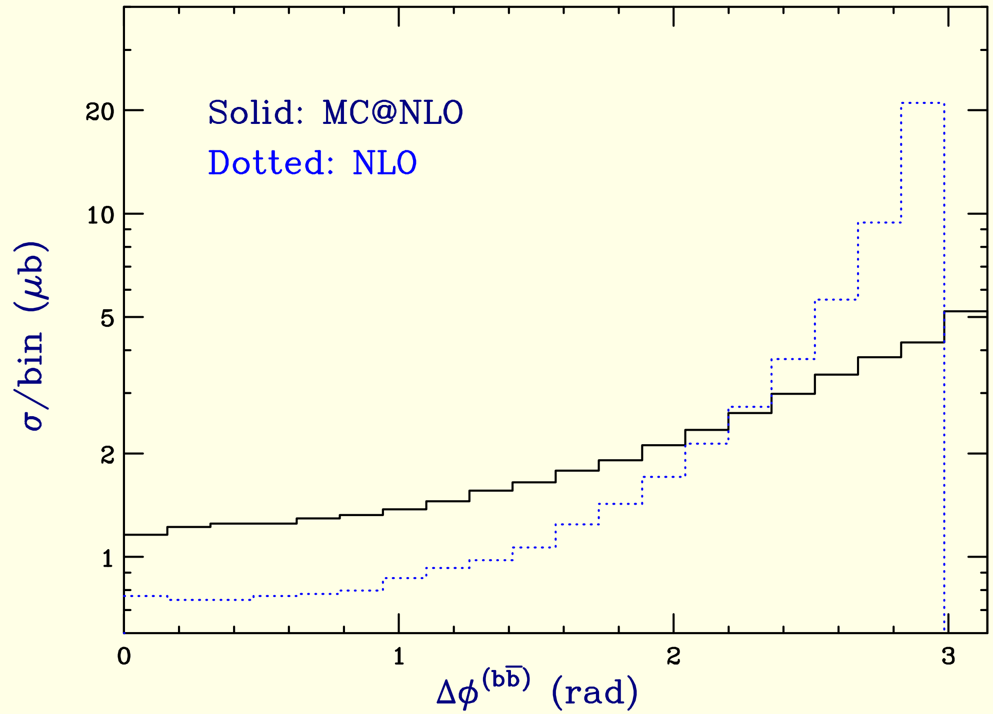
The difference may be negative; thus events with negative weights appear. It is however not singular, if the MC describes exactly the collinear and soft region. One can thus unweight positive and negative weighted events separately, ending up with events weighted with 1 or -1.

MC@NLO

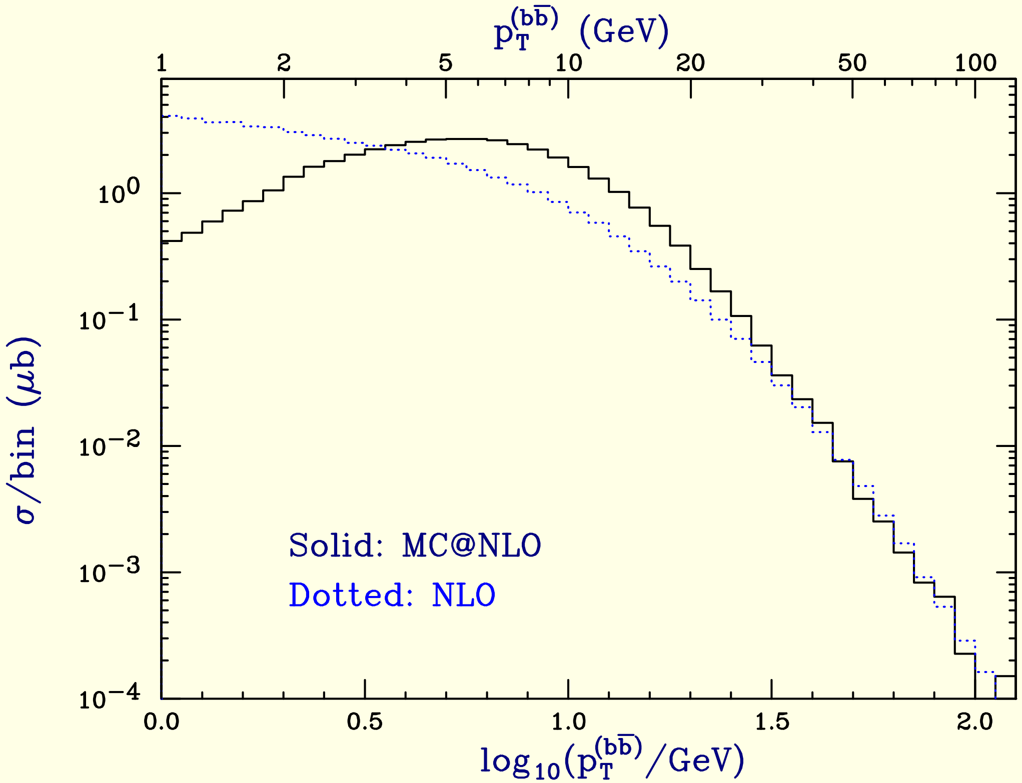




pair p_T wrong at small p_T at NLO,
wrong at large p_T in MC,
right in both regions in MC@NLO.



Solid: MC@NLO
Dotted: NLO
σ/bin (μb)
Δφ^(bb̄) (rad)



p_T^(bb̄) (GeV)
Solid: MC@NLO
Dotted: NLO
σ/bin (μb)
log10(p_T^(bb̄)/GeV)

Alternative MC@NLO

Generate first the hardest event using the exact NLO formula.

Write NLO exact formula as

$$
\begin{aligned}
d\sigma &= B(v_1 \dots v_l) d\Phi_v + V(v_1 \dots v_l) d\Phi_v \\
&+ [R(v_1 \dots v_l, \theta, z, \phi) d\Phi_v d\Phi_e - C(v_1 \dots v_l, \theta, z, \phi) d\Phi_v d\Phi_e \mathbb{P}] \\
&= [V(v_1 \dots v_l) + (R(v_1 \dots v_l, \theta, z, \phi) - C(v_1 \dots v_l, \theta, z, \phi)) \, \mathbb{P} d\Phi_e] \, d\Phi_v \\
&+ B(v_1 \dots v_l) d\Phi_v \left[1 + \left(\frac{R(v_1 \dots v_l, \theta, z, \phi)}{B(v_1 \dots v_l)} - \frac{R(v_1 \dots v_l, \theta, z, \phi)}{B(v_1 \dots v_l)} \mathbb{P} \right) d\Phi_e \right]
\end{aligned}
$$

Turn it into a shower formula

$$
d\sigma = B'(v_1 \dots v_l) d\Phi_v \left[\Delta_R^{(\mathrm{NLO})}(0) + \Delta_R^{(\mathrm{NLO})}(p_\mathrm{T}) \frac{R(v_1 \dots v_l, \theta, z, \phi)}{B(v_1 \dots v_l)} d\Phi_e \right]
$$

$$
B' = B(v_1 \dots v_l) + V(v_1 \dots v_l) + \int (R(v_1 \dots v_l, \theta, z, \phi) - C(v_1 \dots v_l, \theta, z, \phi)) \, d\Phi_e
$$

where we have defined

$$
\Delta_R^{(\mathrm{NLO})}(p_\mathrm{T}) = e^{-\int d\Phi_e \frac{R(v_1 \dots v_l, \theta, z, \phi)}{B(v_1 \dots v_l)} \theta(k_T(v_1 \dots v_l, \theta, z, \phi) - p_\mathrm{T})}
$$

After the hardest event,

- Compute the initial showering angle for each leg
- Shower each leg with the p_T veto
- Perform soft-truncated vetoed shower from nearby pairs of partons.

Advantages

- Generation of hard event independent upon the detailed MC implementation
- If the MC treats the soft-collinear region in an approximate way, no left-over divergences.
- No negative weights
- May be generalized to higher order (e.g. single out two hardest emissions, truncated showers from combination of two or three partons, etc.)

Practical implementation: work in progress Frixione, Gieseke, Webber, P.N.

Conclusions

- Interesting new developments in shower algorithms
- Promising field: many things to study
- Challenging theoretical problems

Single out largest p_T in the shower

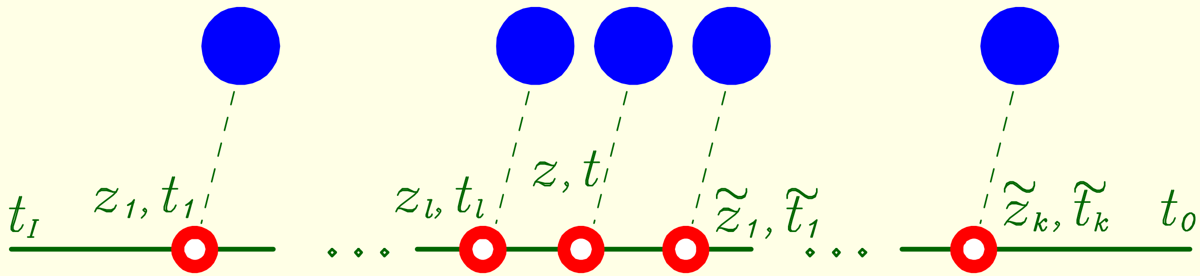
Largest p_T emission always along the hardest line

(actually $z > 3/4$, easy to prove. . .)

Solve HERWIG's equation by iteration along hardest line,

single out largest p_T

$$\mathbb{S}(t_I) = \Delta(t_I)\mathbb{I} + \sum_{l,k=0}^{\infty}$$



.

- Thick lines: $\Delta(t_I, t_1)$, $\Delta(z_1^2 t_1, t_2), \ldots, \Delta(z_l^2 t_l, t)$, $\Delta(z^2 t, \tilde{t}_1), \ldots, \Delta(\tilde{z}_k^2 \tilde{t}_k)$
- Blue blobs: $\mathbb{S}((1-\bar{z})^2 \bar{t})$,
- Red blobs: $2F(\bar{z}, \bar{t})\, \theta(\bar{z} - 1/2)\, \theta(p_\mathrm{T} - \sqrt{\bar{t}}\bar{z}(1-\bar{z}))$ where $p_\mathrm{T} = \sqrt{t}z(1-z)$.
- All intermediate $\bar{z}\bar{t}$ are integrated.

No longer MC equation in present form!

Split Sudakov form factors:

$$\Delta(t_i z_i^2, t_{i+1}) = e^{-\int_{t_{i+1}}^{t_i z_i^2} dt' \int dz' F(t',z')} =$$

$$e^{-\int_{t_{i+1}}^{t_i z_i^2} dt' \int_{1/2}^{1} dz' 2F(t',z')\,\theta(p_{\mathsf{T}} - z'(1-z')\sqrt{t'})} \times e^{-\int_{t_{i+1}}^{t_i z_i^2} dt' \int dz' F(t',z')\theta(z'(1-z')\sqrt{t'} - p_{\mathsf{T}})}$$

First factor matches the z, p_T limited splitting vertex (good for MC!).

Second factor:

$$e^{-\int_{t_{i+1}}^{t_i} dt' \int dz' F(t',z')\theta(z'(1-z')\sqrt{t'} - p_{\mathsf{T}})} \times e^{\int_{t_i z_i^2}^{t_i} dt' \int dz' F(t',z')\theta(z'(1-z')\sqrt{t'} - p_{\mathsf{T}})}$$

$$\approx e^{-\int_{t_{i+1}}^{t_i} dt' \int dz' F(t',z')\theta(z'(1-z')\sqrt{t'} - p_{\mathsf{T}})}$$

Second factor in first line negligible; ($z_i > 1/2$, no collinear log!)

Product of all remnants:

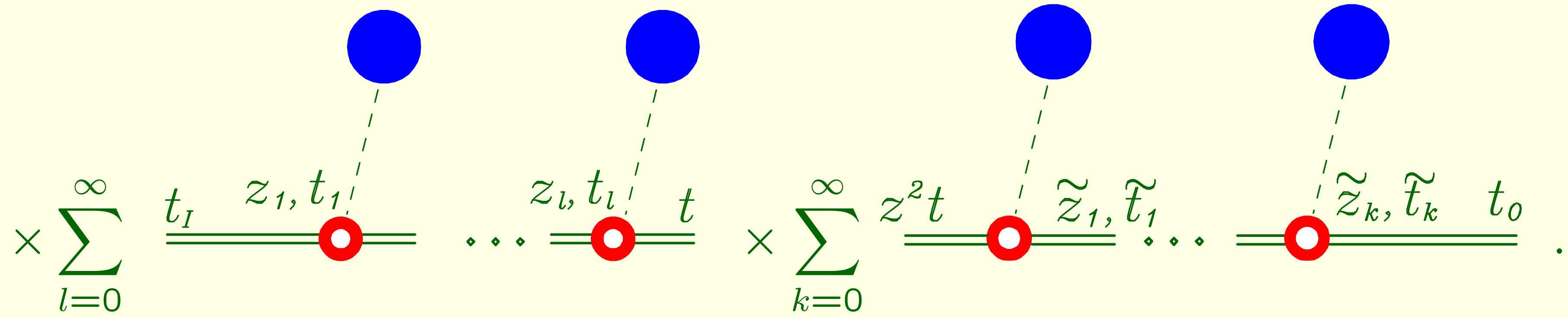
$$\Delta_R(t_I, p_{\mathsf{T}}) = e^{-\int_0^{t_I} dt' \int dz' F(t',z')\theta(z'(1-z')\sqrt{t'} - p_{\mathsf{T}})}$$

probability for not emitting a particle

with transverse momentum larger than p_{T}

Can be used to generate the largest p_{T} event first.

$$\mathbb{S}(t_I) = \Delta(t_I)\mathbb{I} + \theta(t < t_I)dt\,\theta(z > 1/2)dz\,\Delta_R(t_I, p_\top)2F(z,t)\,\mathbb{S}((1-z)^2 t)$$



Double lines: $\Delta_V(t_i, t_{i+1}) = e^{-\int_{t_{i+1}}^{t_i z_i^2} dt' \int_{1/2}^{1} dz' 2F(t',z')\,\theta(p_\top - z'(1-z')\sqrt{t'})}$

Shower equation:

$$\mathbb{S}(t_I) = \Delta(t_I)\mathbb{I} + \theta(t < t_I)dt\,\theta(z > 1/2)dz\,\Delta_R(t_I, p_\top)2F(z,t)\,\mathbb{S}((1-z)^2 t)$$
$$\times \mathbb{S}_{VT}(t_I, t, p_\top)\,\mathbb{S}_V(z^2 t, p_\top)$$

where V stands for vetoed and T for truncated.

Vetoed shower

The following procedure

- generate t' using $\Delta(t, t') = r$
- generate z according to $F(z, t)$
- if $z^2(1-z)^2\, t > p_T$ disregard the splitting, and continue starting from the scale t'

is the same as using $\Delta_R(t, p_\mathsf{T})$ as Sudakov form factor.
So, no complications arise because of the vetoed Sudakov.

proof:

Equation for a vetoed shower:

$$\begin{aligned}\mathbb{S}_V(t_I) &= \Delta(t_I)\,\mathbb{I} + \int_0^{t_I} \Delta(t_I,t)\,F(z,t)\,\theta(g(z,t))\,\mathbb{S}_V(z^2 t)\,\mathbb{S}_V(\,(1-z)^2\,t\,)\,dt\,dz \\ &+ \int_0^{t_I} \Delta(t_I,t)\,F(z,t)\,\left[1-\theta(g(z,t))\right]\mathbb{S}_V(t)\,dt\,dz\,.\end{aligned}$$

Rewrite it as

$$\begin{aligned}&\int_0^{t_I} \left[\delta(t-t_I) - \Delta(t_I,t)h(t)\right]\mathbb{S}_V(t)\,dt \\ &= \Delta(t_I)\,\mathbb{I} + \int_0^{t_I} \Delta(t_I,t)\,F(z,t)\,\theta(g(z,t))\,\mathbb{S}_V(z^2 t)\,\mathbb{S}_V(\,(1-z)^2\,t\,)\,dt\,dz\,,\end{aligned}$$

with

$$h(t) = \int dz\,F(z,t)\,\left[1-\theta(g(z,t))\right]\,.$$

We thus have

$$\mathbb{S}_V(t_I) = \Delta'(t_I)\,\mathbb{I} + \int_0^{t_I} \Delta'(t_I,t)\,F(z,t)\,\theta(g(z,t))\,\mathbb{S}_V(z^2 t)\,\mathbb{S}_V(\,(1-z)^2\,t\,)\,dt\,dz$$

with

$$\int_0^{t_I} \left[\delta(t - t_I) - \Delta(t_I, t)h(t)\right] \Delta'(t) = \Delta(t_I) \; .$$

Using $\Delta(t_I, t) = \Delta(t_I)/\Delta(t)$, and defining $r(t) = \Delta'(t)/\Delta(t)$ we find

$$r(t_I) - \int_0^{t_I} h(t)\, r(t)\, dt = 1 \;\Rightarrow\; r(t) = e^{\int_0^{t_I} h(t)}$$

or

$$\Delta'(t_I) = e^{-\int_0^{t_I} F(z,t)\, \theta(g(z,t))\, dz\, dt} \; .$$

consistent with unitarity