Introduction into Standard Model and Precision Physics – Lecture I –

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General overview

Lecture I – Standard Model (part 1)

1 Electroweak phenomenology before the GSW model

2 The principle of local gauge invariance

3 The Standard Model of electroweak interaction — matter, Yang–Mills, and Higgs sector

Lecture II – Standard Model (part 2)

Lecture III – Quantum Corrections

Lecture IV – Unstable Particles (part 1)

Lecture V – Unstable Particles (part 2)

1 Electroweak phenomenology before the GSW model

Some phenomenological facts:

- discovery of the weak interaction via radioactive β-decay of nuclei:
  $\mathrm{n} \to \mathrm{p} + \mathrm{e}^- + \bar{\nu}_\mathrm{e}$, $\quad$ $\mathrm{p} \to \mathrm{n} + \mathrm{e}^+ + \nu_\mathrm{e}$ (not possible for free protons)
- terminology "weak": interaction at low energy has very short range
  $\hookrightarrow$ long life time of weakly decaying particles:

strong int.:	$\rho \to 2\pi$,	$\tau \sim 10^{-22}$s
elmg. int.:	$\pi \to 2\gamma$,	$\tau \sim 10^{-16}$s
weak int.:	$\pi^- \to \mu^- + \bar{\nu}_\mu$	$\tau \sim 10^{-8}$s
	$\mu^- \to \mathrm{e}^- + \bar{\nu}_\mathrm{e} + \nu_\mu$,	$\tau \sim 10^{-6}$s

- lepton-number conservation: $\quad \mu^- \not\to \mathrm{e}^- + \gamma \quad$ (BR $\lesssim 10^{-11}$)
  $\Rightarrow$ $L_\mathrm{e}, L_\mu, L_\tau$ individually conserved:
  $L_\mathrm{e} = +1$ for $\mathrm{e}^-, \nu_\mathrm{e}$, $\quad$ $L_\mathrm{e} = -1$ for $\mathrm{e}^+, \bar{\nu}_\mathrm{e}$, $\quad$ etc.

  (For massive ν's with different masses, only $L_\mathrm{e}+L_\mu+L_\tau$ is conserved.)
- parity violation (Wu et al. 1957):
  e.g.: $\quad K^+ \to \underbrace{2\pi, 3\pi}_{\text{final states of different parity}}$

  $^{60}\mathrm{Co} \to {}^{60}\mathrm{Ni}^* + \mathrm{e}^- + \bar{\nu}_\mathrm{e}$
  $\hookrightarrow$ polarization inversion does not yield inversion of spectra

The Fermi model

(Fermi 1933, further developed by Feynman, Gell-Mann and others after 1958)

Lagrangian for "current–current interaction" of four fermions:

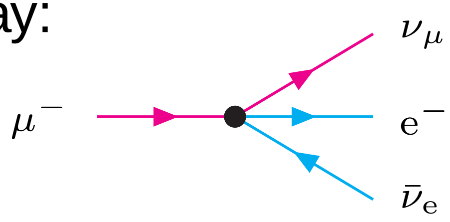
$$\mathcal{L}_{\mathrm{Fermi}}(x) = -2\sqrt{2}G_\mu J_\rho^\dagger(x) J^\rho(x), \qquad G_\mu = 1.16639 \times 10^{-5}\,\mathrm{GeV}^{-2}$$

with $J_\rho(x) = J_\rho^{\mathrm{lep}}(x) + J_\rho^{\mathrm{had}}(x) =$ charged weak current

- Leptonic part J_ρ^{lep} of J_ρ:

$$J_\rho^{\mathrm{lep}} = \overline{\psi_{\nu_\mathrm{e}}}\gamma_\rho\omega_-\psi_\mathrm{e} + \overline{\psi_{\nu_\mu}}\gamma_\rho\omega_-\psi_\mu \qquad \omega_\pm = \tfrac{1}{2}(1\pm\gamma_5) = \text{chirality projectors}$$

  - ◇ only left-handed fermions $(\omega_-\psi)$, right-handed anti-fermions $(\overline{\psi}\omega_+)$ feel (charged-current) weak interactions $\Rightarrow$ maximal P-violation
  - ◇ doublet structure: $\begin{pmatrix}\nu_\mathrm{e}\\ \mathrm{e}^-\end{pmatrix}$, $\begin{pmatrix}\nu_\mu\\ \mu^-\end{pmatrix}$, later completed by $\begin{pmatrix}\nu_\tau\\ \tau^-\end{pmatrix}$
  - ◇ $(J^{\mathrm{lep},\rho})^\dagger J_\rho^{\mathrm{lep}}$ induces muon decay:



- Hadronic part $J_\rho^{\rm had}$ of J_ρ:

  Relevant quarks for energies $\lesssim 1\,\mathrm{GeV}$: $\quad \mathrm{u,d,s,c}$
  $\hookrightarrow$ meson ($q\bar{q}$) and baryon (qqq) spectra

  Question: doublet structure $\begin{pmatrix}\mathrm{u}\\ \mathrm{d}\end{pmatrix}, \begin{pmatrix}\mathrm{c}\\ \mathrm{s}\end{pmatrix}$?

  Problem: e.g. annihilation of $\mathrm{u\bar{s}}$ pair would not be allowed,
  but is observed: $\underbrace{K^+}_{\mathrm{u\bar{s}}\ \text{pair in quark model}} \to \mu^+\nu_\mu$

  Solution (Cabibbo 1963):

  u-c-mixing and d-s-mixing in weak interaction

  $\hookrightarrow$ doublets $\begin{pmatrix}\mathrm{u}\\ \mathrm{d}'\end{pmatrix}, \begin{pmatrix}\mathrm{c}\\ \mathrm{s}'\end{pmatrix}$ with $\begin{pmatrix}\mathrm{d}'\\ \mathrm{s}'\end{pmatrix} = U_\mathrm{C}\begin{pmatrix}\mathrm{d}\\ \mathrm{s}\end{pmatrix}$,

  orthogonal Cabbibo matrix $\quad U_\mathrm{C} = \begin{pmatrix}\cos\theta_\mathrm{C} & \sin\theta_\mathrm{C}\\ -\sin\theta_\mathrm{C} & \cos\theta_\mathrm{C}\end{pmatrix}$,

  empirical result: $\quad \theta_\mathrm{C} \approx 13^\circ$

$$J_\rho^{\rm had} = \overline{\psi_\mathrm{u}}\gamma_\rho\omega_-\psi_{\mathrm{d}'} + \overline{\psi_\mathrm{c}}\gamma_\rho\omega_-\psi_{\mathrm{s}'}$$

Remarks on the Fermi model:

- universal coupling G_μ for all transitions
  ($U_{\mathrm{C}}^\dagger U_{\mathrm{C}} = \mathbf{1}$ is part of universality)
- no (pseudo-)scalar or tensor couplings, such as $(\overline{\psi}\psi)(\overline{\psi}\psi)$, $(\overline{\psi}\psi)(\overline{\psi}\gamma_5\psi)$, etc., necessary to describe low-energy experiments ($E \lesssim 1\,\mathrm{GeV}$)
- Problems:
  - ◇ cross sections for $\nu_\mu \mathrm{e} \to \nu_\mathrm{e}\mu$, etc., grow for energy $E \to \infty$ as E^2

    $\hookrightarrow$ unitarity violation !
  - ◇ no consistent evaluation of higher perturbative orders possible
    (no cancellation of UV divergences)

    $\hookrightarrow$ non-renormalizability !

“Intermediate-vector-boson (IVB) model”

Idea: “resolution” of four-fermion interaction by vector-boson exchange

Lagrangian:

$$\mathcal{L}_{\mathrm{IVB}} = \mathcal{L}_{0,\mathrm{ferm}} + \mathcal{L}_{0,\mathrm{W}} + \mathcal{L}_{\mathrm{int}},$$

$$\mathcal{L}_{0,\mathrm{ferm}} = \overline{\psi_f}(\mathrm{i}\partial\!\!\!/ - m_f)\psi_f, \qquad \text{(summation over } f \text{ assumed)}$$

$$\mathcal{L}_{0,\mathrm{W}} = -\frac{1}{2}(\partial_\mu W_\nu^+ - \partial_\nu W_\mu^+)(\partial^\mu W^{-,\nu} - \partial^\nu W^{-,\mu}) + M_\mathrm{W}^2 W_\mu^+ W^{-,\mu},$$

$$\text{with } W_\mu^\pm = \frac{1}{\sqrt{2}}(W_\mu^1 \mp \mathrm{i}W_\mu^2), \quad W_\mu^i \text{ real}$$

$\mathrm{W}^\pm$ are vector bosons with electric charge $\pm e$ and mass M_W.

Propagator:

$$G_{\mu\nu}^{\mathrm{WW}}(k) = \frac{-\mathrm{i}}{k^2 - M_\mathrm{W}^2}\left(g_{\mu\nu} - \frac{k_\mu k_\nu}{M_\mathrm{W}^2}\right), \qquad k = \text{momentum}$$

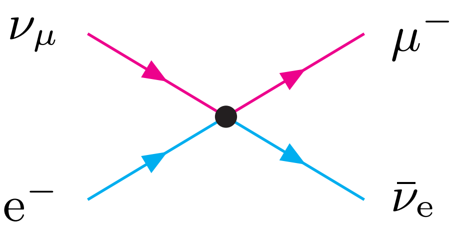
Interaction Lagrangian:

$$\mathcal{L}_{\mathrm{int}} = \frac{g_\mathrm{W}}{\sqrt{2}}\left(J^\rho W_\rho^+ + J^{\rho\dagger} W_\rho^-\right),$$

J^ρ = charged weak current as in Fermi model

Four-fermion interaction in process $\nu_\mu \mathrm{e}^- \to \mu^- \nu_\mathrm{e}$

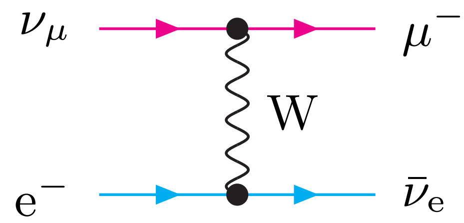
Fermi model:



$$-\mathrm{i}2\sqrt{2}G_\mu \, g_{\rho\sigma}$$

$$\times \left[\bar{u}_{\mu^-}\gamma^\rho\omega_- u_{\nu_\mu}\right] \left[\bar{u}_{\nu_\mathrm{e}}\gamma^\sigma\omega_- u_{\mathrm{e}^-}\right]$$

IVB model:



$$\tfrac{\mathrm{i}}{2} g_\mathrm{W}^2 \frac{1}{k^2 - M_\mathrm{W}^2} \left(g_{\rho\sigma} - \frac{k_\rho k_\sigma}{M_\mathrm{W}^2} \right)$$

$$\times \left[\bar{u}_{\mu^-}\gamma^\rho\omega_- u_{\nu_\mu}\right] \left[\bar{u}_{\nu_\mathrm{e}}\gamma^\sigma\omega_- u_{\mathrm{e}^-}\right]$$

$\Rightarrow$ identification for $|k| \ll M_\mathrm{W}$: $\quad 2\sqrt{2}G_\mu = \dfrac{g_\mathrm{W}^2}{2M_\mathrm{W}^2}$

Consequences for the high-energy behaviour:

- k^ρ terms: $\quad \bar{u}_{\nu_\mathrm{e}} \not{k} \omega_- u_{\mathrm{e}^-} = \bar{u}_{\nu_\mathrm{e}} (\not{p}_\mathrm{e} - \not{p}_{\nu_\mathrm{e}}) \omega_- u_{\mathrm{e}^-} = m_\mathrm{e} \bar{u}_{\nu_\mathrm{e}} \omega_- u_{\mathrm{e}^-}$
  $\hookrightarrow$ no extra factors of scattering energy E
- propagator $1/(k^2 - M_\mathrm{W}^2) \;\sim\; 1/E^2$ for $|k| \sim E \gg M_\mathrm{W}$
  $\hookrightarrow$ damping of amplitude in high-energy limit by factor $1/E^2$

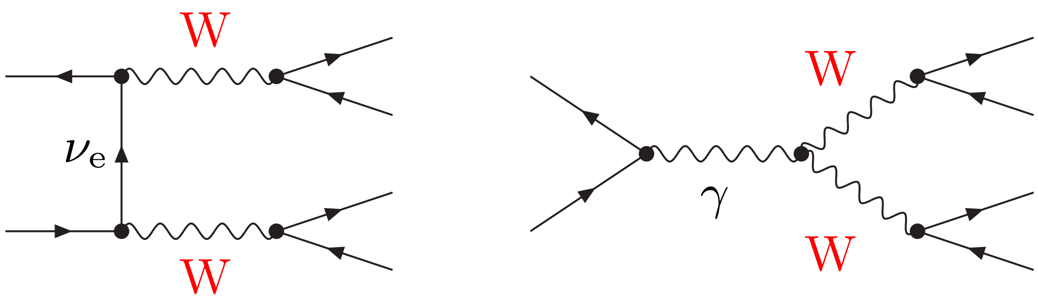
$\Rightarrow$ cross section $\underset{E\to\infty}{\sim}$ const$/E^2$, $\qquad \Rightarrow$ No unitarity violation !

Comments on the IVB model:

- Formal similiarity with QED interaction: $J^\rho W_\rho^+$ + h.c. $\quad\longleftrightarrow\quad j^\rho_{\mathrm{elmg.}} A_\rho$
- Intermediate vector bosons can be produced, e.g.

$$\underbrace{\mathrm{u}\bar{\mathrm{d}}}_{\text{in pp collision}} \longrightarrow \underbrace{\mathrm{W}^+ \to f\bar{f}'}_{\mathrm{W}^\pm \text{ unstable}}$$ (discovery 1983 at CERN)

- Problems:
  - ◇ unitarity violations in cross sections with longitudinal W bosons, e.g.



  - ◇ non-renormalizability
    (no consistent treatment of higher perturbative orders)

$\hookrightarrow$ Solution by spontaneously broken gauge theories !

2 The principle of local gauge invariance

QED as U(1) gauge theory:

Lagrangian $\mathcal{L}_{0,\mathrm{ferm}} = \overline{\psi_f}(\mathrm{i}\partial\!\!\!/ - m_f)\psi_f$ has global phase symmetry:

$$\psi_f \to \psi'_f = \exp\{-\mathrm{i}Q_f e\theta\}\psi_f, \quad \overline{\psi_f} \to \overline{\psi'_f} = \overline{\psi_f}\exp\{+\mathrm{i}Q_f e\theta\}$$

with space-time-independent group parameter θ

“Gauging the symmetry”: demand local symmetry, $\theta \to \theta(x)$

To maintain local symmetry, extend theory by “minimal substitution”:

$\partial^\mu \to D^\mu = \partial^\mu + \mathrm{i}Q_f e A^\mu(x)$ = “covariant derivative”,

$A^\mu(x)$ = spin-1 gauge field (photon).

Transformation property of photon $A_\mu(x) \to A'_\mu(x) = A_\mu(x) + \partial_\mu\theta(x)$ ensures

- $D_\mu\psi_f \to (D_\mu\psi_f)' = D'_\mu\psi'_f = \exp\{-\mathrm{i}Q_f e\theta\}(D_\mu\psi_f)$
- gauge invariance of field-strength tensor $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$

Gauge-invariant Lagrangian of QED:

$$\mathcal{L}_{\mathrm{QED}} = \overline{\psi_f}(\mathrm{i}\partial\!\!\!/ - Q_f e A\!\!\!/ - m_f)\psi_f - \frac{1}{4}F_{\mu\nu}F^{\mu\nu}$$

Non-Abelian gauge theory (Yang–Mills theory):

Starting point:

Lagrangian $\mathcal{L}_\Phi(\Phi, \partial_\mu \Phi)$ of free or self-interacting fields with “internal symmetry”:

- $\Phi = \begin{pmatrix} \phi_1 \\ \vdots \\ \phi_n \end{pmatrix}$ = multiplet of a compact Lie group G:

  $\Phi \to \Phi' = U(\theta)\Phi, \quad U(\theta) = \exp\{-\mathrm{i}gT^a\theta^a\}$ = unitary,

  T^a = group generators, $\quad [T^a, T^b] = \mathrm{i}C^{abc}T^c, \quad \mathrm{Tr}\left\{T^aT^b\right\} = \frac{1}{2}\delta^{ab}$

- $\mathcal{L}_\Phi$ is invariant under G: $\quad \mathcal{L}_\Phi(\Phi, \partial_\mu\Phi) = \mathcal{L}_\Phi(\Phi', \partial_\mu\Phi')$

Example: self-interacting (complex) boson multiplet

$$\mathcal{L}_\Phi = (\partial_\mu\Phi)^\dagger(\partial^\mu\Phi) - m^2\Phi^\dagger\Phi + \lambda(\Phi^\dagger\Phi)^2$$

(m = common boson mass, λ = coupling strength)

Gauging the symmetry by minimal substitution:

$$\mathcal{L}_\Phi(\Phi, \partial_\mu\Phi) \to \mathcal{L}_\Phi(\Phi, D_\mu\Phi) \quad \text{with} \quad D_\mu = \partial_\mu + \mathrm{i}gT^aA^a_\mu(x),$$

g = gauge coupling, T^a = generator of G in Φ representation, $A^a_\mu(x)$ = gauge fields

Transformation property of gauge fields:

- $\mathcal{L}_\Phi(\Phi, D_\mu\Phi)$ local invariant if $\quad D_\mu\Phi \;\to\; (D_\mu\Phi)' = D'_\mu\Phi' = U(\theta)(D_\mu\Phi)$

  $\Rightarrow\; T^a A'^a_\mu \;=\; UT^aA^a_\mu U^\dagger - \frac{\mathrm{i}}{g}U(\partial_\mu U^\dagger),\qquad A^a_\mu A^{a,\mu}$ = not gauge invariant

  infinitesimal form: $\quad \delta A^a_\mu = gC^{abc}\delta\theta^b A^c_\mu + \partial_\mu\delta\theta^a$

- covariant definition of field strength: $\quad [D_\mu, D_\nu] = \mathrm{i}gT^aF^a_{\mu\nu}$

  $\Rightarrow\; T^aF^a_{\mu\nu} \;\to\; T^aF'^a_{\mu\nu} = UT^aF^a_{\mu\nu}U^\dagger,\qquad F^a_{\mu\nu}F^{a,\mu\nu}$ = gauge invariant

  explicit form: $\quad F^a_{\mu\nu} = \partial_\mu A^a_\nu - \partial_\nu A^a_\mu - gC^{abc}A^b_\mu A^c_\nu$

Yang–Mills Lagrangian for gauge and matter fields:

$$\mathcal{L}_{\mathrm{YM}} = -\frac{1}{4}F^a_{\mu\nu}F^{a,\mu\nu} + \mathcal{L}_\Phi(\Phi, D_\mu\Phi)$$

- Lagrangian contains terms of order $(\partial A)A^2$, A^4 in F^2 part

  $\hookrightarrow$ cubic and quartic gauge-boson self-interactions

- gauge coupling determines gauge-boson–matter and gauge-boson self-interaction $\to$ unification of interactions

- mass term $M^2(A^a_\mu A^{a,\mu})$ for gauge bosons forbidden by gauge invariance

  $\hookrightarrow$ gauge bosons of unbroken Yang–Mills theory are massless

Quantum chromodynamics — gauge theory of strong interactions

- **Gauge group:** SU(3)$_c$, dim. $= 8$

  structure constants f^{abc}, gauge coupling g_s, $\alpha_\mathrm{s} = \dfrac{g_\mathrm{s}^2}{4\pi}$

- **Gauge bosons:** 8 massless gluons g with fields $A_\mu^a(x)$, $a = 1, \dots, 8$

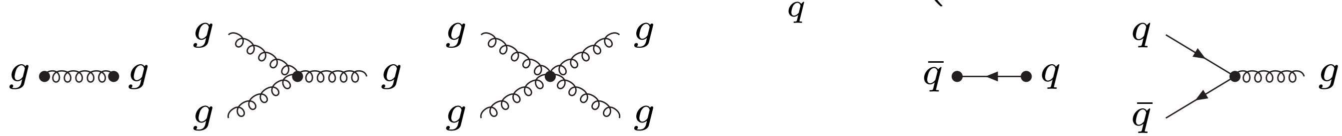
- **Matter fermions:** quarks q (spin-$\frac{1}{2}$) with flavours $q = \mathrm{d}, \mathrm{u}, \mathrm{s}, \mathrm{c}, \mathrm{b}, \mathrm{t}$
  in fundamental representation:

$$\psi_q(x) \;\equiv\; q(x) \;=\; \begin{pmatrix} q_r(x) \\ q_g(x) \\ q_b(x) \end{pmatrix} \;=\; \text{colour triplet}$$

$$T^a = \frac{\lambda^a}{2}, \quad \text{Gell-Mann matrices } \lambda^1 = \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \text{ etc.}$$

- **Lagrangian:**

$$\begin{aligned} \mathcal{L}_\mathrm{QCD} &= -\frac{1}{4} F^a_{\mu\nu} F^{a,\mu\nu} + \sum_q \overline{\psi}_q (\mathrm{i}\not{D} - m_q)\psi_q \\ &= -\frac{1}{4}\left(\partial_\mu A^a_\nu - \partial_\nu A^a_\mu - g_\mathrm{s} f^{abc} A^b_\mu A^c_\nu\right)^2 + \sum_q \overline{\psi}_q \left(\mathrm{i}\not{\partial} - g_\mathrm{s} \frac{\lambda^a}{2} \not{A}^a - m_q\right)\psi_q \end{aligned}$$



3 The Standard Model of electroweak interaction (Glashow–Salam–Weinberg model)

— matter, Yang–Mills, and Higgs sector

3.1 The gauge group for electroweak interaction

Why unification of weak and elmg. interaction ?

- similiarity: spin-1 fields couple to matter currents formed by spin-$\frac{1}{2}$ fields
- elmg. coupling of charged $\mathrm{W}^\pm$ bosons

$\gamma, \mathrm{W}^+, \mathrm{W}^-$ as gauge bosons of group SU(2) ? – No!

Reason: charge operator Q cannot be SU(2) generator, since Tr $\{Q\} \neq 0$

for fermion doublets: $Q = \begin{pmatrix} 0 & 0 \\ 0 & -1 \end{pmatrix}$ for $\begin{pmatrix} \nu_\mathrm{e} \\ \mathrm{e}^- \end{pmatrix}$, etc.

Possible way out: additional heavy fermions like E^+ as partner to e^- ?
$\hookrightarrow$ no experimental confirmation !

Minimal solution: $\mathrm{SU(2)_I} \times \mathrm{U(1)_Y}$

- $\mathrm{SU(2)_I}$ $\rightarrow$ weak isospin group with gauge bosons $\mathrm{W^+, W^-, W^0}$
- $\mathrm{U(1)_Y}$ $\rightarrow$ weak hypercharge with gauge boson B

$\mathrm{W^0}$ and B carry identical quantum numbers

$\hookrightarrow$ two neutral gauge bosons γ, Z as mixed states

Experiment: 1973 discovery of neutral weak currents at CERN

$\hookrightarrow$ indirect confirmation of Z exchange

1983 discovery of $\mathrm{W^\pm}$ and Z bosons at CERN

3.2 Fermion sector and minimal substitution

Multiplet structure:

Distinguish between left-/right-handed parts of fermions: $\psi^{\mathrm{L}} = \omega_-\psi$, $\psi^{\mathrm{R}} = \omega_+\psi$

- ψ^{L} couple to $\mathrm{W}^\pm$ $\quad\rightarrow$ group ψ^{L} into SU(2)$_{\mathrm{I}}$ doublets, weak isospin $T^a_{\mathrm{I}} = \frac{\sigma^a}{2}$
- ψ^{R} do not couple to $\mathrm{W}^\pm$ $\quad\rightarrow$ ψ^{R} are SU(2)$_{\mathrm{I}}$ singlets, weak isospin $T^a_{\mathrm{I}} = 0$
- $\psi^{\mathrm{L/R}}$ couple to γ in the same way

  $\hookrightarrow$ adjust coupling to U(1)$_{\mathrm{Y}}$ (i.e. fix weak hypercharges $Y^{\mathrm{L/R}}$ for $\psi^{\mathrm{L/R}}$) such that elmg. coupling results: $\mathcal{L}_{\mathrm{int,QED}} = -Q_f e \overline{\psi_f} \not{A} \psi_f$

Fermion content of the SM:
(ignoring possible right-handed neutrinos)

$$
\begin{array}{llcccccc}
 & & & & & & T^3_{\mathrm{I}} & Q \\
\text{leptons:} & \Psi^{\mathrm{L}}_L = & \begin{pmatrix} \nu^{\mathrm{L}}_{\mathrm{e}} \\ \mathrm{e}^{\mathrm{L}} \end{pmatrix}, & \begin{pmatrix} \nu^{\mathrm{L}}_{\mu} \\ \mu^{\mathrm{L}} \end{pmatrix}, & \begin{pmatrix} \nu^{\mathrm{L}}_{\tau} \\ \tau^{\mathrm{L}} \end{pmatrix}, & & \begin{matrix} +\frac{1}{2} \\ -\frac{1}{2} \end{matrix} & \begin{matrix} 0 \\ -1 \end{matrix} \\
 & \psi^{\mathrm{R}}_l = & \mathrm{e}^{\mathrm{R}}, & \mu^{\mathrm{R}}, & \tau^{\mathrm{R}}, & & 0 & -1 \\
\text{quarks:} & \Psi^{\mathrm{L}}_Q = & \begin{pmatrix} \mathrm{u}^{\mathrm{L}} \\ \mathrm{d}^{\mathrm{L}} \end{pmatrix}, & \begin{pmatrix} \mathrm{c}^{\mathrm{L}} \\ \mathrm{s}^{\mathrm{L}} \end{pmatrix}, & \begin{pmatrix} \mathrm{t}^{\mathrm{L}} \\ \mathrm{b}^{\mathrm{L}} \end{pmatrix}, & & \begin{matrix} +\frac{1}{2} \\ -\frac{1}{2} \end{matrix} & \begin{matrix} +\frac{2}{3} \\ -\frac{1}{3} \end{matrix} \\
 & \psi^{\mathrm{R}}_u = & \mathrm{u}^{\mathrm{R}}, & \mathrm{c}^{\mathrm{R}}, & \mathrm{t}^{\mathrm{R}}, & & 0 & +\frac{2}{3} \\
 & \psi^{\mathrm{R}}_d = & \mathrm{d}^{\mathrm{R}}, & \mathrm{s}^{\mathrm{R}}, & \mathrm{b}^{\mathrm{R}}, & & 0 & -\frac{1}{3}
\end{array}
$$

(Each quark exists in 3 colours!)

Free Lagrangian of (still massless) fermions:

$$\mathcal{L}_{0,\text{ferm}} = \mathrm{i}\overline{\psi_f}\not\partial\psi_f = \mathrm{i}\overline{\Psi_L^{\mathrm{L}}}\not\partial\Psi_L^{\mathrm{L}} + \mathrm{i}\overline{\Psi_Q^{\mathrm{L}}}\not\partial\Psi_Q^{\mathrm{L}} + \mathrm{i}\overline{\psi_l^{\mathrm{R}}}\not\partial\psi_l^{\mathrm{R}} + \mathrm{i}\overline{\psi_u^{\mathrm{R}}}\not\partial\psi_u^{\mathrm{R}} + \mathrm{i}\overline{\psi_d^{\mathrm{R}}}\not\partial\psi_d^{\mathrm{R}}$$

Minimal substitution:

$$\partial_\mu \to D_\mu = \partial_\mu - \mathrm{i}g_2 T_{\mathrm{I}}^a W_\mu^a + \mathrm{i}g_1 \tfrac{1}{2} Y B_\mu = D_\mu^{\mathrm{L}}\omega_- + D_\mu^{\mathrm{R}}\omega_+,$$

$$D_\mu^{\mathrm{L}} = \partial_\mu - \frac{\mathrm{i}g_2}{\sqrt{2}}\begin{pmatrix} 0 & W_\mu^+ \\ W_\mu^- & 0 \end{pmatrix} - \frac{\mathrm{i}}{2}\begin{pmatrix} g_2 W_\mu^3 - g_1 Y^{\mathrm{L}} B_\mu & 0 \\ 0 & -g_2 W_\mu^3 - g_1 Y^{\mathrm{L}} B_\mu \end{pmatrix},$$

$$D_\mu^{\mathrm{R}} = \partial_\mu + \mathrm{i}g_1 \tfrac{1}{2} Y^{\mathrm{R}} B_\mu$$

Photon identification:

"Weinberg rotation": $\begin{pmatrix} Z_\mu \\ A_\mu \end{pmatrix} = \begin{pmatrix} c_{\mathrm{W}} & s_{\mathrm{W}} \\ -s_{\mathrm{W}} & c_{\mathrm{W}} \end{pmatrix}\begin{pmatrix} W_\mu^3 \\ B_\mu \end{pmatrix}$, $\quad c_{\mathrm{W}} = \cos\theta_{\mathrm{W}},\ s_{\mathrm{W}} = \sin\theta_{\mathrm{W}}$,

θ_{W} = weak mixing angle

$$D_\mu^{\mathrm{L}}\Big|_{A_\mu} = -\frac{\mathrm{i}}{2}A_\mu\begin{pmatrix} -g_2 s_{\mathrm{W}} - g_1 c_{\mathrm{W}} Y^{\mathrm{L}} & 0 \\ 0 & g_2 s_{\mathrm{W}} - g_1 c_{\mathrm{W}} Y^{\mathrm{L}} \end{pmatrix} \overset{!}{=} \mathrm{i}eA_\mu \begin{pmatrix} Q_1 & 0 \\ 0 & Q_2 \end{pmatrix}$$

- charged difference in doublet $Q_1 - Q_2 = 1 \quad \to \quad g_2 = \dfrac{e}{s_{\mathrm{W}}}$
- normalize $Y^{\mathrm{L/R}}$ such that $\quad g_1 = \dfrac{e}{c_{\mathrm{W}}}$

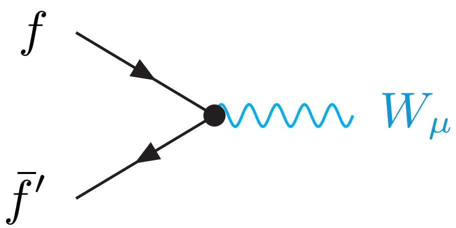
  $\hookrightarrow$ Y fixed by "Gell-Mann–Nishijima relation": $\quad Q = T_{\mathrm{I}}^3 + \dfrac{Y}{2}$

Fermion–gauge-boson interaction:

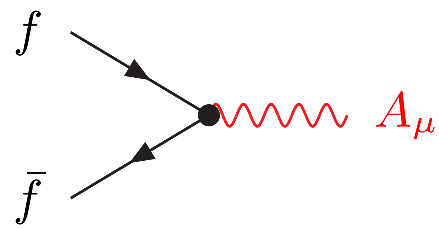
$$\mathcal{L}_{\mathrm{ferm,YM}} = \frac{e}{\sqrt{2}s_{\mathrm{W}}}\overline{\Psi_F^{\mathrm{L}}}\begin{pmatrix} 0 & \not{W}^+ \\ \not{W}^- & 0 \end{pmatrix}\Psi_F^{\mathrm{L}} + \frac{e}{2c_{\mathrm{W}}s_{\mathrm{W}}}\overline{\Psi_F^{\mathrm{L}}}\sigma^3\not{Z}\Psi_F^{\mathrm{L}}$$

$$- e\frac{s_{\mathrm{W}}}{c_{\mathrm{W}}}Q_f\overline{\psi_f}\not{Z}\psi_f - eQ_f\overline{\psi_f}\not{A}\psi_f \qquad (f\text{=all fermions, } F\text{= all doublets})$$

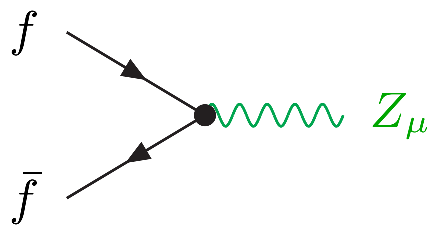
Feynman rules:



$$\frac{\mathrm{i}e}{\sqrt{2}s_{\mathrm{W}}}\gamma_\mu\omega_-$$



$$-\mathrm{i}Q_f e\gamma_\mu$$



$$\mathrm{i}e\gamma_\mu(g_f^+\omega_+ + g_f^-\omega_-) = \mathrm{i}e\gamma_\mu(v_f - a_f\gamma_5)$$

with $g_f^+ = -\frac{s_{\mathrm{W}}}{c_{\mathrm{W}}}Q_f, \quad g_f^- = -\frac{s_{\mathrm{W}}}{c_{\mathrm{W}}}Q_f + \frac{T^3_{\mathrm{I},f}}{c_{\mathrm{W}}s_{\mathrm{W}}},$

$$v_f = -\frac{s_{\mathrm{W}}}{c_{\mathrm{W}}}Q_f + \frac{T^3_{\mathrm{I},f}}{2c_{\mathrm{W}}s_{\mathrm{W}}}, \quad a_f = \frac{T^3_{\mathrm{I},f}}{2c_{\mathrm{W}}s_{\mathrm{W}}}$$

3.3 Gauge-boson sector

Yang–Mills Lagrangian for gauge fields:

$$\mathcal{L}_{\mathrm{YM}} = -\frac{1}{4} W^a_{\mu\nu} W^{a,\mu\nu} - \frac{1}{4} B_{\mu\nu} B^{\mu\nu}$$

Field-strength tensors:

$$W^a_{\mu\nu} = \partial_\mu W^a_\nu - \partial_\nu W^a_\mu + g_2 \epsilon^{abc} W^b_\mu W^c_\nu, \qquad B_{\mu\nu} = \partial_\mu B_\nu - \partial_\nu B_\mu$$

Lagrangian in terms of “physical” fields:

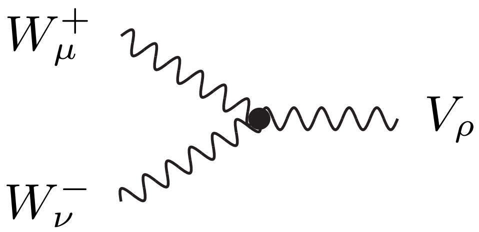
$$\begin{aligned}
\mathcal{L}_{\mathrm{YM}} = {} & -\frac{1}{2}(\partial_\mu W^+_\nu - \partial_\nu W^+_\mu)(\partial^\mu W^{-,\nu} - \partial^\nu W^{-,\mu}) \\
& - \frac{1}{4}(\partial_\mu Z_\nu - \partial_\nu Z_\mu)(\partial^\mu Z^\nu - \partial^\nu Z^\mu) - \frac{1}{4}(\partial_\mu A_\nu - \partial_\nu A_\mu)(\partial^\mu A^\nu - \partial^\nu A^\mu)
\end{aligned}$$

$+$ (trilinear interaction terms involving AW^+W^-, ZW^+W^-)

$+$ (quadrilinear interaction terms involving
AAW^+W^-, AZW^+W^-, ZZW^+W^-, $W^+W^-W^+W^-$)

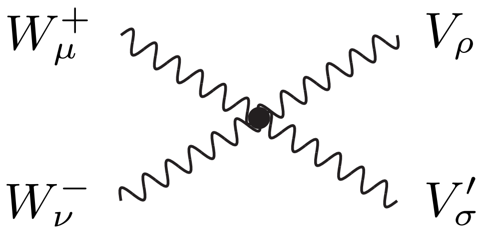
Feynman rules for gauge-boson self-interactions:

(fields and momenta incoming)



$$\mathrm{i}eC_{WWV}\Big[g_{\mu\nu}(k_+ - k_-)_\rho + g_{\nu\rho}(k_- - k_V)_\mu + g_{\rho\mu}(k_V - k_+)_\nu\Big]$$

with $C_{WW\gamma} = 1, \quad C_{WWZ} = -\dfrac{c_\mathrm{W}}{s_\mathrm{W}}$



$$\mathrm{i}e^2C_{WWVV'}\Big[2g_{\mu\nu}g_{\rho\sigma} - g_{\mu\rho}g_{\sigma\nu} - g_{\mu\sigma}g_{\nu\rho}\Big]$$

with $C_{WW\gamma\gamma} = -1, \qquad C_{WW\gamma Z} = \dfrac{c_\mathrm{W}}{s_\mathrm{W}},$

$$C_{WWZZ} = -\frac{c_\mathrm{W}^2}{s_\mathrm{W}^2}, \quad C_{WWWW} = \frac{1}{s_\mathrm{W}^2}$$

3.4 Higgs sector and spontaneous symmetry breaking

Idea: spontaneous breakdown of $\mathrm{SU}(2)_\mathrm{I}\times\mathrm{U}(1)_\mathrm{Y}$ symmetry $\to$ $\mathrm{U}(1)_\mathrm{elmg}$ symmetry

$\hookrightarrow$ masses for $\mathrm{W}^\pm$ and Z bosons, but γ remains massless

Note: choice of scalar extension of massless model involves freedom

GSW model:

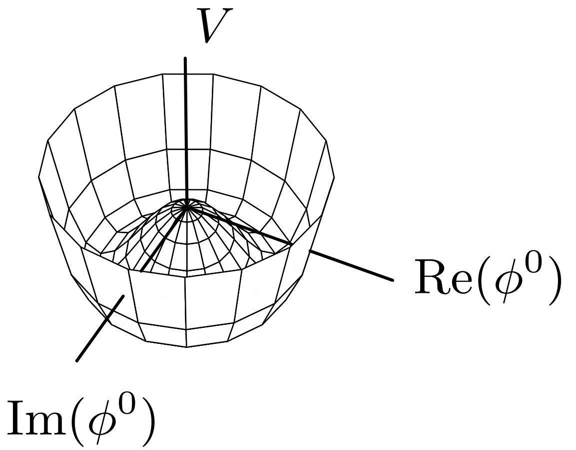
Minimal scalar sector with complex scalar doublet $\Phi = \begin{pmatrix} \phi^+ \\ \phi^0 \end{pmatrix}$, $Y_\Phi = 1$

Scalar self-interaction via Higgs potential:

$$V(\Phi) = -\mu^2\Phi^\dagger\Phi + \frac{\lambda}{4}(\Phi^\dagger\Phi)^2, \quad \mu^2, \lambda > 0,$$

$$= \mathrm{SU}(2)_\mathrm{I}\times\mathrm{U}(1)_\mathrm{Y} \text{ symmetric}$$

$$V(\Phi) = \text{minimal for} \quad |\Phi| = \sqrt{\frac{2\mu^2}{\lambda}} \equiv \frac{v}{\sqrt{2}} > 0$$



ground state Φ_0 (=vacuum expectation value of Φ) not unique

specific choice $\Phi_0 = \begin{pmatrix} 0 \\ \frac{v}{\sqrt{2}} \end{pmatrix}$ not gauge invariant $\Rightarrow$ spontaneous symmetry breaking

elmg. gauge invariance unbroken, since $\quad Q\Phi_0 = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix} \Phi_0 = 0$

Field excitations in Φ:

$$\Phi(x) = \begin{pmatrix} \phi^+(x) \\ \frac{1}{\sqrt{2}}\Big(v + H(x) + \mathrm{i}\chi(x)\Big) \end{pmatrix}$$

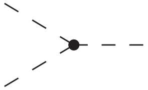
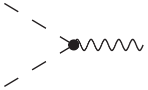
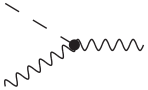
Gauge-invariant Lagrangian of Higgs sector: $\quad (\phi^- = (\phi^+)^\dagger)$

$$\mathcal{L}_\mathrm{H} = (D_\mu\Phi)^\dagger(D^\mu\Phi) - V(\Phi) \qquad \text{with } D_\mu = \partial_\mu - \mathrm{i}g_2\frac{\sigma^a}{2}W_\mu^a + \mathrm{i}\frac{g_1}{2}B_\mu$$

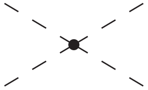
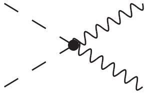
$$= (\partial_\mu\phi^+)(\partial^\mu\phi^-) - \frac{\mathrm{i}ev}{2s_\mathrm{W}}(W_\mu^+\partial^\mu\phi^- - W_\mu^-\partial^\mu\phi^+) + \frac{e^2v^2}{4s_\mathrm{W}^2}W_\mu^+W^{-,\mu}$$

$$+\frac{1}{2}(\partial\chi)^2 + \frac{ev}{2c_\mathrm{W}s_\mathrm{W}}Z_\mu\partial^\mu\chi + \frac{e^2v^2}{4c_\mathrm{W}^2s_\mathrm{W}^2}Z^2 + \frac{1}{2}(\partial H)^2 - \mu^2H^2$$

$+$ (trilinear SSS, SSV, SVV interactions)

$+$ (quadrilinear $SSSS$, $SSVV$ interactions)

Implications:

- gauge-boson masses: $\quad M_\mathrm{W} = \dfrac{ev}{2s_\mathrm{W}}, \quad M_\mathrm{Z} = \dfrac{ev}{2c_\mathrm{W}s_\mathrm{W}} = \dfrac{M_\mathrm{W}}{c_\mathrm{W}}, \quad M_\gamma = 0$
- physical Higgs boson H: $\quad M_\mathrm{H} = \sqrt{2\mu^2}$ = free parameter
- would-be Goldstone bosons $\phi^\pm$, χ: $\quad$ unphysical degrees of freedom

3.5 ρ-parameter and custodial SU(2) symmetry

Observation: Higgs potential of SM invariant under larger symmetry

$$V(\Phi) = f(\Phi^\dagger\Phi), \quad \Phi^\dagger\Phi = \mathrm{Re}\{\phi^+\}^2 + \mathrm{Im}\{\phi^+\}^2 + \mathrm{Re}\{\phi^0\}^2 + \mathrm{Im}\{\phi^0\}^2$$

$$= \text{invariant under O(4) = 4-dim. rotations}$$

Relation between O(4) $\simeq$ SU(2)$\times$SU(2) and SU(2)$_\mathrm{I}\times$U(1)$_\mathrm{Y}$ symmetry

$\hookrightarrow$ matrix notation:

$$\Pi \equiv (\,\tilde{\Phi}, \Phi\,) = \begin{pmatrix} \phi^{0*} & \phi^+ \\ -\phi^- & \phi^0 \end{pmatrix} \qquad \to \ \tfrac{1}{2}\,\mathrm{Tr}\left\{\Pi^\dagger\Pi\right\} = \Phi^\dagger\Phi$$

SU(2)$_\mathrm{I}\times$U(1)$_\mathrm{Y}$ transformation: $\quad U_\mathrm{I} \ = \ \exp\{\mathrm{i}g_2\theta^a T^a_\mathrm{I}\}, \quad U_\mathrm{Y} \ = \ \exp\{-\mathrm{i}g_1\theta^Y T_\mathrm{Y}\}$

$$\Pi \ \to \ \Pi' = U_\mathrm{I}\,\Pi\,U_\mathrm{Y}^\dagger, \qquad T^a_\mathrm{I} = \sigma^a/2, \quad T_\mathrm{Y} = \sigma^3/2$$

covariant derivative:

$$D_\mu\Pi \ = \ \partial_\mu\Pi - \mathrm{i}g_2\mathcal{W}_\mu\Pi - \mathrm{i}g_1\Pi B_\mu T_\mathrm{Y}, \qquad \mathcal{W}_\mu \equiv W^a_\mu T^a_\mathrm{I}$$

transformation of gauge fields:

$$\mathcal{W}_\mu \ \to \ \mathcal{W}'_\mu = U_\mathrm{I}\left(\mathcal{W}_\mu + \tfrac{\mathrm{i}}{g_2}\partial_\mu\right)U_\mathrm{I}^\dagger \qquad B_\mu \ \to \ B'_\mu = B_\mu + \partial_\mu\theta^Y$$

O(4) symmetry: $\quad \Phi^\dagger\Phi$ invariant under SU(2)$_\mathrm{I}\times$SU(2)$_{\mathrm{I}'}$ transformation

$$\Pi \ \to \ \Pi' = U_\mathrm{I}\,\Pi\,U_{\mathrm{I}'}^\dagger, \qquad U_{\mathrm{I}'} \ = \ \exp\{-\mathrm{i}g_1\theta^b T^b_{\mathrm{I}'}\}, \quad T^b_{\mathrm{I}'} \ = \ \sigma^b/2$$

Situation after spontaneous symmetry breaking:

ground state $\Pi_0 = (\tilde{\Phi}_0, \Phi_0) \propto \mathbf{1}$ still "diagonal" SU(2) symmetric:

$\Pi_0 \;\to\; \Pi_0' = U\,\Pi_0\,U^\dagger = \Pi_0,$ i.e. $[T^a, \Pi_0] = 0$ for SU(2) generators T^a

$\hookrightarrow$ under global transformation U

- W_μ^a transforms as 3-vector: $W_\mu^a \to W_\mu'^a = R_U^{ab} W_\mu^b$ (R_U = rotation matrix)
- B_μ transforms as 3rd component of a fictive triplet B_μ^a with R_U

$\hookrightarrow$ mass terms for gauge bosons

$$\mathcal{L}_{\mathrm{WZ,mass}} = \frac{1}{2}\,\mathrm{Tr}\left\{(D_\mu\Pi_0)^\dagger(D^\mu\Pi_0)\right\} \;=\; \frac{1}{2}\mathrm{Tr}\Bigg\{\Pi_0^\dagger\Pi_0\,\underbrace{\left(g_2 W_\mu^a T^a + g_1 T^3 B_\mu\right)^2}_{\substack{\text{invariant under } U \\ \hookrightarrow \text{ length of 3-vector}}}\Bigg\}$$

$$\propto g_2^2(W^1W^1 + W^2W^2) + (g_2 W^3 + g_1 B)^2 \propto c_{\mathrm{W}}^2 W^+W^- + \tfrac{1}{2}Z^2$$

$\Rightarrow$ Relation for the ρ-parameter: $\rho \equiv \dfrac{M_{\mathrm{W}}^2}{M_{\mathrm{Z}}^2 c_{\mathrm{W}}^2} \;=\; 1$

Role of the ρ-parameter in low-energy physics:

effective four-fermion interaction (cf. IVB model) with charged and neutral currents:

$\mathcal{L}_{4f,\mathrm{eff}} \;=\; -2\sqrt{2}G_\mu\left(J_{\mathrm{CC},\mu}^\dagger J_{\mathrm{CC}}^\mu + \rho\, J_{\mathrm{NC},\mu}^0 J_{\mathrm{NC}}^{0,\mu}\right),$ ρ = ratio of NC to CC interaction

Literature

- Böhm/Denner/Joos:
  “Gauge Theories of the Strong and Electroweak Interaction”
- Cheng/Li:
  “Gauge Theory of Elementary Particle Physics”
- Ellis/Stirling/Webber:
  “QCD and Collider Physics”
- Peskin/Schroeder:
  “An Introduction to Quantum Field Theory”
- Weinberg:
  “The Quantum Theory of Fields, Vol. 2: Modern Applications”

Introduction into
Standard Model and Precision Physics
– Lecture III –

Stefan Dittmaier

MPI München

General overview

6 Quantum field theories and higher perturbative orders

6.1 General procedure

Formulate theory: Lagrangian

$\Downarrow$

quantization $\longrightarrow$ gauge fixing, Faddeev–Popov ghosts

$\Downarrow$

Perturbative evaluation: Feynman rules

$\Downarrow$

Feynman graphs

$\Downarrow$

loop integrals $\longrightarrow$ technical problem: divergences (UV, IR)

$\Downarrow$

regularization $\longrightarrow$ divergences mathematically meaningful

$\Downarrow$

Define input parameters: renormalization $\longrightarrow$ eliminates UV divergences

$\Downarrow$

Theoretical predictions: calculation of observables (cross sections, decay widths, etc.)

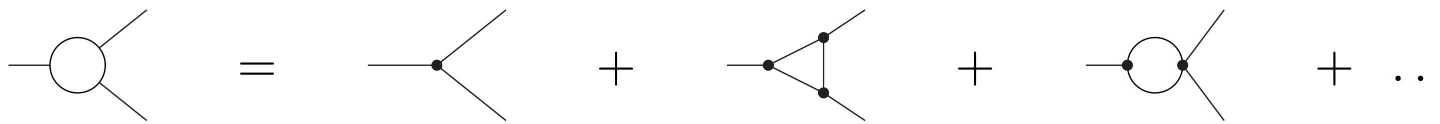
$\hookrightarrow$ IR divergences cancel for sufficiently inclusive quantities
(e.g. inclusion of photon bremsstrahlung)

6.2 Green functions, transition amplitudes, and observables

"Amputated" Green functions $G_{\mathrm{amp}}^{\phi_1 \dots \phi_n}$:

calculated as sum of all connected Feynman diagrams with external n legs $\phi_1, \dots, \phi_n$ with external propagators (and propagator corrections) omitted

$G_{\mathrm{amp}}^{\phi_1\phi_2\phi_3} =$



Transition amplitude $\mathcal{M}_{fi}$ for $|i\rangle \rightarrow |f\rangle$:

calculated from amputated Green functions $G_{\mathrm{amp}}^{\phi_1 \dots \phi_n}$ by "LSZ reduction":

- put external momenta to their mass shell, $p_i^2 = m_i^2$
- contract with wave functions of external particles (Dirac spinors, polarization vectors)

  Note: fields must be normalized: $R_{\phi_i} = 1$ (= residue of propagator pole), otherwise multiply by $\sqrt{R_{\phi_i}}$ for each external leg

Cross section for transition $|i\rangle \rightarrow |f\rangle$:

$$\sigma = \text{flux} \times \int \mathrm{dLIPS}\, |\mathcal{M}_{fi}|^2$$

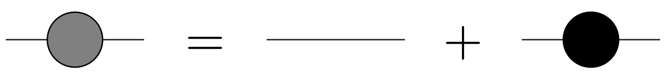
“Vertex functions” $\Gamma^{\phi_1\cdots\phi_n}$ as irreducible building blocks:

- $\Gamma^{\phi_1\phi_2} \;\equiv\; -(G^{\phi_1\phi_2})^{-1} \;=\; -$(inverse propagator)

  example: scalar 2-point function

  $$\Gamma^{\phi\phi}(p) = \mathrm{i}(p^2 - m^2) + \mathrm{i}\Sigma(p^2),$$

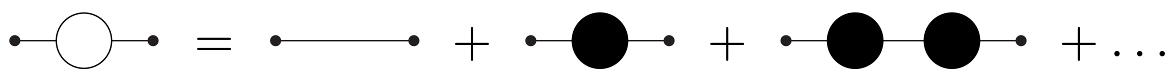
  $\Sigma =$ self-energy = sum of 1PI graphs

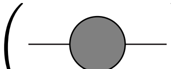
  1PI = 1-particle-irreducible
  (graph cannot be disconnected by cutting *one* line)

  $$G^{\phi\phi}(p) = \frac{\mathrm{i}}{p^2 - m^2} + \frac{\mathrm{i}}{p^2 - m^2}\,\mathrm{i}\Sigma(p^2)\,\frac{\mathrm{i}}{p^2 - m^2} + \dots$$

  (Dyson series)

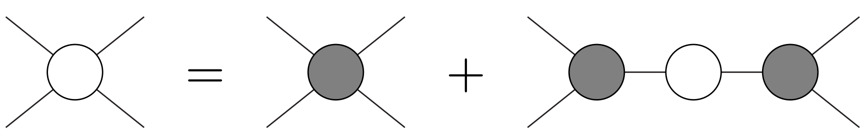
  

  $$= \frac{\mathrm{i}}{p^2 - m^2 + \Sigma(p^2)} \;=\; -\Big(\Gamma^{\phi\phi}(p)\Big)^{-1} \;=\; -\Big(\;\; \Big)^{-1}$$

- $\Gamma^{\phi_1\cdots\phi_n} \;\equiv\; G_{\mathrm{amp}}^{\phi_1\cdots\phi_n}\Big|_{\text{only 1PI graphs}}$

  example:

  $+$ two permutations

  $G_{\mathrm{amp}}^{\phi\phi\phi\phi}$ $\qquad \Gamma^{\phi\phi\phi\phi}$ $\qquad \Gamma^{\phi\phi\phi}G^{\phi\phi}\Gamma^{\phi\phi\phi}$

6.3 Loop integrals and regularization

Regularization of divergences

Observation: loop integrals involve divergences

- UV divergences for $q \to \infty$, e.g.:

$$\int \mathrm{d}^4 q \, \frac{1}{(q^2 - m_0^2)(q^2 - m_1^2)} \sim \int \frac{\mathrm{d}q}{q} \text{ for } q \to \infty \qquad \to \text{logarithmic divergence}$$

- IR divergences for $q \to q_0$, e.g.:

$$\int \mathrm{d}^4 q \, \frac{1}{q^2(q^2 + 2qp_1)(q^2 + 2qp_2)} \sim \int \frac{\mathrm{d}q}{q} \text{ for } q \to 0 \qquad \to \text{logarithmic divergence}$$

"Regularization": extension of theory by free parameter δ such that

- integrals (and thus the theory) become finite, i.e. well defined
- original theory is obtained as limiting case $\delta \to \delta_0$

$\hookrightarrow$ fix input parameters x_i of regularized theory ($\delta \neq \delta_0$) by experiment

$\Rightarrow$ observables must have finite limit $\delta \to \delta_0$ as functions of x_i

(independent of regularization scheme)

Convenient regularization schemes:

- Dimensional regularization: switch to $D \neq 4$ space-time dimensions
  - ◇ regularizes UV (and IR) divergences, respects gauge invariance, easy use
  - ◇ prescription: (μ = arbitrary reference mass, drops out in observables)

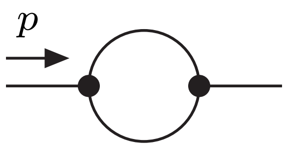
    $$\int \mathrm{d}^4 q \;\to\; (2\pi\mu)^{4-D} \int \mathrm{d}^D q$$ and D-dim. momenta, metric, Dirac algebra

    and analytic continuation to complex D !
  - ◇ divergences appear as poles $\dfrac{1}{4-D}$ in results

    $\hookrightarrow$ define $\Delta \equiv \dfrac{2}{4-D} - \gamma_{\mathrm{E}} + \ln(4\pi) = \dfrac{2}{4-D} +$ const.
- IR regularization by infinitesimal photon mass m_γ
  and (if relevant) by small fermion mass m_f
  - ◇ prescription: photon propagator pole $\dfrac{1}{q^2} \to \dfrac{1}{q^2 - m_\gamma^2}$
  - ◇ divergences appear as $\ln(m_\gamma)$ and $\ln(m_f)$ terms

Standard 1-loop integrals:

- 2-point integrals:

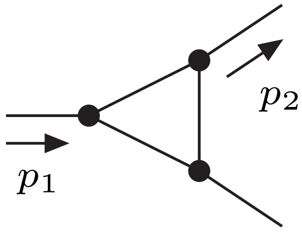


$$B_{0,\mu,\mu\nu,\ldots}(p,m_0,m_1) \;=\; \frac{(2\pi\mu)^{4-D}}{i\pi^2}\int \mathrm{d}^D q\, \frac{1,q_\mu,q_\mu q_\nu,\ldots}{(q^2-m_0^2+\mathrm{i}0)[(q+p)^2-m_1^2+\mathrm{i}0]}$$

scalar integral $B_0 =$ logarithmically UV divergent $= \Delta +$ finite,

vector integral $B_\mu = -\frac{1}{2}p_\mu\Delta +$ finite, etc.

- 3-point integrals:



$$C_{0,\mu,\mu\nu,\ldots}(p_1,p_2,m_0,m_1,m_2)$$
$$= \frac{(2\pi\mu)^{4-D}}{i\pi^2}\int \mathrm{d}^D q\, \frac{1,q_\mu,q_\mu q_\nu,\ldots}{(q^2-m_0^2+\mathrm{i}0)[(q+p_1)^2-m_1^2+\mathrm{i}0][(q+p_2)^2-m_2^2+\mathrm{i}0]}$$

C_0, $C_\mu =$ UV finite,

$C_{\mu\nu} =$ logarithmically UV divergent $= \frac{1}{4}g_{\mu\nu}\Delta +$ finite, etc.

- 4-point integrals: $D_{\ldots}$ functions, etc.

Features of one-loop integrals:

- sign of infinitesimally small imaginary part $\mathrm{i}0$ in mass terms reflects causality
- general results for 1-loop integrals known
  (complicated but straightforward calculation)
  - ◇ momentum integrals can be carried out after "Feynman parametrization"
    $\hookrightarrow$ $(n-1)$-dimensional integrals for n-point functions
  - ◇ B functions $\rightarrow$ can be expressed in terms of log's
  - ◇ C, D, etc. $\rightarrow$ involve dilogarithms $\mathrm{Li}_2(x) = -\int_0^x \frac{\mathrm{d}t}{t}\ln(1-t)$
- tensor integrals can be decomposed into Lorentz covariants:

$$B^\mu = p^\mu B_1, \qquad B^{\mu\nu} = g^{\mu\nu} B_{00} + p^\mu p^\nu B_{11},$$

$$C^\mu = p_1^\mu C_1 + p_2^\mu C_2, \quad C^{\mu\nu} = p_1^\mu p_1^\nu C_{11} + p_2^\mu p_2^\nu C_{22} + (p_1^\mu p_2^\nu + p_1^\nu p_2^\mu) + g^{\mu\nu} C_{00}, \quad \text{etc.}$$

$\hookrightarrow$ tensor coefficients B_1, B_{ij}, C_i, etc. can be obtained as
linear combinations of scalar integrals B_0, C_0, etc.
(e.g. by "Passarino–Veltman reduction")

6.4 Renormalization

Propagators and 2-point functions:

Structure of one-loop self-energies (scalar case as example):

$$\Sigma(p^2) \;=\; C_1\, p^2\, \Delta \;+\; C_2\, \Delta \;+\; \Sigma_{\mathrm{finite}}(p^2) \;=\; \text{UV divergent}$$

Behaviour of propagator near pole for free propagation:

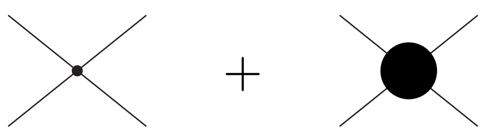
$$G^{\phi\phi}(p^2) \;=\; \frac{\mathrm{i}}{p^2 - m^2 + \Sigma(p^2)} \;\underset{p^2 \to m^2}{\widetilde{\quad}}\; \frac{1}{1+\Sigma'(m^2)}\,\frac{\mathrm{i}}{p^2 - m^2 + \Sigma(m^2)}$$

$\hookrightarrow$ higher-order corrections change location and residue of propagator pole

Interaction vertices:

Example: scalar 4-point interaction $\mathcal{L}_{\phi^4} = \lambda\phi^4/4!$

$$\Gamma^{\phi\phi\phi\phi}(p_1,p_2,p_3) = \quad \mathrm{i}\lambda \quad + \quad \mathrm{i}\Lambda\,(p_1,p_2,p_3)$$



momentum-dependent one-loop correction:

$$\Lambda\,(p_1,p_2,p_3) \;=\; C_3\Delta \;+\; \Lambda_{\mathrm{finite}}\,(p_1,p_2,p_3) \;=\; \text{UV divergent}$$

$\hookrightarrow$ higher-order corrections change coupling strengths

Structure of UV divergences:

- Renormalizable field theories:

  UV divergences in vertex functions have analytical form of elementary vertex structures (directly related to $\mathcal{L}$)

  $\hookrightarrow$ idea: absorb divergences in free parameters

  $\Rightarrow$ Reparametrization of theory (=renormalization)

  Different types of renormalizable theories:

  ◇ theories with unrelated couplings of non-negative mass dimensions

  $\hookrightarrow$ renormalizability proven by power counting and "BPHZ procedure"

  ◇ gauge theories (couplings unified by gauge invariance)

  $\hookrightarrow$ renormalizability non-trivial consequence of gauge symmetry 't Hooft '71

- Non-renormalizable field theories:

  e.g. theories with couplings of negative mass dimensions (cf. Fermi model)

  operators of higher and higher mass dimensions needed to absorb UV divergences

  $\hookrightarrow$ infinitely many free parameters, much less predictive power

Practical procedure for renormalization:

consider original ("bare") parameters and fields as preliminary
(denoted with subscripts "0" in the following)

$\hookrightarrow$ switch to new "renormalized" parameters and fields that obey certain conditions

Propagators and 2-point functions:

- mass renormalization: $m_0^2 = m^2 + \delta m^2$,
  $m^2 \stackrel{!}{=}$ location of propagator pole = "physical mass" $\quad \rightarrow \delta m^2 = \Sigma(m^2)$

- wave-function ren.: rescale fields $\phi_0 = \sqrt{Z_\phi}\phi$, $\quad G^{\phi\phi} = Z_\phi^{-1} G^{\phi_0\phi_0}$
  fix $Z_\phi = 1 + \delta Z_\phi$ such that residue of $G^{\phi\phi}$ at $p^2 = m^2$ equals 1
  $\hookrightarrow \; \delta Z_\phi = -\Sigma'(m^2)$

$\Rightarrow$ Renormalized propagator $G^{\phi\phi}$ is UV finite:

$$G^{\phi\phi}(p^2) = \frac{\mathrm{i}}{p^2 - m^2 + \Sigma_{\mathrm{ren}}(p^2)},$$

$$\begin{aligned} \Sigma_{\mathrm{ren}}(p^2) &= \Sigma(p^2) - \Sigma(m^2) + (p^2 - m^2)\Sigma'(m^2) \;=\; \text{ren. self-energy} \\ &= \Sigma_{\mathrm{finite}}(p^2) - \Sigma_{\mathrm{finite}}(m^2) + (p^2 - m^2)\Sigma'_{\mathrm{finite}}(m^2) \;=\; \text{UV finite} \end{aligned}$$

Vertex functions for interactions:

- coupling renormalization: $\lambda_0 \;=\; \lambda + \delta\lambda$

  fix $\delta\lambda$ such that λ assumes a measured value for special kinematics p_i^{exp}

  note: $\Gamma^{\phi\phi\phi\phi} \;=\; Z_\phi^2 \Gamma^{\phi_0\phi_0\phi_0\phi_0}$

  $\hookrightarrow\;\; \delta\lambda = -2\delta Z_\phi \lambda - \Lambda(p_1^{\mathrm{exp}}, p_2^{\mathrm{exp}}, p_3^{\mathrm{exp}})$

$\Rightarrow$ Renormalized vertex function is UV finite:

$$\Gamma^{\phi\phi\phi\phi}(p_1,p_2,p_3) = \mathrm{i}\lambda + \mathrm{i}\Lambda_{\mathrm{ren}}(p_1,p_2,p_3)\,,$$

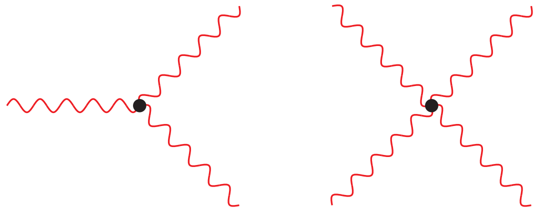
$$\Lambda_{\mathrm{ren}}(p_1,p_2,p_3) = \Lambda_{\mathrm{finite}}(p_1,p_2,p_3) \;-\; \Lambda_{\mathrm{finite}}(p_1^{\mathrm{exp}}, p_2^{\mathrm{exp}}, p_3^{\mathrm{exp}}) \;=\; \text{UV finite}$$

7 Electroweak Standard Model — radiative corrections

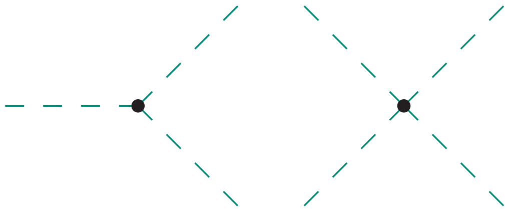
7.1 Loop corrections

Recapitulation of elementary SM couplings (vertices)

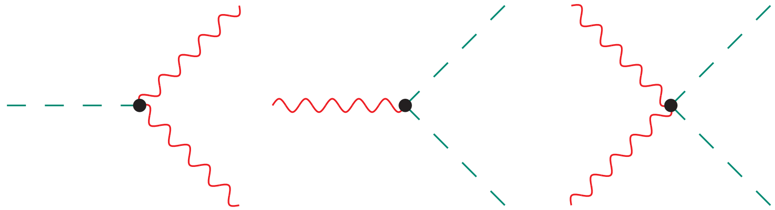
gauge-boson self-couplings:



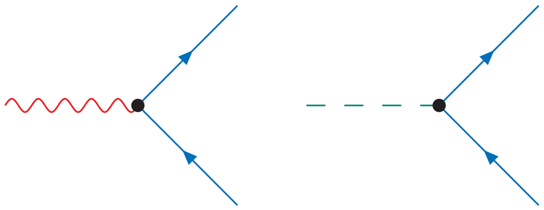
Higgs self-couplings:



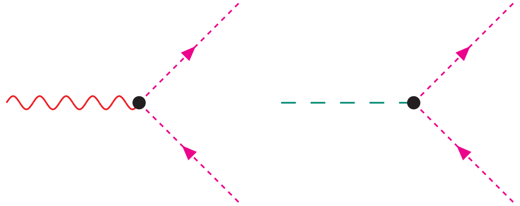
gauge-boson–Higgs couplings:



fermion couplings:



Faddeev–Popov couplings:

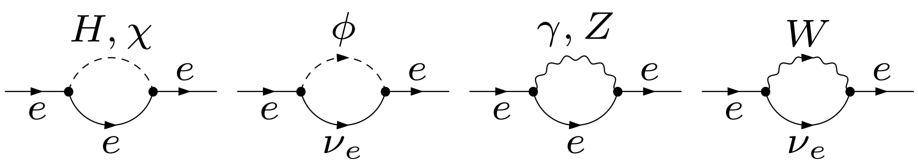


$\Rightarrow$ Large variety of loop diagrams !

Examples for 2-point functions at one loop: ('t Hooft–Feynman gauge)

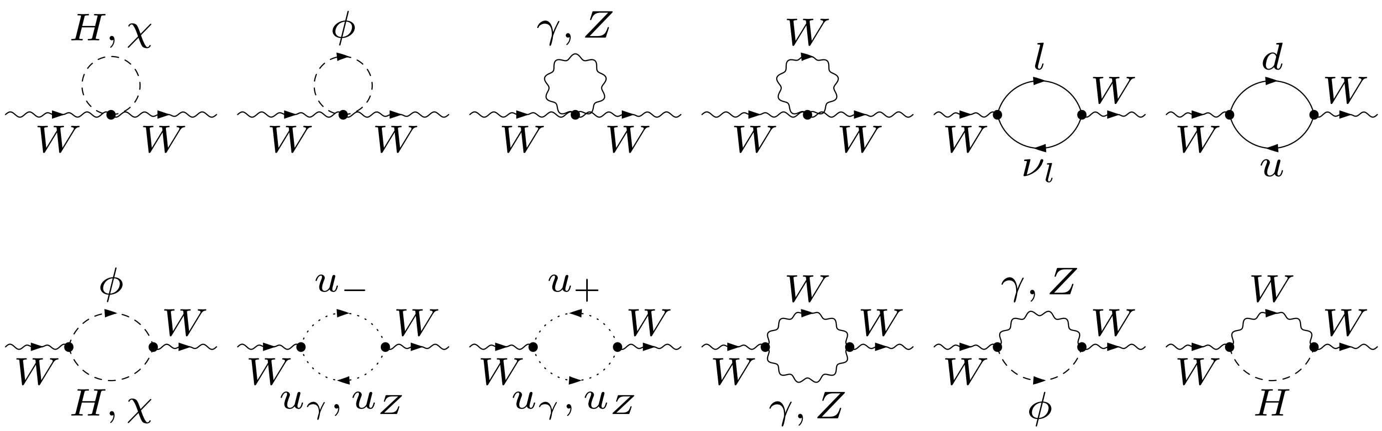
Electron self-energy:

$$\Gamma^{e\bar{e}}(p) = \mathrm{i}(\not{p} - m_e) + \mathrm{i}\not{p}\omega_+ \Sigma^e_R(p^2) + \mathrm{i}\not{p}\omega_- \Sigma^e_L(p^2) + \mathrm{i}m_e \Sigma^e_S(p^2)$$



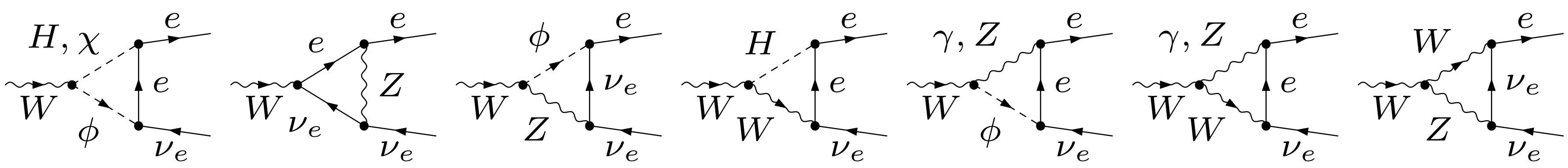
W-boson self-energy:

$$\Gamma^{W^-W^+}_{\mu\nu}(k) = -\mathrm{i}g_{\mu\nu}(k^2 - M_W^2) - \mathrm{i}\left(g_{\mu\nu} - \frac{k_\mu k_\nu}{k^2}\right)\Sigma^W_T(k^2) - \mathrm{i}\frac{k_\mu k_\nu}{k^2}\Sigma^W_L(k^2)$$

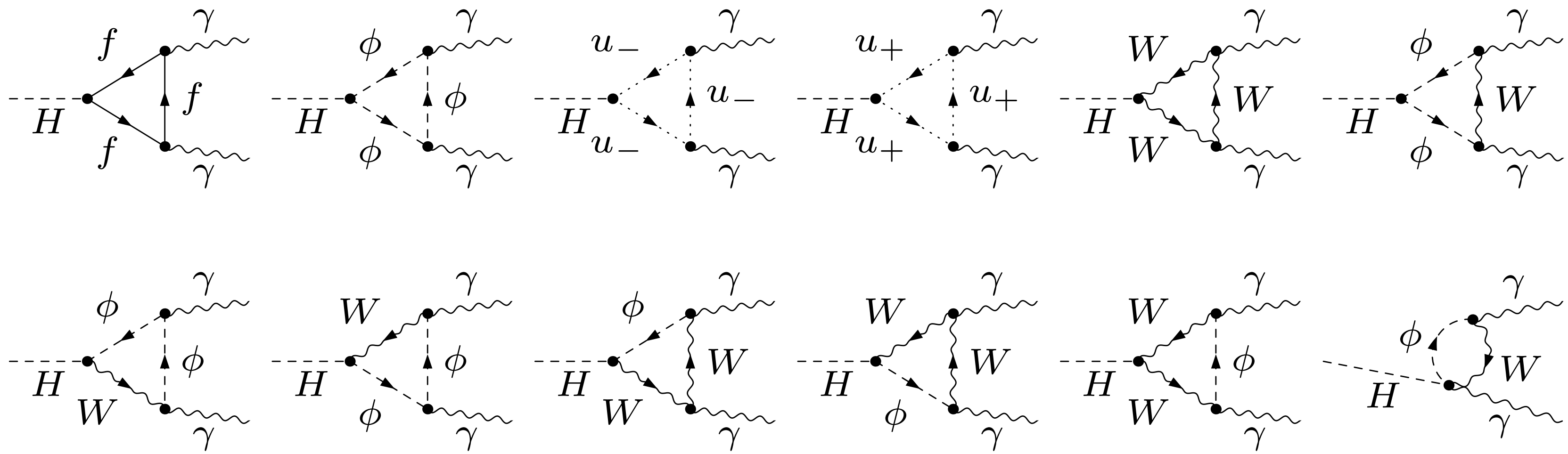


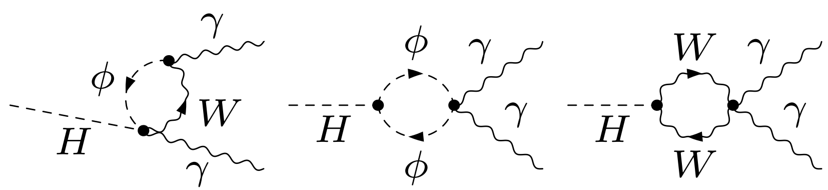
Examples for 3-point functions at one loop:

$\mathrm{We}\nu_\mathrm{e}$ vertex correction:



$\mathrm{H}\gamma\gamma$ vertex (loop induced):





$+$ diagrams (if different) with photons interchanged

7.2 Renormalization

Bare input parameters: $e_0, M_{\mathrm{W},0}, M_{\mathrm{Z},0}, M_{\mathrm{H},0}, m_{f,0}, V_{ij,0}$

Renormalization transformation:

- Parameter renormalization:

$$e_0 = (1+\delta Z_e)e,$$

$$M_{\mathrm{W},0}^2 = M_{\mathrm{W}}^2 + \delta M_{\mathrm{W}}^2, \quad M_{\mathrm{Z},0}^2 = M_{\mathrm{Z}}^2 + \delta M_{\mathrm{Z}}^2, \quad M_{\mathrm{H},0}^2 = M_{\mathrm{H}}^2 + \delta M_{\mathrm{H}}^2,$$

$$m_{f,0} = m_f + \delta m_f, \quad V_{ij,0} = V_{ij} + \delta V_{ij}, \quad \text{(both } V_{ij,0}, V_{ij} \text{ unitary)}$$

Note: renormalization of $c_{\mathrm{W}}, s_{\mathrm{W}}$ fixed by on-shell condition $c_{\mathrm{W}} = \dfrac{M_{\mathrm{W}}}{M_{\mathrm{Z}}}$
(s_{W} is *not* a free parameter if M_{W}, M_{Z} are used as input parameters)

- Field renormalization

$$W_0^\pm = \sqrt{Z_W}\,W^\pm, \quad \begin{pmatrix} Z_0 \\ A_0 \end{pmatrix} = \begin{pmatrix} \sqrt{Z_{ZZ}} & \sqrt{Z_{ZA}} \\ \sqrt{Z_{AZ}} & \sqrt{Z_{AA}} \end{pmatrix} \begin{pmatrix} Z \\ A \end{pmatrix}, \quad H_0 = \sqrt{Z_H}\,H,$$

$$\psi_{f,0}^{\mathrm{L}} = \sqrt{Z_{ff'}^{\mathrm{L}}}\,\psi_{f'}^{\mathrm{L}}, \quad \psi_{f,0}^{\mathrm{R}} = \sqrt{Z_{ff'}^{\mathrm{R}}}\,\psi_{f'}^{\mathrm{R}}$$

Note: matrix renormalization necessary to account for loop-induced mixing

Renormalization conditions:

- Mass renormalization:

  on-shell definition: mass^2 is location of pole in propagator

  $\hookrightarrow \; \delta M_\mathrm{W}^2 = \mathrm{Re}\{\Sigma_\mathrm{T}^W(M_\mathrm{W}^2)\}$, similar expressions for $\delta M_\mathrm{Z}^2, \delta M_\mathrm{H}^2, \delta m_f$

  Note: $\diamond$ location of pole is complex for unstable particles

  $\hookrightarrow$ subtlety in all-orders definition, but not relevant at one loop

  (gauge-invariant definition: mass^2 as real part of pole location)

  $\diamond$ other definitions of quark masses often more appropriate

  (running masses, masses in effective field theories)

- Field renormalization: (bosons and leptons)

  $\diamond$ residues of propagators (diagonal, transverse parts) normalized to 1

  $\hookrightarrow \; \delta Z_W = -\mathrm{Re}\{\Sigma_\mathrm{T}^{W\,\prime}(M_\mathrm{W}^2)\}$,

  similar expressions for $\delta Z_{AA}, \delta Z_{ZZ}, \delta Z_H, \delta Z_{ff}^{\mathrm{L/R}}$

  $\diamond$ suppression of mixing propagators on particle poles

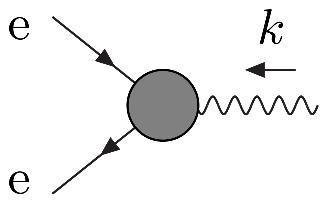
  $\hookrightarrow$ fixes non-diagonal constants $\delta Z_{AZ}, \delta Z_{ZA}, \delta Z_{ff'}^{\mathrm{L/R}} \; (f \neq f')$

  Note: problems for unstables particles beyond one loop

  (field-renormalization constants become complex)

Renormalization conditions: (continued)

- Charge renormalization: define e in Thomson limit



$A_\mu \quad \xrightarrow{k\to 0} \quad \mathrm{i}e\gamma_\mu$ for on-shell electrons

$\Rightarrow$ $e =$ elementary charge of classical electrodynamics

fine-structure constant $\alpha(0) = \dfrac{e^2}{4\pi} = 1/137.03599976$

Gauge invariance relates δZ_e to photon wave-function renormalization:

$$\delta Z_e \;=\; -\frac{1}{2}\delta Z_{AA} - \frac{s_\mathrm{W}}{2c_\mathrm{W}}\delta Z_{ZA}$$

- Quark-field and CKM-matrix renormalization $\to$ fixes $\delta Z_{qq'}^{\mathrm{L/R}}, \delta V_{ij}$

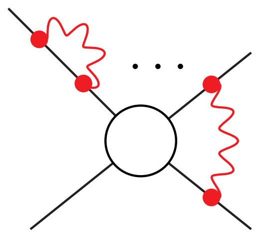
rotation to mass eigenstates;
CKM part requires a careful (non-trivial) investigation
of mixing self-energies, mass eigenstates, LSZ reduction, etc.

General result: all renormalization constants can be obtained from self-energies.

7.3 IR divergences and photon bremsstrahlung

Consider processes with charged external particles, e.g., $\mathrm{e}^+\mathrm{e}^- \to \mu^+\mu^-$

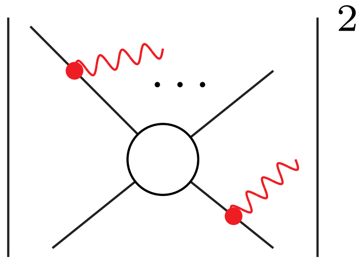
- Virtual corrections: loop diagrams



IR divergences from soft virtual photons ($q \to 0$)

$$\int \frac{\mathrm{d}^4 q \ldots}{(q^2 - m_\gamma^2)(2qp_1)(2qp_2)} \to C \ln(m_\gamma)$$

- “Real” corrections: photon bremsstrahlung

$$\int \frac{\mathrm{d}^3\mathbf{q}}{2q_0}$$ 

IR divergences from soft real photons ($\mathbf{q} \to 0$)

$$\int \frac{\mathrm{d}^3 \mathbf{q} \ldots}{\sqrt{\mathbf{q}^2 + m_\gamma^2}(2qp_1)(2qp_2)} \to -C \ln(m_\gamma)$$

Bloch–Nordsieck theorem:

IR divergences of virtual and real corrections cancel in the sum

$\hookrightarrow$ virtual and soft-photonic corrections cannot be discussed separately

$\leftrightarrow$ related to limited experimental resolution of soft photons

$\Rightarrow$ Cross-section predictions necessarily depend on treatment of photon emission (energy and angular cuts)

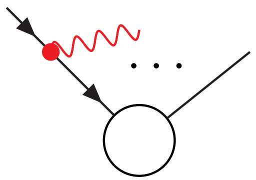
Separation of soft and hard photons:

Why? cancellation of $\ln(m_\gamma)$ terms delicate in practice, but terms are universal

- soft photons, $m_\gamma < E_\gamma < \Delta E \ll Q$ = typical scale of the process
  $\hookrightarrow$ correction is universal factor δ_{soft} to Born cross section
  relatively simple analytical expression with explicit $C\ln(\Delta E/m_\gamma)$ terms
- hard photons, $E_\gamma > \Delta E$
  $\hookrightarrow$ Monte Carlo integration of full radiative process, but with $m_\gamma = 0$
  $-C\ln(\Delta E)$ terms emerge numerically

$\ln(\Delta E)$ contributions cancel numerically in sum for small ΔE up to $\mathcal{O}(\Delta E/E)$

Calculation of soft-photon factor:



$$
\begin{aligned}
&= A(p-q)\,\frac{\mathrm{i}(\not{p}-\not{q}+m_f)}{(p-q)^2-m_f^2}\,(\mathrm{i}Q_f e)\not{\varepsilon}^*_\gamma u_f(p) \\
\underset{q\to 0}{\sim}\; & -Q_f e\,\frac{\varepsilon^*_\gamma p}{qp}\,A(p)u_f(p) \;=\; -Q_f e\,\frac{\varepsilon^*_\gamma p}{qp}\,\mathcal{M}_{\mathrm{Born}}
\end{aligned}
$$

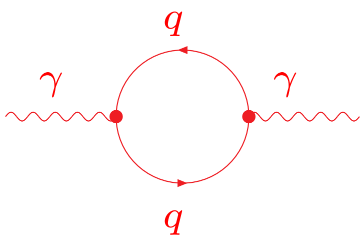
"Eikonal factorization" holds for all charged particles (spin $0, \frac{1}{2}, 1$)

$$\Rightarrow\quad \delta_{\mathrm{soft}} = -\frac{\alpha}{2\pi^2}\int_{m_\gamma<q_0<\Delta E}\frac{\mathrm{d}^3\mathbf{q}}{2q_0}\sum_{i,j}\frac{(\pm Q_i)(\pm Q_j)(p_i p_j)}{(qp_i)(qp_j)}$$

(i = particle with charge Q_i incoming($+$) or outgoing ($-$))

7.4 The universal radiative corrections $\Delta\alpha$ and $\Delta\rho$

Running electromagnetic coupling $\alpha(s)$:



becomes sensitive to unphysical quark masses m_q

for $|s|$ in GeV range and below (non-perturbative regime)

$\hookrightarrow$ charge-renormalization constant δZ_e sensitive to m_q

Solution: fit hadronic part of $\Delta\alpha(s) \;=\; -\mathrm{Re}\{\Sigma^{AA}_{\mathrm{T,ren}}(s)/s\}$ and thus of δZ_e

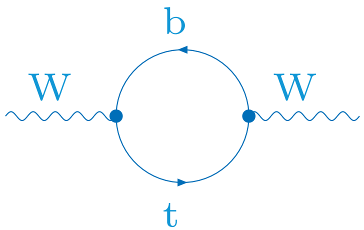
via dispersion relations to $R(s) \;=\; \dfrac{\sigma(\mathrm{e^+e^-} \to \text{hadrons})}{\sigma(\mathrm{e^+e^-} \to \mu^+\mu^-)}$ Jegerlehner et al.

$\Rightarrow$ Running elmg. coupling: $\quad \alpha(s) \;=\; \dfrac{\alpha(0)}{1-\Delta\alpha_{\mathrm{ferm}\neq\mathrm{top}}(s)}$

Leading correction to the ρ-parameter:

mass differences in fermion doublets break custodial SU(2) symmetry

$\hookrightarrow$ large effects from bottom–top loops in W self-energy Veltman '77



$$\Delta\rho_{\mathrm{top}} \;\sim\; \frac{\Sigma^{ZZ}_{\mathrm{T}}(0)}{M_{\mathrm{Z}}^2} - \frac{\Sigma^{WW}_{\mathrm{T}}(0)}{M_{\mathrm{W}}^2} \;\sim\; \frac{3G_\mu m_{\mathrm{t}}^2}{8\sqrt{2}\pi^2}$$

8 Radiative corrections to muon decay

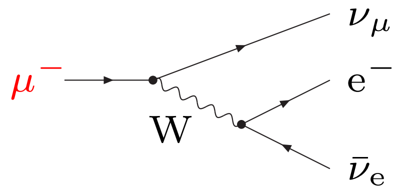
Precision calculation of M_W via μ decay

$\hookrightarrow$ M_W as function of $\alpha(0)$, G_μ, M_Z and the quantity Δr

$$M_\mathrm{W}^2\left(1-\frac{M_\mathrm{W}^2}{M_\mathrm{Z}^2}\right) = \frac{\pi\alpha(0)}{\sqrt{2}G_\mu}(1+\Delta r)$$

Δr comprises quantum corrections to μ decay
(beyond electromagnetic corrections in Fermi model)

Lowest order:



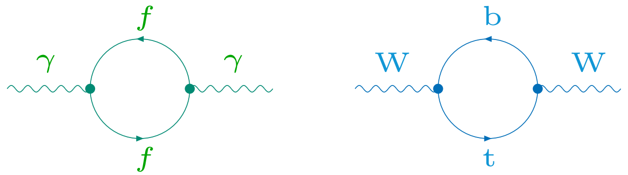
$\mathcal{O}(\alpha)$ corrections:

Sirlin '80, Marciano, Sirlin '80

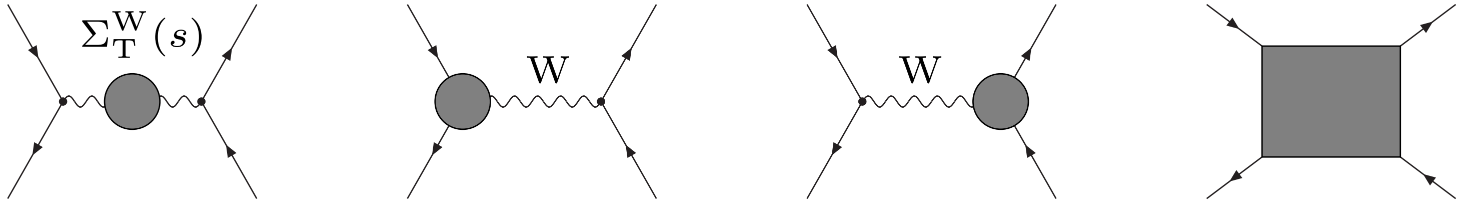
$$\Delta r_{1-\mathrm{loop}} = \Delta\alpha(M_\mathrm{Z}^2) \quad - \frac{c_\mathrm{W}^2}{s_\mathrm{W}^2}\Delta\rho_\mathrm{top} \quad + \quad \Delta r_\mathrm{rem}(M_\mathrm{H})$$

$$\sim 6\% \qquad\qquad \sim 3\% \qquad\qquad \sim 1\%$$

$$\alpha\ln(m_f/M_\mathrm{Z}) \qquad G_\mu m_\mathrm{t}^2 \qquad \alpha\ln(M_\mathrm{H}/M_\mathrm{Z})$$

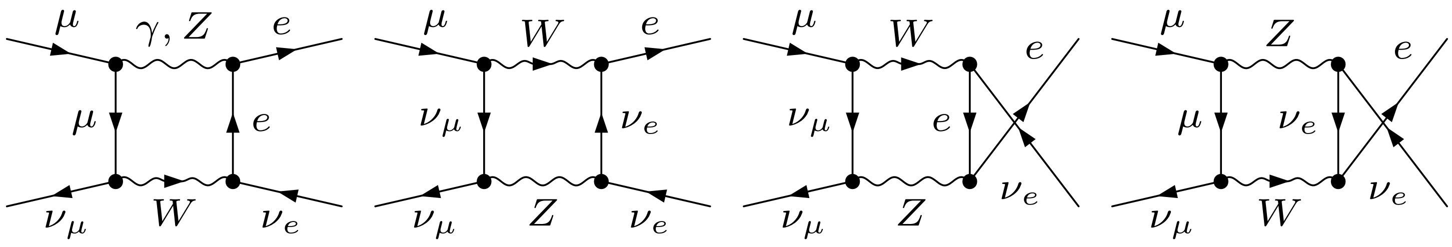


Virtual correction – 1-loop diagrams:

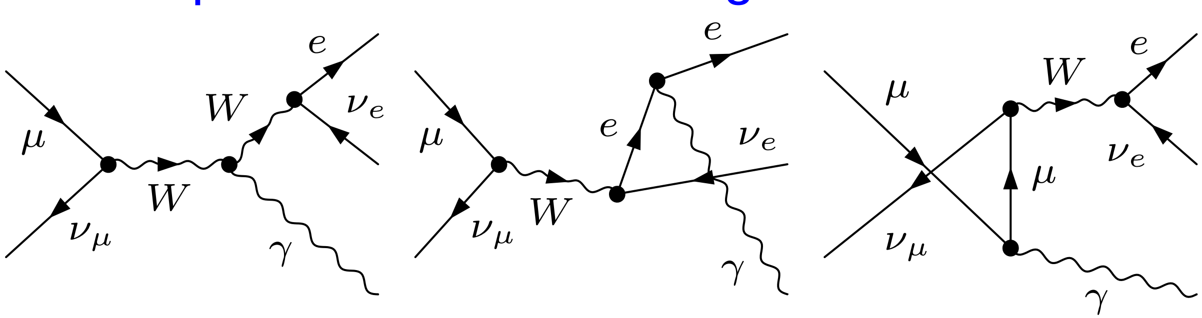


W self-energy $\quad$ $\mathrm{W}l\nu_l$ vertex correction $\quad$ box diagrams

e.g.:

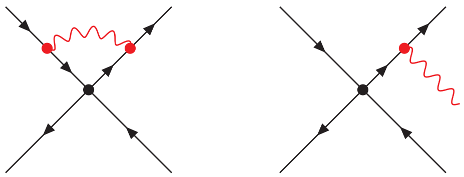


Real correction – 1-photon bremsstrahlung:



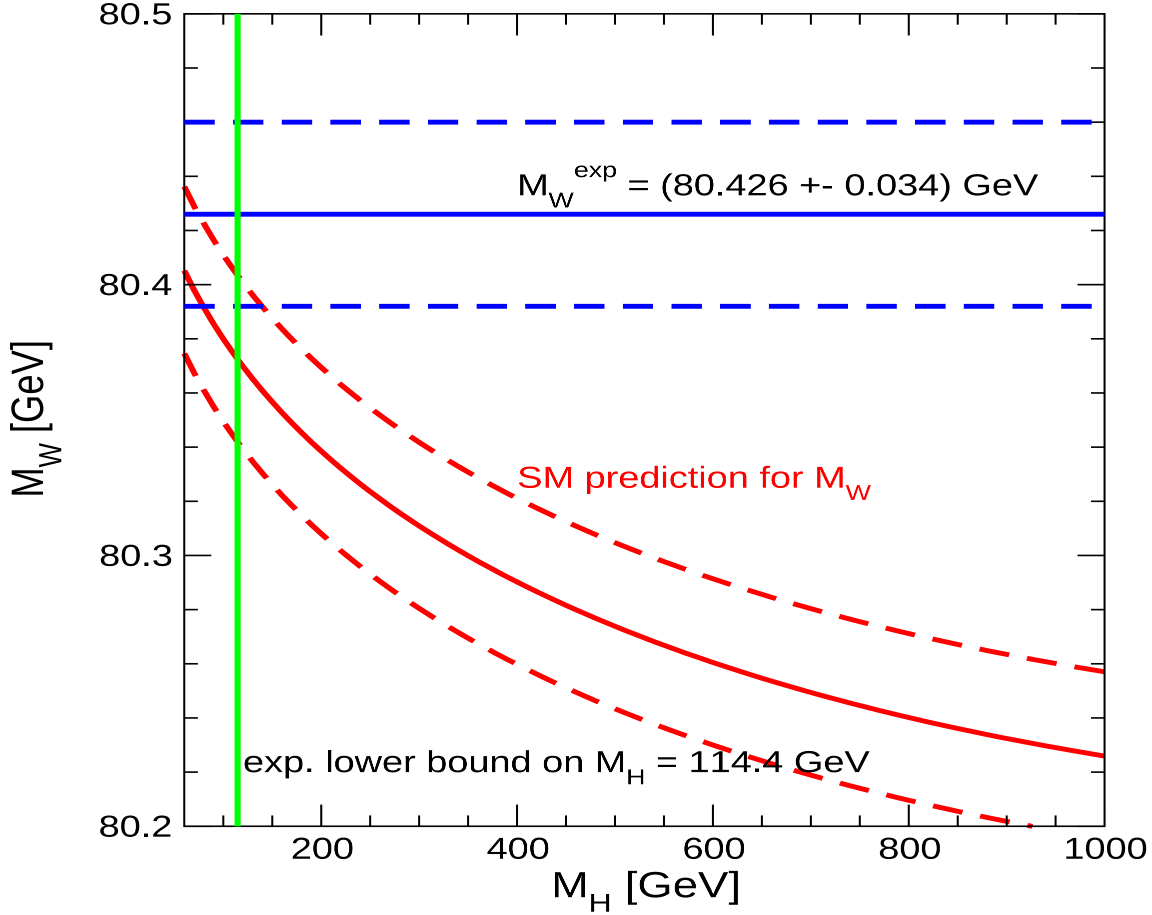
Consistent use of G_μ:

Photonic QED corrections are treated in the Fermi model and subtracted from Δr



etc.

State-of-the-art prediction of M_W from muon decay:



Hollik et al. '03

Theoretical uncertainty:

status '00: $\Delta M_\mathrm{W} \sim 6\,\mathrm{MeV}$

status '06: $\Delta M_\mathrm{W} \sim 4\,\mathrm{MeV}$

Experimental error:

status '06: $\Delta M_\mathrm{W} \sim 29\,\mathrm{MeV}$

ILC(?): $\Delta M_\mathrm{W} \sim 7\,\mathrm{MeV}$

Prediction includes:

- full electroweak corrections of $\mathcal{O}(\alpha)$ (1-loop level)
- full electroweak corrections of $\mathcal{O}(\alpha^2)$ (2-loop level)
  (v.Ritbergen,Stuart '98; Seidensticker,Steinhauser '99;
  Freitas,Hollik,Walter,Weiglein '00-'02; Awramik,Czakon '02/'03; Onishchenko,Veretin '02)
- various improvements by universal corrections to ρ-parameter

Literature

- Textbooks:
  - Böhm/Denner/Joos: “Gauge Theories of the Strong and Electroweak Interaction”
  - Collins: “Renormalization”
  - Itzykson/Zuber: “Quantum Field Theory”
  - Peskin/Schroeder: “An Introduction to Quantum Field Theory”
  - Weinberg: “The Quantum Theory of Fields, Vol. 1: Foundations”;
    “The Quantum Theory of Fields, Vol. 2: Modern Applications”
- (Incomplete) list of articles on techniques for radiative corrections:
  - one-loop integrals:
    G. ’t Hooft and M. Veltman, Nucl. Phys. B 153 (1979) 365;
    G. Passarino and M. Veltman, Nucl. Phys. B 160 (1979) 151;
    W. Beenakker and A. Denner, Nucl. Phys. B 338 (1990) 349;
    A. Denner, U. Nierste and R. Scharf, Nucl. Phys. B 367 (1991) 637;
    A. Denner and S. Dittmaier, Nucl. Phys. B 734 (2006) 62 and references therein
  - renormalization of the electroweak SM:
    K. I. Aoki, Z. Hioki, M. Konuma, R. Kawabe and T. Muta, Prog. Theor. Phys. Suppl. 73 (1982) 1;
    M. Böhm, W. Hollik and H. Spiesberger, Fortsch. Phys. 34 (1986) 687;
    W. F. Hollik, Fortsch. Phys. 38 (1990) 165;
    A. Denner, Fortsch. Phys. 41 (1993) 307;
    A. Denner, S. Dittmaier and G. Weiglein, Nucl. Phys. B 440 (1995) 95
  - IR structure of photon radiation:
    D. R. Yennie, S. C. Frautschi and H. Suura, Annals Phys. 13 (1961) 379.

Introduction into
Standard Model and Precision Physics
– Lecture IV –

Stefan Dittmaier

MPI München

General overview

9 Unstable particles in quantum field theory

9.1 Introduction

Almost all interesting elementary particles are unstable:

- known: leptons μ, τ and massive gauge bosons Z, $\mathrm{W}^\pm$, etc.
- Higgs bosons: H_{SM}, $\{\mathrm{h}, \mathrm{H}, \mathrm{A}, \mathrm{H}^\pm\}_{\mathrm{MSSM}}$
- postulated new particles, e.g. in SUSY: $\tilde{l}$, $\tilde{q}$, $\tilde{g}$, $\tilde{\chi}$ (maybe apart from LSP)

Lifetimes τ too short for detection (e.g. $\tau_{\mathrm{W,Z}} \sim 10^{-25}\mathrm{s} \;\to\; \Delta l = c\tau \sim 10^{-16}\mathrm{m}$)

$\hookrightarrow$ only decay products detected,
unstable particles appear as resonances in certain observables

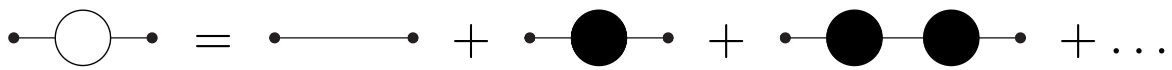
Examples: $\mathrm{e}^+\mathrm{e}^- \to \mathrm{Z} \to f\bar{f}$, $\mathrm{e}^+\mathrm{e}^- \to \mathrm{WW} \to 4f$, $\mathrm{e}^+\mathrm{e}^- \to \mathrm{t}\bar{\mathrm{t}} \to 6f$,
$\mathrm{pp} \to \mathrm{W/Z} \to 2l$, $\mathrm{pp} \to \mathrm{H}{+}2q \to \mathrm{ZZ}{+}2q \to 4l{+}2\mathrm{jets}$, etc.

$\Rightarrow$ Consistent treatment of unstable particles needed
in perturbative evaluation of quantum field theories

9.2 Mass and width of unstable particles

Dyson series and propagator poles

Propagator near resonance: (scalar example)



$$G^{\phi\phi}(p) = \frac{\mathrm{i}}{p^2-m^2} + \frac{\mathrm{i}}{p^2-m^2}\,\mathrm{i}\Sigma(p^2)\,\frac{\mathrm{i}}{p^2-m^2} + \ldots = \frac{\mathrm{i}}{p^2-m^2+\Sigma(p^2)}$$

$\Sigma(p^2) =$ renormalized self-energy, $m =$ ren. mass

Stable particle: $\mathrm{Im}\{\Sigma(p^2)\} = 0$ at $p^2 \sim m^2$

$\hookrightarrow$ propagator pole for real value of p^2,
renormalization condition for physical mass m: $\Sigma(m^2) = 0$

Unstable particle: $\mathrm{Im}\{\Sigma(p^2)\} \neq 0$ at $p^2 \sim m^2$

$\hookrightarrow$ propagator pole shifted into complex p^2 plane,
definition of mass and width non-trivial

Commonly used mass/width definitions:

- "on-shell mass/width" $M_{\mathrm{OS}}/\Gamma_{\mathrm{OS}}$: $\quad M_{\mathrm{OS}}^2 - m^2 + \mathrm{Re}\{\Sigma(M_{\mathrm{OS}}^2)\} \stackrel{!}{=} 0$

$$\hookrightarrow \quad G^{\phi\phi}(p) \quad \underset{p^2 \to M_{\mathrm{OS}}^2}{\widetilde{\qquad}} \quad \frac{1}{(p^2 - M_{\mathrm{OS}}^2)(1 + \mathrm{Re}\{\Sigma'(M_{\mathrm{OS}}^2)\}) + \mathrm{i}\,\mathrm{Im}\{\Sigma(M_{\mathrm{OS}}^2)\}}$$

comparison with form of Breit–Wigner resonance $\quad \dfrac{R_{\mathrm{OS}}}{p^2 - m^2 + \mathrm{i}m\Gamma}$

yields: $\quad M_{\mathrm{OS}}\Gamma_{\mathrm{OS}} \equiv \mathrm{Im}\{\Sigma(M_{\mathrm{OS}}^2)\} \;/\; (1 + \mathrm{Re}\{\Sigma'(M_{\mathrm{OS}}^2)\}), \qquad \Sigma'(p^2) \equiv \frac{\partial\Sigma(p^2)}{\partial p^2}$

- "pole mass/width" M/Γ: $\quad \mu^2 - m^2 + \Sigma(\mu^2) \stackrel{!}{=} 0$

complex pole position: $\quad \mu^2 \equiv M^2 - \mathrm{i}M\Gamma$

$$\hookrightarrow \quad G^{\phi\phi}(p) \quad \underset{p^2 \to \mu^2}{\widetilde{\qquad}} \quad \frac{1}{(p^2 - \mu^2)[1 + \Sigma'(\mu^2)]} = \frac{R}{p^2 - M^2 + \mathrm{i}M\Gamma}$$

Note: $\quad \mu =$ gauge independent for any particle (pole location is property of S-matrix)

$M_{\mathrm{OS}} =$ gauge dependent at 2-loop order $\quad$ Sirlin '91; Stuart '91; Gambino, Grassi '99; Grassi, Kniehl, Sirlin '01

Relation between "on-shell" and "pole" definitions:

Subtraction of defining equations yields:

$$M_{\mathrm{OS}}^2 + \mathrm{Re}\{\Sigma(M_{\mathrm{OS}}^2)\} \;=\; M^2 - \mathrm{i}M\Gamma + \Sigma(M^2 - \mathrm{i}M\Gamma)$$

Equation can be uniquely solved via recursion in powers of coupling α:

ansatz:
$$\begin{aligned} M_{\mathrm{OS}}^2 &= M^2 + c_1\alpha^1 + c_2\alpha^2 + \dots \\ M_{\mathrm{OS}}\Gamma_{\mathrm{OS}} &= M\Gamma + d_2\alpha^2 + d_3\alpha^3 + \dots, \qquad c_i, d_i = \text{real} \end{aligned}$$

counting in α: $\quad M_{\mathrm{OS}}, M = \mathcal{O}(\alpha^0), \quad \Gamma_{\mathrm{OS}}, \Gamma, \Sigma(p^2) = \mathcal{O}(\alpha^1)$

Result:

$$\begin{aligned} M_{\mathrm{OS}}^2 &= M^2 + \mathrm{Im}\{\Sigma(M^2)\}\,\mathrm{Im}\{\Sigma'(M^2)\} \;+\; \mathcal{O}(\alpha^3) \\ M_{\mathrm{OS}}\Gamma_{\mathrm{OS}} &= M\Gamma + \mathrm{Im}\{\Sigma(M^2)\}\,\mathrm{Im}\{\Sigma'(M^2)\}^2 \\ &\qquad + \tfrac{1}{2}\,\mathrm{Im}\{\Sigma(M^2)\}^2\,\mathrm{Im}\{\Sigma''(M^2)\} \;+\; \mathcal{O}(\alpha^4) \end{aligned}$$

i.e. $\{M_{\mathrm{OS}}, \Gamma_{\mathrm{OS}}\} \;=\; \{M, \Gamma\} \;+$ gauge-dependent 2-loop corrections

Important examples: W and Z bosons

In good approximation: $\mathrm{W} \to f\bar{f}'$, $\mathrm{Z} \to f\bar{f}$ with masses fermions f, f'

so that: $$\mathrm{Im}\{\Sigma_\mathrm{T}^\mathrm{V}(p^2)\} = p^2 \times \frac{\Gamma_\mathrm{V}}{M_\mathrm{V}}\,\theta(p^2), \qquad \mathrm{V} = \mathrm{W}, \mathrm{Z}$$

$$\hookrightarrow\; M_\mathrm{OS}^2 = M^2 + \Gamma^2 + \mathcal{O}(\alpha^3) \qquad M_\mathrm{OS}\Gamma_\mathrm{OS} = M\Gamma + \frac{\Gamma^3}{M} + \mathcal{O}(\alpha^4)$$

In terms of measured numbers:

W boson: $M_\mathrm{W} \approx 80\,\mathrm{GeV}$, $\Gamma_\mathrm{W} \approx 2.1\,\mathrm{GeV}$

$\hookrightarrow\; M_\mathrm{W,OS} - M_\mathrm{W,pole} \approx 28\,\mathrm{MeV}$

Z boson: $M_\mathrm{Z} \approx 91\,\mathrm{GeV}$, $\Gamma_\mathrm{Z} \approx 2.5\,\mathrm{GeV}$

$\hookrightarrow\; M_\mathrm{Z,OS} - M_\mathrm{Z,pole} \approx 34\,\mathrm{MeV}$

Exp. accuracy: $\Delta M_\mathrm{W,exp} = 29\,\mathrm{MeV}$, $\Delta M_\mathrm{Z,exp} = 2.1\,\mathrm{MeV}$

$\hookrightarrow$ Difference in definitions phenomenologically important !

A closer look into resonance shapes:

- "on-shell mass/width" $M_{\mathrm{OS}}/\Gamma_{\mathrm{OS}}$: $\qquad M_{\mathrm{OS}}^2 - m^2 + \mathrm{Re}\{\Sigma(M_{\mathrm{OS}}^2)\} \stackrel{!}{=} 0$

$$
\begin{aligned}
G^{\phi\phi}(p) \;&=\; \frac{1}{p^2 - M_{\mathrm{OS}}^2 + \Sigma(p^2) - \mathrm{Re}\{\Sigma(M_{\mathrm{OS}}^2)\}} \\
&\underset{p^2\to M_{\mathrm{OS}}^2}{\widetilde{}}\; \frac{1}{(p^2 - M_{\mathrm{OS}}^2)[1+\mathrm{Re}\{\Sigma'(M_{\mathrm{OS}}^2)\}] + \mathrm{i}\,\mathrm{Im}\{\Sigma(p^2)\} \;+\; \mathcal{O}[(p^2-M_{\mathrm{OS}}^2)^2]} \\
&=\; \frac{R_{\mathrm{OS}}}{p^2 - M_{\mathrm{OS}}^2 + \mathrm{i} M_{\mathrm{OS}}\Gamma_{\mathrm{OS}}(p^2) \;+\; \mathcal{O}[(p^2-M_{\mathrm{OS}}^2)^2]}
\end{aligned}
$$

with the "running on-shell width" $\quad \Gamma_{\mathrm{OS}}(p^2) \;=\; \dfrac{\mathrm{Im}\{\Sigma(p^2)\}}{M_{\mathrm{OS}}[1+\mathrm{Re}\{\Sigma'(M_{\mathrm{OS}}^2)\}]}$

- "pole mass/width" M/Γ: $\qquad \mu^2 - m^2 + \Sigma(\mu^2) \stackrel{!}{=} 0$

$$
\begin{aligned}
G^{\phi\phi}(p) \;&=\; \frac{1}{p^2 - \mu^2 + \Sigma(p^2) - \Sigma(\mu^2)} \\
&\underset{p^2\to \mu^2}{\widetilde{}}\; \frac{1}{(p^2-\mu^2)[1+\Sigma'(\mu^2)] \;+\; \mathcal{O}[(p^2-\mu^2)^2]} \;=\; \frac{R}{(p^2-\mu^2) \;+\; \mathcal{O}[(p^2-\mu^2)^2]}
\end{aligned}
$$

Example of W and Z bosons continued:

Approximation of massless decay fermions:

$$\Gamma_{\mathrm{V,OS}}(p^2) \;=\; \Gamma_{\mathrm{V,OS}} \times \frac{p^2}{M^2_{\mathrm{V,OS}}}\,\theta(p^2), \qquad \mathrm{V} = \mathrm{W}, \mathrm{Z}$$

Fit of W/Z resonance shapes to experimental data:

- ansatz $\left|\dfrac{R'}{p^2 - m'^2 + \mathrm{i}\gamma' p^2/m'}\right|^2$ yields: $m' = M_{\mathrm{V,OS}}, \quad \gamma' = \Gamma_{\mathrm{V,OS}}$
- ansatz $\left|\dfrac{R}{p^2 - m^2 + \mathrm{i}\gamma m}\right|^2$ yields: $m = M_{\mathrm{V,pole}}, \quad \gamma = \Gamma_{\mathrm{V,pole}}$

Note: the two forms are equivalent:

$$R = \frac{R'}{1 + \mathrm{i}\gamma'/m'}, \quad m^2 = \frac{m'^2}{1 + \gamma'^2/m'^2}, \quad m\gamma = \frac{m'\gamma'}{1 + \gamma'^2/m'^2}$$

$\hookrightarrow$ consistent with relation between “on-shell” and “pole” definitions !

9.3 Complex mass and decay widths

Free propagator with finite width:

$$G(x-y) = \int \frac{\mathrm{d}^4 p}{(2\pi)^4}\, \mathrm{e}^{-\mathrm{i}p(x-y)}\, \frac{\mathrm{i}}{p^2 - M^2 + \mathrm{i}M\Gamma}, \qquad \tilde{E}_p = \sqrt{\mathbf{p}^2 + M^2 - \mathrm{i}M\Gamma}$$

$$= \int \frac{\mathrm{d}^3 \mathbf{p}}{(2\pi)^3}\, \mathrm{e}^{\mathrm{i}\mathbf{p}(\mathbf{x}-\mathbf{y})} \int \frac{\mathrm{d}p_0}{2\pi}\, \mathrm{e}^{-\mathrm{i}p_0(x_0-y_0)}\, \frac{\mathrm{i}}{2\tilde{E}_p} \left(\frac{1}{p_0 - \tilde{E}_p} - \frac{1}{p_0 + \tilde{E}_p} \right)$$

Contour integration in p_0 plane yields

$$G(x-y) = \int \frac{\mathrm{d}^3 \mathbf{p}}{(2\pi)^3}\, \mathrm{e}^{\mathrm{i}\mathbf{p}(\mathbf{x}-\mathbf{y})}\, \frac{1}{2\tilde{E}_p} \left[\theta(x_0 - y_0)\mathrm{e}^{-\mathrm{i}(x_0-y_0)\tilde{E}_p} + \theta(y_0 - x_0)\mathrm{e}^{\mathrm{i}(x_0-y_0)\tilde{E}_p} \right]$$

For $\Gamma \ll M$: $\quad \tilde{E}_p \approx E_p - \mathrm{i}\Gamma M/(2E_p), \quad E_p = \sqrt{\mathbf{p}^2 + M^2}$

$$G(x-y) = \int \frac{\mathrm{d}^3 \mathbf{p}}{(2\pi)^3}\, \mathrm{e}^{\mathrm{i}\mathbf{p}(\mathbf{x}-\mathbf{y})}\, \frac{1}{2E_p} \Big[\theta(x_0 - y_0)\mathrm{e}^{-\mathrm{i}(x_0-y_0)E_p}\, \mathrm{e}^{-(x_0-y_0)\Gamma m/(2E_p)}$$
$$+ \theta(y_0 - x_0)\mathrm{e}^{\mathrm{i}(x_0-y_0)E_p}\, \mathrm{e}^{-(y_0-x_0)\Gamma m/(2E_p)} \Big]$$

Exponential decay in particle and antiparticle propagation $x_0 - y_0 \gtrless 0$:

$|G|^2 \propto \mathrm{e}^{\mp(x_0-y_0)\Gamma_p}$ with $\Gamma_p = \Gamma M/E_p = \Gamma/\gamma =$ width of particle with momentum p

Conventional definition of decay widths via amplitudes:

Partial decay widths for $\phi \to f$:

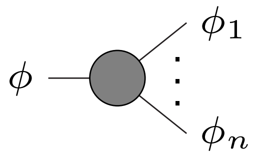
$$\Gamma_{\phi\to f,\mathrm{conv}} = \underbrace{\frac{1}{2m}}_{\text{flux factor}} \int \mathrm{d}\Phi_{\phi\to f}\, \left|\mathcal{M}_{\phi\to f}\right|^2$$

Comments:

- Lorentz-invariant phase space for final state $|f\rangle = |\phi_1(k_1),\dots,\phi_n(k_n)\rangle$:

$$\int \mathrm{d}\Phi_{\phi\to f} = \left[\prod_{l=1}^{n} \int \frac{\mathrm{d}^4 k_l}{(2\pi)^4}\,(2\pi)\delta(k_l^2-m_l^2)\theta(k_l^0)\right](2\pi)^4\delta\!\left(p-\textstyle\sum_{m=1}^{n} k_m\right)$$

- Transition matrix element $\mathcal{M}_{\phi\to f}$ calculated from diagrams



  Note: $\mathcal{M}_{\phi\to f}$ involves external unstable particle ϕ

  $\hookrightarrow$ problems expected in higher orders !

- Mass definition of ϕ relevant

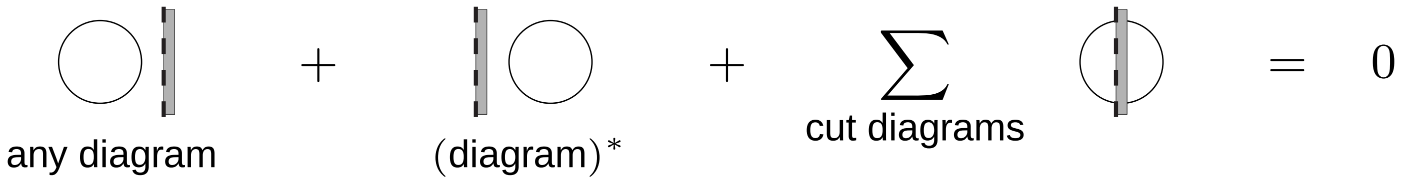
  $\hookrightarrow$ usual choice at 1–2 loops: $m = M_{\mathrm{OS}}$

Total decay width: $\Gamma_{\mathrm{conv}} = \sum_f \Gamma_{\phi\to f,\mathrm{conv}}$

$\hookrightarrow$ Relation between Γ_{conv} and “on-shell” / pole definitions ? Answer by unitarity...

9.4 Unstable particles and unitarity in (perturbative) QFT

Causality implies Cutkowsky cut rules for diagrams:



cut propagators: $2\pi\,\delta(p^2-m^2)\,\theta(p_0)$

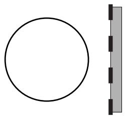
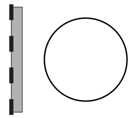
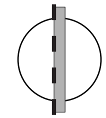
$=$ phase space for free propagation

$\hookrightarrow$ sum over connected diagrams: $\quad (S-\mathbf{1}) + (S-\mathbf{1})^\dagger + (S-\mathbf{1})(S-\mathbf{1})^\dagger = 0$

$\Rightarrow$ unitarity of S-matrix: $\quad SS^\dagger = \mathbf{1}$

9.4 Unstable particles and unitarity in (perturbative) QFT

Causality implies Cutkowsky cut rules for diagrams:

 $+$  $+$ $\sum_{\text{cut diagrams}}$  $= \quad 0$

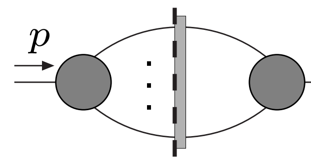
any diagram (diagram)*

cut propagators: $2\pi\,\delta(p^2-m^2)\,\theta(p_0)$
$=$ phase space for free propagation

$\hookrightarrow$ sum over connected diagrams: $(S-\mathbf{1}) + (S-\mathbf{1})^\dagger + (S-\mathbf{1})(S-\mathbf{1})^\dagger = 0$

$\Rightarrow$ unitarity of S-matrix: $SS^\dagger = \mathbf{1}$

Application to self-energy $\rightarrow$ relation to conventional decay width Γ_{conv}

$$\mathrm{Im}\{\Sigma^{\phi\phi}(p^2)\}\Big|_{p^2=m^2} = \frac{1}{2}\sum_{\text{cut diagrams}} \phi$$ 

ϕ

$$= \frac{1}{2}\sum_{\text{decay channels } f} \underbrace{\int \mathrm{d}\Phi_{\phi\to f}\,\left|\mathcal{M}_{\phi\to f}\right|^2}_{=2m\,\Gamma_{\phi\to f,\text{conv}}} = m\,\Gamma_{\text{conv}}$$

$\Rightarrow \quad \Gamma_{\phi\to f,\text{conv}} = \Gamma_{\phi\to f,\text{OS}}$ for mass definition $m = M_{\text{OS}}$

Note: “derivation” quite sloppy (ignores problems in defining $\mathcal{M}_{\phi\to f}$ in higher orders !)

Subtleties with S-matrix elements and unitarity

S-matrix elements:

Definition for $|i\rangle \to |f\rangle$: $$\langle f|S|i\rangle = \lim_{\substack{t_0\to-\infty \\ t_1\to+\infty}} \langle f,t_1|S|i,t_0\rangle$$

But: for unstable states $|f\rangle$: $$\lim_{t_1\to+\infty} \langle f,t_1| = 0$$

$\hookrightarrow$ S-matrix elements for external unstable particles do not exist,
application of LSZ reduction not justified !

Unitarity:

Cut equations not consistent for external or internal unstable particles !

But: important result of Veltman '63

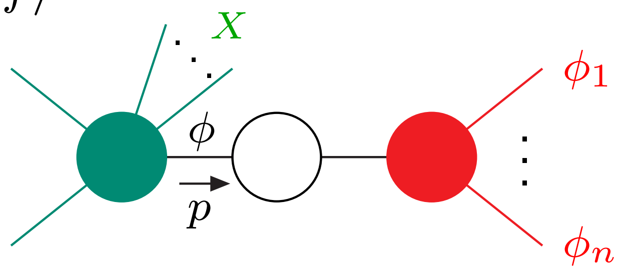
Toy field theory with stable and unstable scalar fields
$\hookrightarrow$ theory is unitary, causal, and renormalizable on space of stable external states

Comments on Veltman's result:

- cut equations: no cuts of internal propagators for unstable particles
- statement rests on consideration of "complete" (resummed) propagators
  $\hookrightarrow$ does not provide a practical method for standard perturbation theory

9.5 Resonances – factorization into production and decay subprocesses

Transition rate near a resonance: $|i\rangle \;\to\; |X,\phi(p)\rangle \;\to\; |X,f\rangle$



$$\int \mathrm{d}\Phi_{i\to Xf}\,|\mathcal{M}_{i\to Xf}|^2 \;\underset{p^2\to m^2}{\sim}\; \int \mathrm{d}\Phi_{i\to Xf}\,|\mathcal{M}_{i\to X\phi\to Xf}|^2$$

Phase-space factorization:

$$\int \mathrm{d}\Phi_{i\to Xf} \;=\; \int \frac{\mathrm{d}p^2}{2\pi} \int \mathrm{d}\Phi_{i\to X\phi(p)} \int \mathrm{d}\Phi_{\phi(p)\to f}$$

Decomposition of resonance diagrams:

$$\mathcal{M}_{i\to X\phi\to Xf} \;=\; \sum_\lambda \mathcal{M}^{(\lambda)}_{i\to X\phi}\,\frac{1}{p^2-m^2+\mathrm{i}m\Gamma}\,\mathcal{M}^{(\lambda)}_{\phi\to f}, \qquad \lambda = \text{polarization index of } \phi$$

$\hookrightarrow$ total rate proportional to (hat on $\hat{\Phi}$, $\hat{\mathcal{M}}$ means $p^2{=}m^2$ used)

$$\int \mathrm{d}\Phi_{i\to Xf}\,|\mathcal{M}_{i\to Xf}|^2 \;\underset{p^2\to m^2}{\sim}\; \sum_{\lambda,\lambda'} \int \mathrm{d}\hat{\Phi}_{i\to X\phi(p)}\,\hat{\mathcal{M}}^{(\lambda)}_{i\to X\phi}\big(\hat{\mathcal{M}}^{(\lambda')}_{i\to X\phi}\big)^*$$

$$\times \int \frac{\mathrm{d}p^2}{2\pi}\,\frac{1}{|p^2-m^2+\mathrm{i}m\Gamma|^2}\;\underbrace{\int \mathrm{d}\hat{\Phi}_{\phi(p)\to f}\,\hat{\mathcal{M}}^{(\lambda)}_{\phi\to f}\big(\hat{\mathcal{M}}^{(\lambda')}_{\phi\to f}\big)^*}_{=D_{\lambda\lambda'}\ \text{(“decay correlation”)}}$$

Manipulations for total rate:

- Rotational invariance in ϕ rest frame implies for full integral $\int \mathrm{d}\Phi_{\phi(p)\to f}$

  $D_{\lambda\lambda'} = \delta_{\lambda\lambda'}\,\bar{D}$, with spin average $\bar{D} = 2m\Gamma_{\phi\to f,\mathrm{conv}}$

- "Narrow-width approximation" (NWA) for resonance factor:

$$\frac{1}{|p^2 - m^2 + \mathrm{i}m\Gamma|^2} = \frac{1}{(p^2-m^2)^2 + m^2\Gamma^2} \underset{\Gamma\to 0}{\widetilde{\quad}} \frac{\pi}{m\Gamma}\,\delta(p^2-m^2)$$

Resulting NWA for total rate and total cross section:

$$\int \mathrm{d}\Phi_{i\to Xf}\,|\mathcal{M}_{i\to Xf}|^2 \underset{p^2\to m^2}{\longrightarrow} \sum_\lambda \int \mathrm{d}\hat{\Phi}_{i\to X\phi(p)}\,|\hat{\mathcal{M}}^{(\lambda)}_{i\to X\phi}|^2 \underbrace{\frac{\Gamma_{\phi\to f,\mathrm{conv}}}{\Gamma}}_{=\mathrm{BR}_{\phi\to f}}$$
("branching ratio")

$\Rightarrow\ \sigma^{\mathrm{NWA}}_{i\to Xf} = \sigma_{i\to X\phi}\,\mathrm{BR}_{\phi\to f}$ with $\sum_f \mathrm{BR}_{\phi\to f} = 1$ if $\Gamma = \Gamma_{\mathrm{conv}}$

Note: NWA insufficient to describe

- invariant-mass distributions of decay products (needed for resonance shape)
- angular distributions of decay products (needed for spin determination)
- "off-shell effects" resulting from regions with $|p^2 - m^2| \gg m\Gamma$
  (in particular because of neglect of non-resonant diagrams)

9.6 The issue of gauge invariance

Gauge invariance implies...

- Slavnov–Taylor or Ward identites
  = algebraic relations of or between Greens functions

  $\hookrightarrow$ guarantee cancellation of unitarity-violating terms,
  crucial for proof of unitarity of S-matrix

- compensation of gauge-fixing artefacts
  = gauge-parameter independence of S-matrix

  although Greens function (e.g. self-energies) are gauge dependent

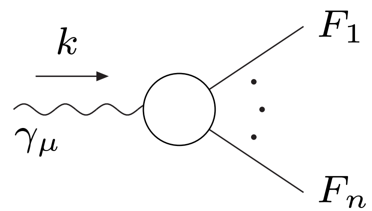
Both statements hold order by order in standard perturbation theory !

But: Resonances require Dyson summation of resonant propagators
$\hookrightarrow$ perturbative orders mixed
$\hookrightarrow$ gauge invariance jeopardized !

Note: Gauge-invariance-violating terms are formally of higher order,
but can be dramatically enhanced

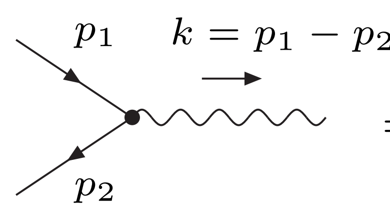
Important Ward identities for processes with EW gauge bosons:

Elmg. U(1) gauge invariance implies

k^μ 

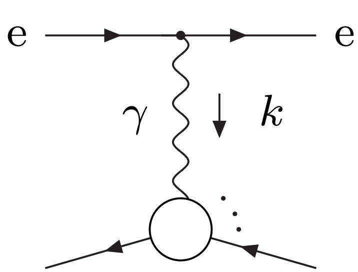
$= \; 0$ for any on-shell fields F_l

$\hookrightarrow$ Identity becomes crucial for collinear light fermions:

for fermion momenta $p_1 \sim c\, p_2$: 

$= \; \bar{u}_2(p_2)\gamma^\mu u_1(p_1) \; \propto \; k^\mu$

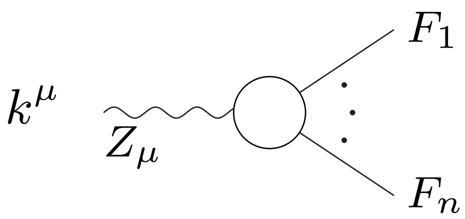
A typical situation: quasi-real space-like photons



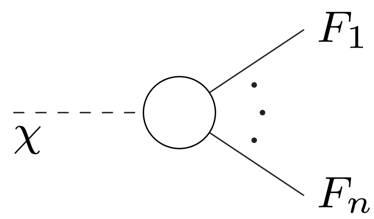
$\sim \; \dfrac{1}{k^2}\, k^\mu\, T^\gamma_\mu$ for $k^2 \; \to \; \mathcal{O}(m_{\mathrm{e}}^2) \; \ll \; E^2$

Identity $k^\mu\, T^\gamma_\mu \; = \; 0$ needed to cancel $1/k^2$,
otherwise gauge-invariance-breaking terms enhanced by E^2/m_{e}^2 ($\sim 10^{10}$ for LEP2)

Electroweak SU(2) gauge invariance implies

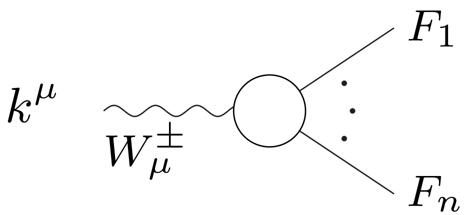


$= \quad \mathrm{i}M_\mathrm{Z}$

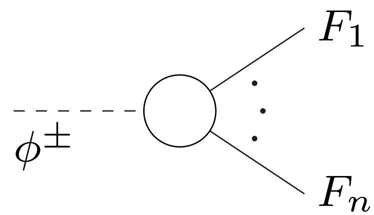


F_l = on-shell fields

$\chi, \phi^\pm$ = would-be Goldstone fields

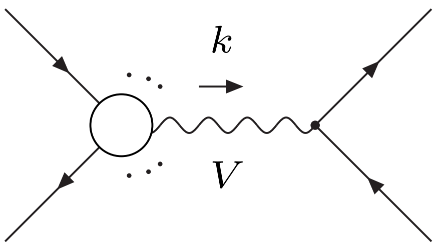


$= \quad \pm M_\mathrm{W}$



A typical situation: high-energetic quasi-real longitudinal vector bosons

$\hookrightarrow$ fermion current attached to $\mathrm{V}(k)$ again $\propto k^\mu$



$$\sim \frac{1}{k^2 - M_\mathrm{V}^2}\, k^\mu\, T^V_\mu \qquad \text{for } \; k^0 \gg M_\mathrm{V}$$

Identity $k^\mu T^V_\mu = c_V\, M_\mathrm{V} T^S$ needed to cancel factor k^0,
otherwise gauge-invariance/unitarity-breaking terms enhanced by k^0/M_V

10 Lowest-order descriptions of resonance processes

10.1 Motivation

The final aim: a method to describe resonance processes in lowest order that

- is mathematically consistent, but simple to apply
- valid in resonant and non-resonant regions of phase space
- supports arbitrary differential distributions
- respects gauge invariance (at least controls breaking effects)
- respects unitarity (at least controls breaking effects)
- can be generalized to higher orders

$\hookrightarrow$ Aim is highly demanding,
different solutions proposed (with different strengths and weaknesses)

Discussed in the following:

naive "solutions" (propagator modifications, fudge factors, etc.),
"complex-mass scheme", "fermion-loop scheme", pole expansions

Not discussed:

proposals of effective field theories Beenakker et al. '00,'03; Beneke et al. '03,'04; Hoang,Reisser '04

Counting of orders in resonance processes:

- self-energy = loop effect: $\Sigma(p^2) = \mathcal{O}(\alpha)$
- width = higher-order effect: $m\Gamma = m^2\,\mathcal{O}(\alpha)$

$\hookrightarrow$ Propagator in resonance region and in the continuum:

$$\frac{m\Gamma}{p^2 - m^2 + \Sigma(p^2)} \sim \frac{m\Gamma}{p^2 - m^2 + \mathrm{i}m\Gamma} = \begin{cases} \mathcal{O}(1) & \text{for } |p^2 - m^2| \sim m\Gamma \\ \mathcal{O}(\alpha) & \text{for } |p^2 - m^2| \gg m\Gamma \end{cases}$$

Implications: [resonant part counted as $\mathcal{O}(1)$]

- higher-order corrections to resonant parts are of $\mathcal{O}(\alpha)$

  $\hookrightarrow$ (virtual+real) corrections to scattering matrix elements and to total width Γ in resonant denominator

- off-shell effects are generically of $\mathcal{O}(\Gamma/m) = \mathcal{O}(\alpha)$

  or in presence of phase-space cuts:

$$\int_{(m-\Delta)^2}^{(m+\Delta)^2} \mathrm{d}p^2\, |\mathcal{M}_{\mathrm{res}}|^2 \propto \int_{(m-\Delta)^2}^{(m+\Delta)^2} \mathrm{d}p^2\, \frac{1}{(p^2-m^2)^2 + m^2\Gamma^2} \sim \frac{\pi}{m\Gamma}$$

$$\int_{(m-\Delta)^2}^{(m+\Delta)^2} \mathrm{d}p^2\, |\mathcal{M}_{\text{non-res}}|^2 \propto \mathcal{O}(\Delta) \qquad \Rightarrow \sigma_{\text{non-res}}/\sigma_{\text{res}} \sim \frac{\Delta\Gamma}{m^2} \quad \text{for } \Delta \gg \Gamma$$

10.2 Naive approaches

Naive propagator substitutions in full tree-level amplitudes:

$$\frac{1}{k^2 - m^2} \to \frac{1}{k^2 - m^2 + im\Gamma(k^2)} \qquad \text{for resonant or all propagators}$$

- constant width $\Gamma(k^2) = \text{const.}$ $\to$ U(1) respected (if all propagators dressed), SU(2) "mildly" violated
- step width $\Gamma(k^2) \propto \theta(k^2)$ $\to$ U(1) and SU(2) violated
- running width $\Gamma(k^2) \propto \theta(k^2) \times k^2$ $\to$ U(1) and SU(2) violated

  $\hookrightarrow$ results can be totally wrong !

Fudge factor approaches:

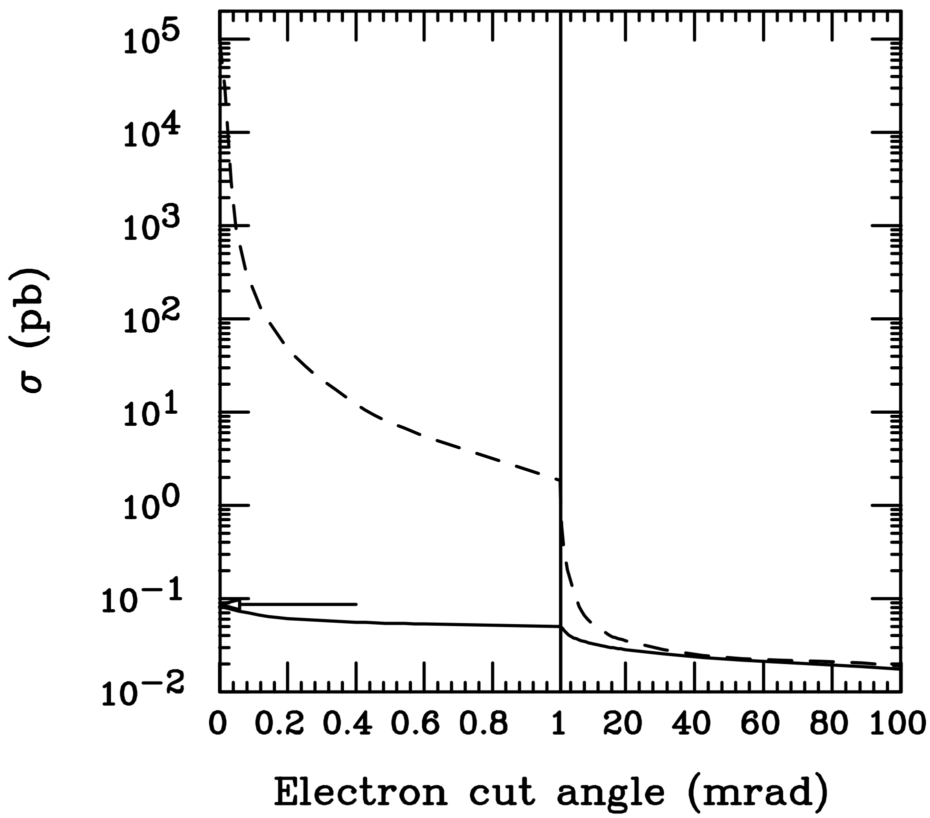
Multiply full amplitudes without widths with

factors $\dfrac{p^2 - m^2}{p^2 - m^2 + im\Gamma}$ for each potentially resonant propagator

$\hookrightarrow$ procedure preserves gauge invariance,
but introduces spurious factors of $\mathcal{O}(\Gamma/m)$

Note: none of these schemes preserves unitarity

An example: $e^-e^+ \to e^-\bar{\nu}_e u\bar{d}$ result of Kurihara, Perret-Gallix, Shimizu '95



$\sqrt{s} = 180\,\mathrm{GeV}$

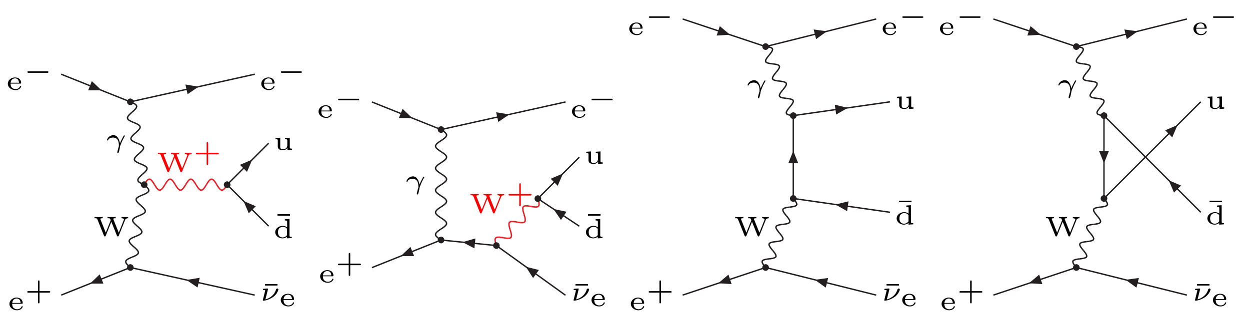
solid: gauge-invariant (fudge factor) scheme

dashed: constant width only in resonant propagator

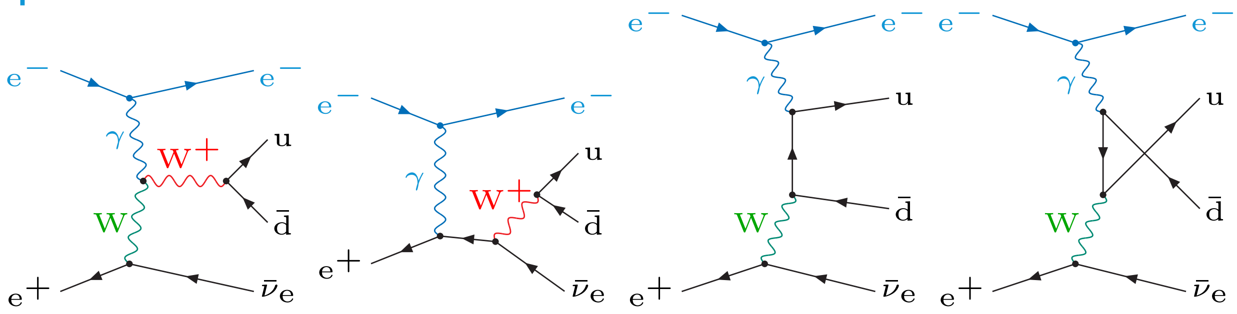
$\hookrightarrow$ crude U(1) gauge-invariance violation

Dominant diagrams:

nearly real photon !



Example continued:



Partial amplitude from above "photon diagrams":

$$\mathcal{M}_\gamma = Q_e e\, \bar{u}_e(k_e)\gamma^\mu u_e(p_e)\, \frac{1}{k_\gamma^2}\, T_\mu^\gamma$$

Elmg. Ward identity:

$$0 \stackrel{!}{=} k_\gamma^\mu T_\mu^\gamma \propto (p_+^2 - p_-^2) Q_W P_w(p_+^2) P_w(p_-^2) + Q_e P_w(p_+^2) - (Q_d - Q_u) P_w(p_-^2)$$

With $Q_W = Q_e = Q_d - Q_u$ and $P_w(p^2) = [p^2 - M_W^2 + \mathrm{i} M_W \Gamma_W(p^2)]^{-1}$

one obtains: $\Gamma_W(p_+^2) \stackrel{!}{=} \Gamma_W(p_-^2)$

$\hookrightarrow$ Elmg. gauge invariance demands
common width on s- and t-channel propagators in "naive fixed width scheme"

10.3 Complex-mass scheme at tree level

Denner, Dittmaier, Roth, Wackeroth '99

Basic idea: mass^2 = location of propagator pole in complex p^2 plane

$\hookrightarrow$ consistent use of complex masses everywhere !

Application to gauge-boson resonances:

- replace $M_\mathrm{W}^2 \;\to\; \mu_\mathrm{W}^2 = M_\mathrm{W}^2 - \mathrm{i} M_\mathrm{W}\Gamma_\mathrm{W}, \qquad M_\mathrm{Z}^2 \;\to\; \mu_\mathrm{Z}^2 = M_\mathrm{Z}^2 - \mathrm{i} M_\mathrm{Z}\Gamma_\mathrm{Z}$

  and define (complex) weak mixing angle via $c_\mathrm{W}^2 = 1 - s_\mathrm{W}^2 = \frac{\mu_\mathrm{W}^2}{\mu_\mathrm{Z}^2}$

  $\hookrightarrow$ preserves all algebraic relations among parameters and amplitudes

- virtue: gauge-invariant result !

  (Slavnov–Taylor identities and gauge-parameter independence)

  $\hookrightarrow$ unitarity cancellations respected !

- drawbacks:
  - ◇ spurios terms of $\mathcal{O}(\frac{\Gamma}{m}) = \mathcal{O}(\alpha)$ (from off-shell propagators and complex mixing angle)

    $\hookrightarrow$ but these terms are beyond tree-level accuracy !
  - ◇ cut equations not valid anymore (reformulation not yet worked out)

    $\hookrightarrow$ unitarity not yet understood, but possible unitarity violation is of $\mathcal{O}(\frac{\Gamma}{m})$

Examples: results from RacoonWW (Denner et al. '99-'01) and Lusifer (Dittmaier,Roth '02)

- $\sigma[\,\mathrm{fb}]$ for $\mathrm{e^+e^-} \to \mathrm{u\bar{d}}\mu^-\bar{\nu}_\mu$

$\sqrt{s}$	189 GeV	500 GeV	2 TeV	10 TeV
constant width	703.5(3)	237.4(1)	13.99(2)	0.624(3)
running width	703.4(3)	238.9(1)	34.39(3)	498.8(1)
complex mass	703.1(3)	237.3(1)	13.98(2)	0.624(3)

- $\sigma[\,\mathrm{fb}]$ for $\mathrm{e^+e^-} \to \mathrm{u\bar{d}}\mu^-\bar{\nu}_\mu + \gamma$ (separation cuts for "visible" γ: $E_\gamma, \theta_{\gamma f} >$ cut)

$\sqrt{s} =$	189 GeV	500 GeV	2 TeV	10 TeV
constant width	224.0(4)	83.4(3)	6.98(5)	0.457(6)
running width	224.6(4)	84.2(3)	19.2(1)	368(6)
complex mass	223.9(4)	83.3(3)	6.98(5)	0.460(6)

- $\sigma[\,\mathrm{fb}]$ for $\mathrm{e^+e^-} \to \nu_\mathrm{e}\bar{\nu}_\mathrm{e}\mu^-\bar{\nu}_\mu\mathrm{u\bar{d}}$ (phase-space cuts applied)

$\sqrt{s}$	500 GeV	800 GeV	2 TeV	10 TeV
constant width	1.633(1)	4.105(4)	11.74(2)	26.38(6)
running width	1.640(1)	4.132(4)	12.88(1)	12965(12)
complex mass	1.633(1)	4.104(3)	11.73(1)	26.39(6)

10.4 Fermion-loop scheme

Argyres et al. '95; Beenakker et al. '96; Passarino '99; Accomando et al. '99

Procedure: Dyson summation of *all* closed fermion-loop graphs

Benefits of the scheme:

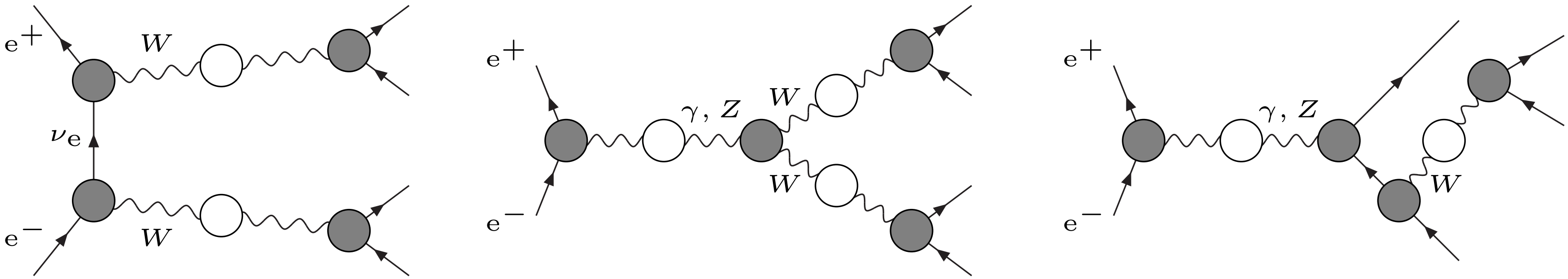
- introduction of widths via resummed self-energies
  for particles that decay into fermions only, e.g. W and Z bosons
- Ward identites (WI) maintained,
  because full set of diagrams of the form $\sum_f N_f^{\mathrm{colour}}$ is considered
- gauge-parameter independence,
  because gauge parameters do not enter loops, and WI are valid for "trees"
- natural inclusion of running-coupling effects possible
- no spurious terms included (selection of diagrams!)
- scheme has natural generalization to remaining "bosonic loops"
  $\hookrightarrow$ background-field quantization Denner, Dittmaier '96

Drawbacks / limitations:

- width in one-loop self-energy is tree-level quantity
  $\hookrightarrow$ scheme does not include fermion-loop corrections to width
- no applicability to unstable particles that decay into bosons (top, Higgs)

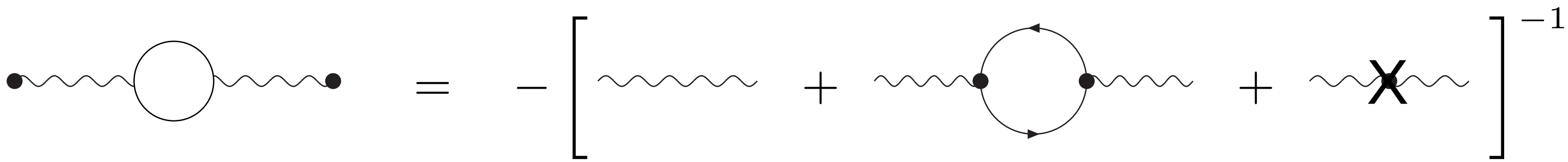
Example: $\mathrm{e}^-\mathrm{e}^+ \to 4f$

Structural diagrams:

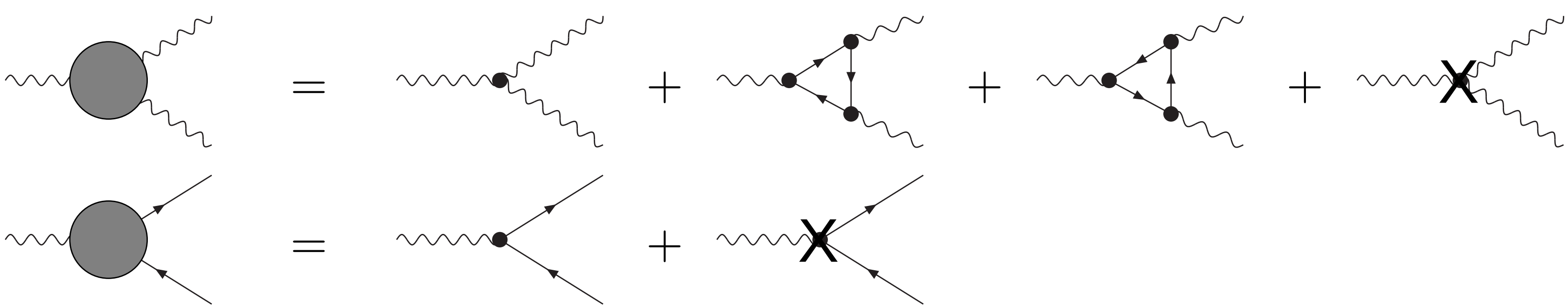


Building blocks:

- resummed propagators:



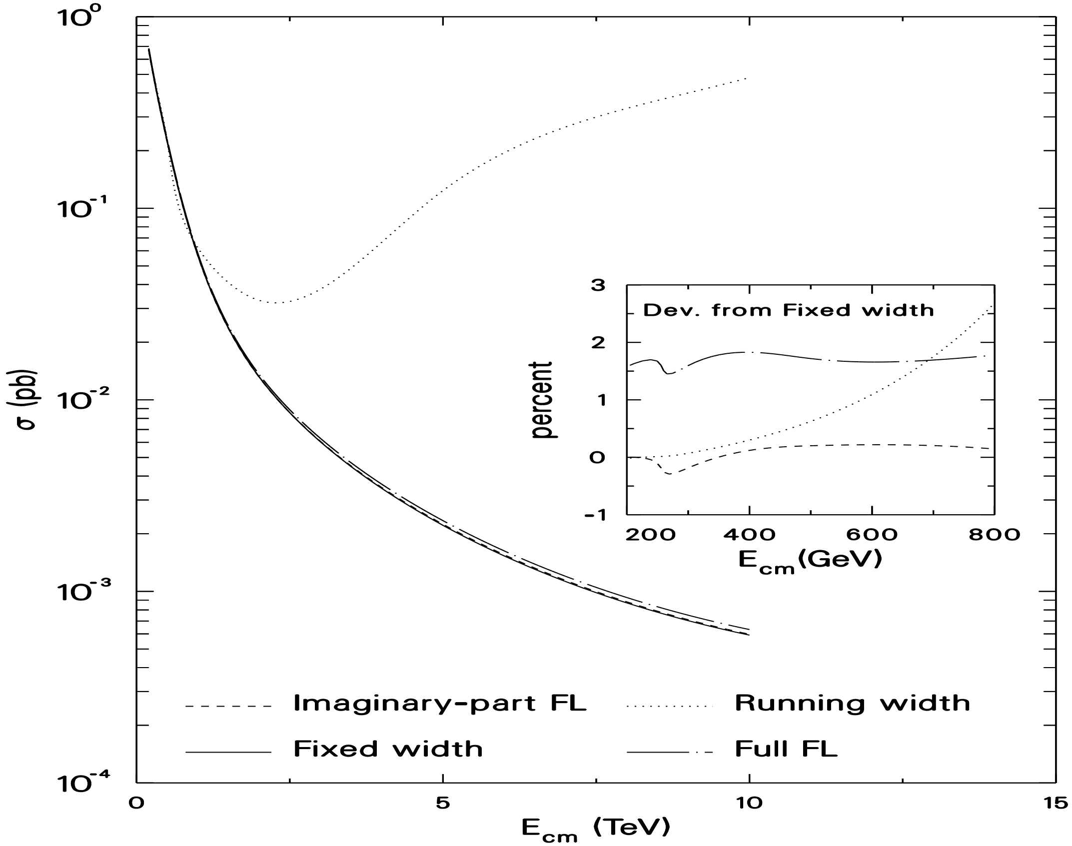
- corrected vertices:



Specific example: $\sigma[\mathrm{fb}]$ for $\mathrm{e}^-\mathrm{e}^+ \to \mu^-\bar{\nu}_\mu \mathrm{u}\bar{\mathrm{d}}$ at high energies

WTO (Passarino '96)

$\sqrt{s}$	200 GeV	500 GeV	1 TeV	2 TeV	5 TeV	10 TeV
running width	672.96(3)	225.45(3)	62.17(1)	33.06(1)	123.759(8)	481.18(5)
constant width	673.08(4)	224.05(3)	56.90(1)	13.19(1)	2.212(6)	0.591(4)
imaginary-part FLS	673.1(1)	224.5(7)	56.8(1)	13.18(4)	2.24(3)	0.597(6)
full FLS	683.7(1)	227.9(2)	58.0(1)	13.57(4)	2.34(3)	0.632(6)



A (not exhaustive) selection of literature

- Unstable particles in quantum field theory
  - ◇ mass and width of unstable particles:
    A. Sirlin, Phys. Rev. Lett. 67 (1991) 2127; Phys. Lett. B 267 (1991) 240;
    R. G. Stuart, Phys. Lett. B 262 (1991) 113; Phys. Rev. Lett. 70 (1993) 3193;
    M. Passera and A. Sirlin, Phys. Rev. Lett. 77 (1996) 4146 [hep-ph/9607253];
    P. Gambino and P. A. Grassi, Phys. Rev. D 62 (2000) 076002 [hep-ph/9907254];
    P. A. Grassi, B. A. Kniehl and A. Sirlin, Phys. Rev. D 65 (2002) 085001 [hep-ph/0109228].
  - ◇ unitarity and causality:
    M. J. G. Veltman, Physica 29 (1963) 186.
- Schemes for treating unstable particles in lowest-order amplitudes (see also references therein)
  - ◇ complex-mass scheme:
    A. Denner, S. Dittmaier, M. Roth and D. Wackeroth, Nucl. Phys. B 560 (1999) 33 [hep-ph/9904472].
  - ◇ fermion-loop scheme:
    E. N. Argyres *et al.*, Phys. Lett. B 358(1995) 339 [hep-ph/9507216];
    W. Beenakker *et al.*, Nucl. Phys. B 500 (1997) 255 [hep-ph/9612260];
    G. Passarino, Nucl. Phys. B 574 (2000) 451 [hep-ph/9911482];
    E. Accomando, A. Ballestrero and E. Maina, Phys. Lett. B 479 (2000) 209 [hep-ph/9911489].
  - ◇ more discussions of gauge-invariance violation:
    Y. Kurihara, D. Perret-Gallix and Y. Shimizu, Phys. Lett. B 349 (1995) 367 [hep-ph/9412215];
    S. Dittmaier and M. Roth, Nucl. Phys. B 642 (2002) 307 [hep-ph/0206070].

Introduction into
Standard Model and Precision Physics
– Lecture V –

Stefan Dittmaier

MPI München

General overview

Lecture I – Standard Model (part 1)

Lecture II – Standard Model (part 2)

Lecture III – Quantum Corrections

Lecture IV – Unstable Particles (part 1)

Lecture V – Unstable Particles (part 2)

11 The pole scheme for radiative corrections to resonance processes

12 Single-W production at hadron colliders

13 $e^+e^- \to WW \to 4f$: double-pole approximation vs. complex-mass scheme

11 The pole scheme for radiative corrections to resonance processes

11.1 General strategy for a single resonance

Stuart '91; H.Veltman '92; Aeppli, v.Oldenborgh, Wyler '94

The idea: expansion about resonance pole

$$\mathcal{M} = \frac{R(p^2)}{p^2-m^2} + N(p^2) = \frac{R(m^2)}{p^2-m^2} + \frac{R(p^2)-R(m^2)}{p^2-m^2} + N(p^2)$$

$$\hookrightarrow \underbrace{\frac{R(m^2)}{p^2-m^2+\mathrm{i}m\Gamma}}_{\text{resonant}} + \underbrace{\frac{R(p^2)-R(m^2)}{p^2-m^2} + N(p^2)}_{\text{non-resonant}}$$

Benefits / drawbacks / subtleties:

- procedure is gauge invariant, because residue $R(m^2)$ is gauge invariant
- scheme is applicable to higher orders
- $R(p^2)$ in general not analytic at $p^2 = m^2$

  $\hookrightarrow$ "non-factorizable corrections" (i.e. not of the form const.$\times$Breit–Wigner)
- $R(m^2)$ is "ambiguous", because it depends on other phase-space variables

  $\hookrightarrow$ $R(m^2)$ depends on choice of phase-space parametrization
- reliability questionable in presence of small scales,
  e.g. γ radiation with $E_\gamma \sim \Gamma$, vicinity of thresholds: $E - E_{\text{threshold}} \sim \Gamma$

The pole expansion including higher orders:

Starting point: complete matrix element
$$\mathcal{M} = \underbrace{\frac{W(p^2)}{p^2 - m^2 + \Sigma(p^2)}}_{=\mathcal{M}'} + N(p^2)$$

Isolation of pole structure:

recall:
$$\begin{aligned} p^2 - m^2 + \Sigma(p^2) &= p^2 - M^2 + \Sigma(p^2) - \Sigma(M^2) \\ &= (p^2 - M^2)[1 + \Sigma'(M^2)] \; + \; \mathcal{O}((p^2 - M^2)^2) \end{aligned}$$

$$\begin{aligned} \Rightarrow \mathcal{M}' &= \frac{W(M^2)}{p^2 - M^2} \frac{1}{1 + \Sigma'(M^2)} + \left[\frac{W(p^2)}{p^2 - m^2 + \Sigma(p^2)} - \frac{W(M^2)}{p^2 - M^2} \frac{1}{1 + \Sigma'(M^2)} \right] \\ &\equiv \frac{w}{p^2 - M^2} + n(p^2) \end{aligned}$$

Comments:

- complex pole mass M as well as w and $\Sigma(M^2)$ are gauge invariant
- evaluation of $W(M^2)$ for complex $p^2 = M^2$ not straightforward !

  But: w and $n(p^2)$ can be perturbatively obtained from quantities with real momenta

  Aeppli et al. '94

Perturbative evaluation of w and $n(p^2)$:

Alternative expansion of resonant diagrams about real mass m^2:

$$\mathcal{M}' = \frac{W(p^2)}{p^2-m^2}\sum_{n=0}^{\infty}\left(\frac{-\Sigma(p^2)}{p^2-m^2}\right)^n = \bar{N}(p^2) + \frac{W_{-1}}{p^2-m^2} + \sum_{n=2}^{\infty}\frac{W_{-n}}{(p^2-m^2)^n}$$

$\hookrightarrow$ perturbative expansion for coefficients:

$$W_{-1} = W(m^2) + \frac{\mathrm{d}}{\mathrm{d}p^2}\Big[-W(p^2)\Sigma(p^2)\Big]_{p^2=m^2} + \frac{1}{2}\frac{\mathrm{d}^2}{\mathrm{d}(p^2)^2}\Big[W(p^2)\Sigma^2(p^2)\Big]_{p^2=m^2} + \dots$$

$$\bar{N}(p^2) = \frac{W(p^2)-W(m^2)}{p^2-m^2} - \frac{W(p^2)\Sigma(p^2) - W(m^2)\Sigma(m^2) - (p^2-m^2)\frac{\mathrm{d}}{\mathrm{d}p^2}\Big[W(p^2)\Sigma(p^2)\Big]_{p^2=m^2}}{(p^2-m^2)^2} + \dots$$

One can show to all orders: (see next slides)

$$w = W_{-1}, \qquad n(p^2) = \bar{N}(p^2)$$

$\hookrightarrow$ residue and non-resonant remainder can be obtained from perturbative calculation with real $p^2 = m^2$

Proof that $w = W^{-1}$:

$$W_{-1} = \sum_{n=0}^{\infty} \frac{1}{n!} \left[\frac{\mathrm{d}^n}{\mathrm{d}s^n} W(s) \Big(-\Sigma(s) \Big)^n \right]_{s=m^2}$$

Expand $[\ldots]$ with $s = M^2 + (m^2 - M^2)$ about $s = M^2$:

$$W_{-1} = \sum_{n=0}^{\infty} \frac{1}{n!} \sum_{k=0}^{\infty} \frac{1}{k!} \left[\frac{\mathrm{d}^{n+k}}{\mathrm{d}s^{n+k}} W(s) \Big(-\Sigma(s) \Big)^n \right]_{s=M^2} \underbrace{(m^2 - M^2)}_{=\Sigma(M^2)}{}^k$$

$$= \sum_{n,k=0}^{\infty} \frac{1}{(n+k)!} \binom{n+k}{n} \left[\frac{\mathrm{d}^{n+k}}{\mathrm{d}s^{n+k}} W(s) \Big(-\Sigma(s) \Big)^n \Big(\Sigma(M^2) \Big)^k \right]_{s=M^2}$$

$$= \sum_{r=0}^{\infty} \frac{1}{r!} \frac{\mathrm{d}^r}{\mathrm{d}s^r} \left[W(s) \underbrace{\Big(\Sigma(M^2) - \Sigma(s) \Big)^r}_{=[-\Sigma'(M^2)]^r (s-M^2)^r + \ldots} \right]_{s=M^2}, \qquad r = n+k$$

Only the terms $\propto [-\Sigma'(M^2)]^r$ survive after setting $(s - M^2)$:

$$\Rightarrow \quad W_{-1} = \sum_{r=0}^{\infty} W(M^2) [-\Sigma'(M^2)]^r = \frac{W(M^2)}{1 + \Sigma'(M^2)} = w$$

Proof that $n(p^2) = \bar{N}(p^2)$:

Formal manipulations with Taylor series:

$$\bar{N}(s) = \frac{W(s) - W(m^2)}{s - m^2} - \frac{W(s)\Sigma(s) - W(m^2)\Sigma(m^2) - (s-m^2)\frac{\mathrm{d}}{\mathrm{d}s}\Big[W(s)\Sigma(s)\Big]_{s=m^2}}{(s-m^2)^2} + \dots$$

$$= \sum_{n=0}^{\infty} (s-m^2)^{-n-1} \Bigg(W(s)\Big(-\Sigma(s)\Big)^n \underbrace{- \sum_{k=0}^{n} \frac{1}{k!} \left[\frac{\mathrm{d}^k}{\mathrm{d}s^k} W(s)\Big(-\Sigma(s)\Big)^n \right]_{s=m^2} (s-m^2)^k}_{\text{subtraction of first } (n+1) \text{ Taylor terms}} \Bigg)$$

$$= \sum_{n=0}^{\infty} \sum_{k=n+1}^{\infty} \frac{1}{k!} \left[\frac{\mathrm{d}^k}{\mathrm{d}s^k} W(s)\Big(-\Sigma(s)\Big)^n \right]_{s=m^2} (s-m^2)^{k-n-1}, \qquad k = n + \ell$$

$$= \sum_{\ell=1}^{\infty} (s-m^2)^{\ell-1} \sum_{n=0}^{\infty} \frac{1}{(n+\ell)!} \left[\frac{\mathrm{d}^{n+\ell}}{\mathrm{d}s^{n+\ell}} W(s)\Big(-\Sigma(s)\Big)^n \right]_{s=m^2}, \qquad n = r - \ell$$

$$= \sum_{\ell=1}^{\infty} (s-m^2)^{\ell-1} \left\{ \sum_{r=0}^{\infty} - \sum_{r=0}^{\ell-1} \right\} \frac{1}{r!} \left[\frac{\mathrm{d}^r}{\mathrm{d}s^r} \frac{W(s)}{[-\Sigma(s)]^\ell} \Big(-\Sigma(s)\Big)^r \right]_{s=m^2}$$

$$= -\frac{W(M^2)}{s-M^2} \frac{1}{1+\Sigma'(M^2)} + \frac{W(s)}{s-m^2+\Sigma(s)}$$

(see next page)

$$= n(s)$$

Proof that $n(p^2) = \bar{N}(p^2)$: (continued)

First term in curly brackets:

$$\sum_{\ell=1}^{\infty}(s-m^2)^{\ell-1}\underbrace{\sum_{r=0}^{\infty}\frac{1}{r!}\left[\frac{\mathrm{d}^r}{\mathrm{d}s^r}\frac{W(s)}{[-\Sigma(s)]^{\ell}}\Big(-\Sigma(s)\Big)^r\right]_{s=m^2}}_{\text{known from proof that } w=W_{-1}}$$

$$= \sum_{\ell=1}^{\infty}(s-m^2)^{\ell-1}\frac{W(M^2)}{[-\Sigma(M^2)]^{\ell}}\frac{1}{1+\Sigma'(M^2)}$$

$$= \frac{1}{-\Sigma(M^2)}\frac{W(M^2)}{1+\Sigma'(M^2)}\sum_{\ell'=0}^{\infty}\frac{(s-m^2)^{\ell'}}{[-\Sigma(M^2)]^{\ell'}} \quad = \quad -\frac{W(M^2)}{s-M^2}\frac{1}{1+\Sigma'(M^2)}$$

Second term in curly brackets:

$$-\sum_{\ell=1}^{\infty}\sum_{r=0}^{\ell-1}(s-m^2)^{\ell-1}\frac{1}{r!}\left[\frac{\mathrm{d}^r}{\mathrm{d}s^r}\frac{W(s)}{[-\Sigma(s)]^{\ell}}\Big(-\Sigma(s)\Big)^r\right]_{s=m^2}, \qquad \ell=\ell'+r$$

$$= -\sum_{\ell'=1}^{\infty}\sum_{r=0}^{\infty}(s-m^2)^{\ell'+r-1}\frac{1}{r!}\left[\frac{\mathrm{d}^r}{\mathrm{d}s^r}\frac{W(s)}{[-\Sigma(s)]^{\ell'}}\right]_{s=m^2}$$

$$= -\sum_{\ell'=1}^{\infty}(s-m^2)^{\ell'-1}\frac{W(s)}{[-\Sigma(s)]^{\ell'}} \quad = \quad \frac{W(s)}{s-m^2+\Sigma(s)}$$

Perturbative ordering in pole scheme:

First step: calculate M^2 from $M^2 = m^2 - \Sigma^{(1)+\ldots+(n+1)}(M^2)$

$\hookrightarrow$ yields Γ in M^2 up to n-loop order

Expansion of matrix element: ($A^{(n)} \equiv n$-loop contribution to A)

$$\mathcal{M} = \frac{W(p^2)}{p^2 - m^2 + \Sigma(p^2)} + N(p^2)$$

$$= \frac{W(M^2)}{p^2 - M^2} \frac{1}{1 + \Sigma'(M^2)} + n(p^2) + N(p^2)$$

$$= \frac{W^{(0)}(m^2)}{p^2 - M^2}$$

LO: leading order in pole approximation

$$+ \frac{W^{(1)}(m^2)}{p^2 - M^2} - \frac{W^{(0)}(m^2)\Sigma^{(1)\prime}(m^2)}{p^2 - M^2} - \frac{W^{(0)\prime}(m^2)\Sigma^{(1)}(m^2)}{p^2 - M^2}$$

$$+ \underbrace{\frac{W^{(0)}(p^2) - W^{(0)}(m^2)}{p^2 - m^2}}_{= n^{(0)}(p^2)} + N^{(0)}(p^2)$$

$+$ non-factorizable corrections

NLO: correction to residue and leading-order off-shellness and non-fact. corrections

$+$ higher orders

Modified (improved!) version of the pole expansion:

Inclusion of lowest order without pole expansion:

$$
\begin{aligned}
\mathcal{M} = \ & \mathcal{M}^{(0)} && \left.\begin{array}{l}\text{LO:}\\ \text{complete leading order}\end{array}\right. \\
& + \frac{W^{(1)}(m^2)}{p^2 - M^2} - \frac{W^{(0)}(m^2)\Sigma^{(1)\prime}(m^2)}{p^2 - M^2} && \left.\begin{array}{l}\text{NLO:}\\ \text{correction to residue}\\ \text{and}\\ \text{non-fact. corrections}\end{array}\right. \\
& + \text{non-factorizable corrections} && \\
& \quad + \text{higher orders}
\end{aligned}
$$

Comments:

- inclusion of $\mathcal{M}^{(0)}$ is usually easier than its expansion
- wave-function correction $\Sigma^{(1)\prime}(m^2) = 0$ in on-shell renormalization scheme
- naive estimate of relative theoretical uncertainty (TU) in NLO:

$$
\mathrm{TU} \quad \sim \quad \begin{cases} \frac{\alpha}{\pi} \times \frac{\Gamma}{m} \times \text{const.} & \text{in resonance region } |p^2 - m^2| \lesssim m\Gamma \\ \frac{\alpha}{\pi} \times \text{const.} & \text{off resonance } |p^2 - m^2| \gg m\Gamma \end{cases}
$$

Factorizable corrections:

$$\mathcal{M}^{(1)}_{\text{fact.}} = \frac{W^{(1)}(m^2) - W^{(0)}(m^2)\Sigma^{(1)\prime}(m^2)}{p^2 - M^2}$$

$$= \sum_\lambda \frac{\mathcal{M}^{(1)}_{\mathrm{production}}(\lambda)\mathcal{M}^{(0)}_{\mathrm{decay}}(\lambda) + \mathcal{M}^{(0)}_{\mathrm{production}}(\lambda)\mathcal{M}^{(1)}_{\mathrm{decay}}(\lambda)}{p^2 - M^2}$$

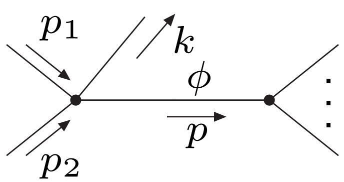
Spin correlations: identical definitions of polarized states $|\phi(\lambda)\rangle$ needed in $\mathcal{M}^{(n)}_{\mathrm{production}}(\lambda)$ and $\mathcal{M}^{(n)}_{\mathrm{decay}}(\lambda)$

Subtlety in kinematics:

gauge invariance of $\mathcal{M}^{(n)}_{\mathrm{production/decay}}$ requires $p^2 = m^2$

$\hookrightarrow$ "on-shell projection" of momenta needed !

Example:



off-shell phase space: $(p_1 + p_2 - k)^2 = p^2 \neq m^2$

$\hookrightarrow$ define $\hat{k}$ (e.g. from angle of k) such that $(p_1 + p_2 - \hat{k})^2 = m^2$

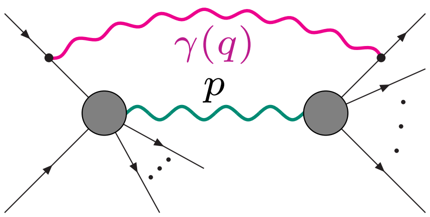
Non-factorizable corrections:

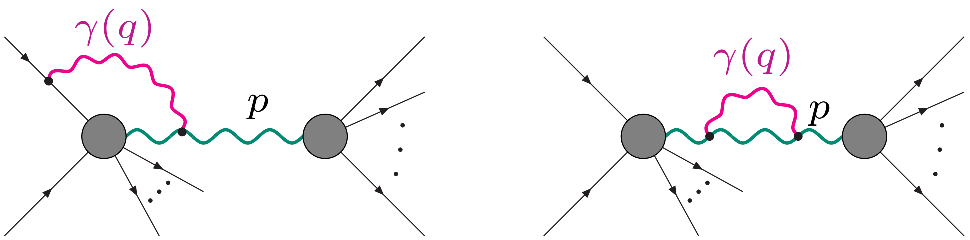
Melnikov, Yakovlev '96; Beenakker, Berends, Chapovsky '97; Denner, Dittmaier, Roth '97,'98

Origin:

on-shell limit ($p^2 \to m^2$) and IR regularization (e.g. $m_\gamma \to 0$) do not commute

in diagrams with exchange of γ/g between external and/or resonant lines:





"manifestly non-factorizable"

- diagram has no explicit propagator factor $(p^2 - m^2)^{-1}$
- resonant IR-divergent contribution in loop integral from region $q \to 0$

"not manifestly non-factorizable" diagrams

- diagram has explicit propagator factor $(p^2 - m^2)^{-1}$ and contributes also to factorizable corrections $W^{(1)}(m^2)$
- non-factorizable part:
  $W^{(1)}_{\text{non-fact.}}(p^2) \equiv \left[W^{(1)}(p^2) - W^{(1)}(m^2)\right]_{p^2 \to m^2}$
  $\hookrightarrow$ receives only contributions from $q \to 0$

Evaluation of NLO non-factorizable corrections:

Only leading behaviour of loop integrands for soft-photon momentum $q \to 0$ relevant

$\hookrightarrow$ "Extended soft-photon (or gluon) approximation":

- neglect q in numerator of diagrams $\quad \to$ scalar loop integrals only
- q only kept in propagators that become singular for $q \to 0$
- resonance propagators are dressed with complex mass: $\quad [(p+q)^2 - M^2]^{-1}$
- take limits $p^2, M^2 \to m^2$ in final result whenever possible

Result factorizes from Born amplitude: $\quad \mathcal{M}^{\mathrm{virt}}_{\text{non-fact.}} = \delta^{\mathrm{virt}}_{\text{non-fact.}} \mathcal{M}^{(0)}$

Features of $\delta^{\mathrm{virt}}_{\text{non-fact.}}$:

- gauge independent by definition
- contains contributions like $\quad \alpha \ln\left(\frac{p^2 - M^2}{m_\gamma M}\right)$

  from non-commutativity of on-shell and soft-photon limits
- free of collinear singularities from external particles
- various cancellations after addition of corresponding real-photon contributions:
  - ◇ no resonant contribution from photon exchange between initial and final states
  - ◇ non-local cancellation of whole effect after integration over p^2

11.2 Real corrections to resonance processes

Calculation of real NLO corrections:

NLO: 1-particle bremsstrahlung in LO (tree-level diagrams)

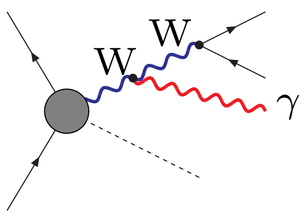
$\hookrightarrow$ LO prescriptions for resonances applicable

But: real $|\mathcal{M}_{i\to f+\gamma/\mathrm{g}}|^2$ is related to $2\,\mathrm{Re}\{\mathcal{M}^{(0)*}_{i\to f}\mathcal{M}^{(1)}_{i\to f}\}$ in soft and collinear limits,

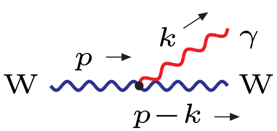
$\hookrightarrow$ matching between resonance descriptions in virtual and real corrections !

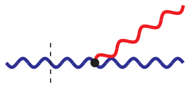
Pole expansions for real corrections:

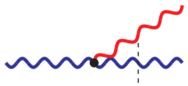
Split diagrams with radiating resonances (2 resonant propagators) as follows:



$$\frac{1}{[(p+k)^2-M^2](p^2-M^2)} = \frac{1}{2pk}\left[\frac{1}{p^2-M^2} - \frac{1}{(p+k)^2-M^2}\right]$$







$E_\gamma \gg \Gamma_\mathrm{W}$ (hard photon): photon can be assigned to production or decay, resonances are well separated in phase space

$\hookrightarrow$ pole-scheme decomposition contains two leading on-shell contributions

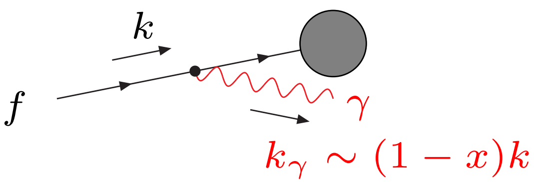
$E_\gamma = \mathcal{O}(\Gamma_\mathrm{W})$ ("semi-soft photon"): two resonances overlap in phase space

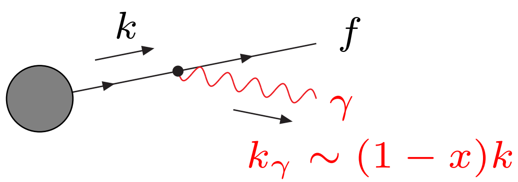
$\hookrightarrow$ definition of leading-pole approximation potentially problematic

(definition depends on specific observable; keep p^2 or $(p-k)^2$ fixed ?)

Enhancement of real-photon emission due to collinear singularities

Collinear photon emission off light particles:





$\hookrightarrow$ leads to mass-singular universal corrections
which can be described via "structure functions" in leading-log approximation:

$$\Gamma_{ff}(x,M^2) \;=\; \delta(1-x) + \frac{Q_f^2\alpha}{2\pi}\,\ln\!\left(\frac{M^2}{m_f^2}\right)\left(\frac{1+x^2}{1-x}\right)_+ + \;\dots$$

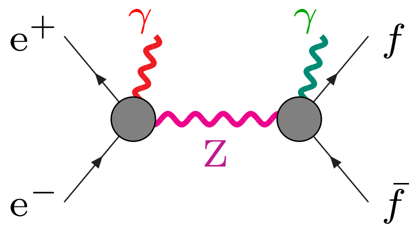
$\hookrightarrow$ e.g. $\sigma^{\mathrm{ISR}}_{\mathrm{e^+e^-}\to X}(p_+,p_-) \approx \int_0^1 \mathrm{d}x_1\,\Gamma_{\mathrm{ee}}(x_1,M^2)\int_0^1 \mathrm{d}x_2\,\Gamma_{\mathrm{ee}}(x_2,M^2)\;\sigma^{\mathrm{Born}}_{\mathrm{e^+e^-}\to X}(x_1p_+,x_2p_-)$

Comments:

- $M =$ QED factorization scale $=$ typical scale of process (set by full calculation)
- structure fucntions Γ_{ff}, etc., known up to $\mathcal{O}(\alpha^5)$ $\oplus$ IR exponentiation
- unitarity / KLN theorem demands $\int_0^1 \mathrm{d}x\,\Gamma_{ff}(x,M^2) = 1$
  $\hookrightarrow$ mass singularities cancel for FSR if $f + n\gamma$ is treated inclusively for collinear γs
- ISR / FSR can lead to large effects, e.g. distortion of resonances

Distortion of resonance shapes by real radiation:

Initial state fixed:



Typical situations: $\mathrm{e^+e^-} \to \mathrm{Z} \to f\bar{f}$,
$\mu^+\mu^- \to \mathrm{Z, H, ?} \to f\bar{f}$

$\hookrightarrow$ scan over s-channel resonance in $\sigma(s)$ by changing CM energy $\sqrt{s}$

Initial-state radiation (ISR):

Z can become resonant for $s = (p_+ + p_-)^2 \; > \; (p_+ + p_- - k_\gamma)^2 \sim M_\mathrm{Z}^2$

$\hookrightarrow$ radiative tail for $s > M_\mathrm{Z}^2$ due to "radiative return"

Final-state radiation (FSR):

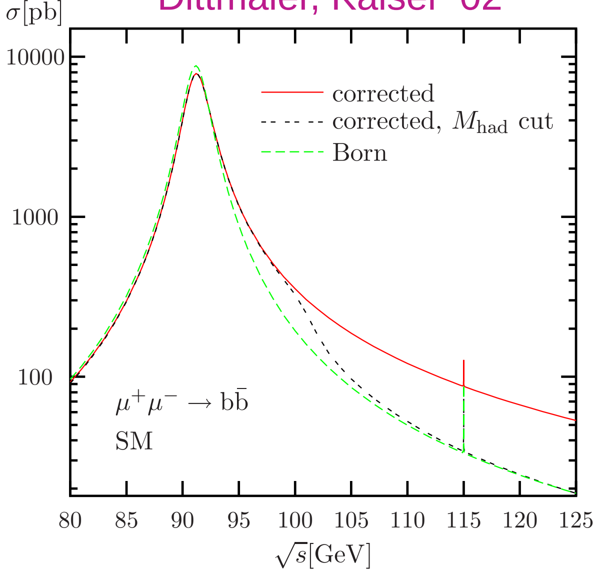
$s = k_\mathrm{Z}^2 \sim M_\mathrm{Z}^2$ for FSR

$\hookrightarrow$ only rescaling of resonance

An example:

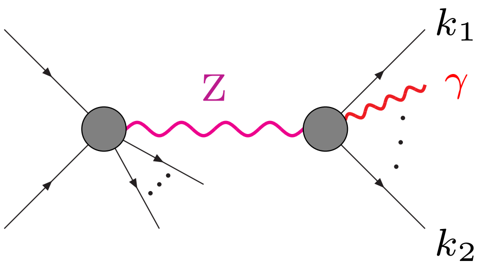
cross section for $\mu^-\mu^+ \to \mathrm{b\bar{b}}$ in lowest order and including photonic and QCD corrections, with and without invariant-mass cut $\sqrt{s} - M(\mathrm{b\bar{b}}) < 10\,\mathrm{GeV}$

Dittmaier, Kaiser '02



Distortion of resonance shapes by real radiation: (continued)

Resonance reconstructed from decay products:



Typical situations: $\mathrm{e^+e^-} \to \mathrm{WW/ZZ} \to 4f$,
$\mathrm{pp} \to \mathrm{Z} \to f\bar{f} + X$

$\hookrightarrow$ resonance in invariant-mass distribution $\dfrac{\mathrm{d}\sigma}{\mathrm{d}M}$ of reconstructed invariant mass M

Final-state radiation (FSR):

resonance for $M^2 = (k_1 + k_2)^2 \; < \; (k_1 + k_2 + k_\gamma)^2 \sim M_\mathrm{Z}^2$

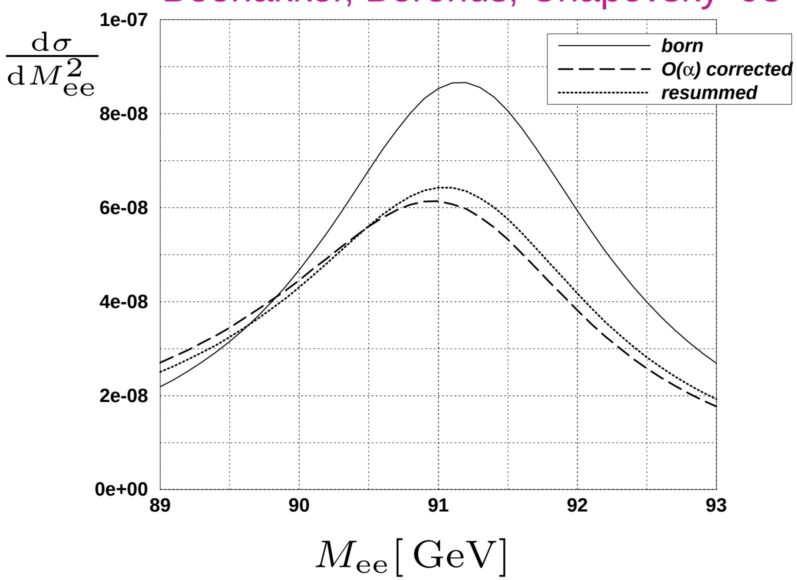
$\hookrightarrow$ radiative tail for $M < M_\mathrm{Z}$

An example:

Z in $\mathrm{e^+e^-} \to \mathrm{ZZ} \to 4l$
reconstructed via $M_\mathrm{ee} = (p_{l_1} + p_{l_2})^2$

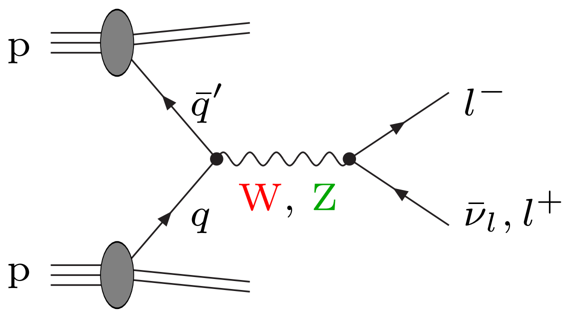
lowest order, $\mathcal{O}(\alpha)$ FSR,
and higher-order FSR beyond $\mathcal{O}(\alpha)$

Beenakker, Berends, Chapovsky '98



12 Single-W production at hadron colliders

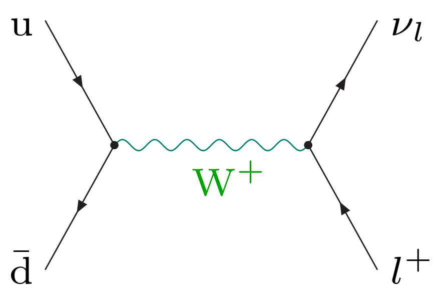
Drell–Yan-like W and Z production:

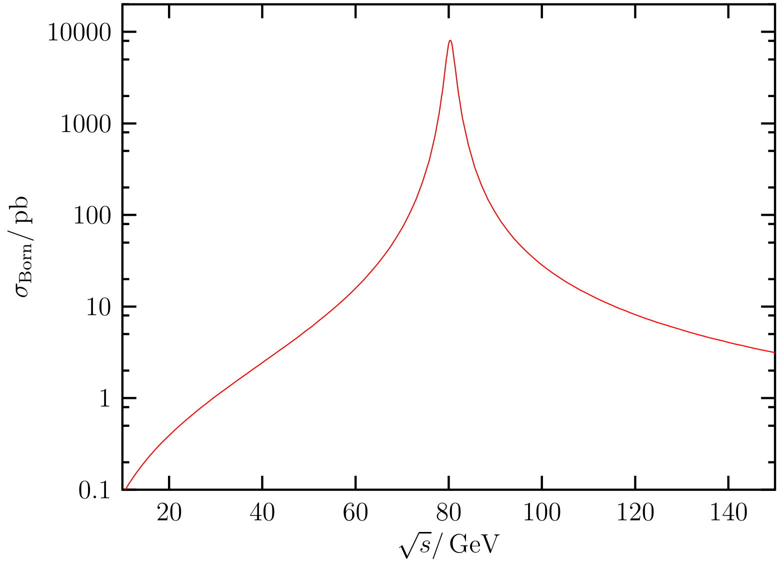


Physics goals:

- $M_Z \;\rightarrow$ detector calibration by comparing with LEP1 result
- $\sin^2\theta^{\mathrm{lept}}_{\mathrm{eff}} \;\rightarrow$ comparison with results of LEP1 and SLC
- $M_W \;\rightarrow$ improvement to $\Delta M_W \sim 15\,\mathrm{MeV}$
- decay widths Γ_Z and Γ_W from M_{ll} or $M_{T,l\nu_l}$ resonance tails
- search for Z' and W' at high M_{ll} or $M_{T,l\nu_l}$
- information on PDFs

Partonic cross section and W-boson resonance:





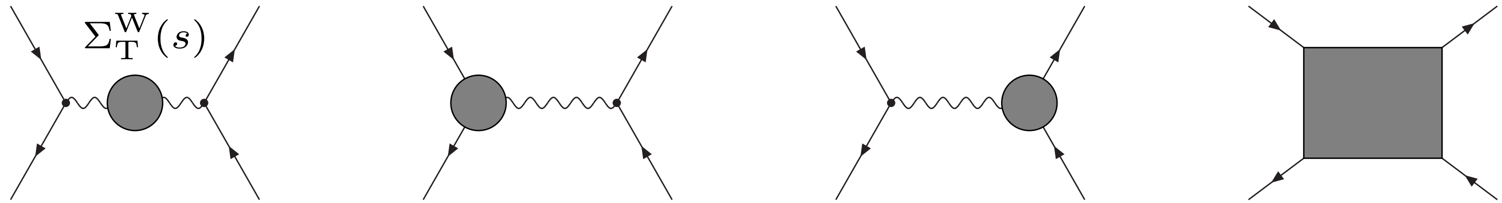
Born amplitude:

$$\mathcal{M}_0 = \frac{e^2}{2s_\mathrm{W}^2} \left[\bar{v}_\mathrm{d}\gamma^\mu \tfrac{1}{2}(1-\gamma_5)u_\mathrm{u}\right] \frac{1}{s - M_\mathrm{W}^2 + \mathrm{i}M_\mathrm{W}\Gamma_\mathrm{W}} \left[\bar{u}_{\nu_l}\gamma_\mu \tfrac{1}{2}(1-\gamma_5)v_l\right]$$

Electroweak corrections:

Dittmaier, Krämer '02; Baur, Wackeroth '04
Arbuzov et al. '05; Carloni Calame et al. '06

- virtual corrections:



W self-energy | Wud and $\mathrm{W}l\nu_l$ vertex corrections | box diagrams

inclusion in factorized form: $|\mathcal{M}_0 + \mathcal{M}_1|^2 = (1 + 2\,\mathrm{Re}\{\delta^\mathrm{virt}\})|\mathcal{M}_0|^2 + \dots$

with $\delta^\mathrm{virt} = \delta_\mathrm{self}(\hat{s}) + \delta_{Wdu}(\hat{s}) + \delta_{W\nu_l l}(\hat{s}) + \delta_\mathrm{box}(\hat{s},\hat{t})$

$\hookrightarrow$ δ^virt gauge independent in limit $\Gamma_\mathrm{W} \to 0$,
non-analytic terms in δ^virt described via $\ln(\hat{s} - M_\mathrm{W}^2) \to \ln(\hat{s} - M_\mathrm{W}^2 + \mathrm{i}M_\mathrm{W}\Gamma_\mathrm{W})$

- real photon corrections:

full amplitude calculation for $\mathrm{u\bar{d}} \to \nu_l l^+ \gamma$ with complex W mass
$\hookrightarrow$ gauge invariant with correct IR (soft and collinear) limits

Electroweak corrections in Pole Approximation (PA):

Baur, Keller Wackeroth '99
Dittmaier, Krämer '02

$\hookrightarrow$ decomposition into factorizable and non-factorizable contributions:

$$\delta^{\mathrm{virt}}_{\mathrm{PA}} = \delta^{\mathrm{virt}}_{\mathrm{fact}} + \delta^{\mathrm{virt}}_{\mathrm{nonfact}}(\hat{s},\hat{t})$$

$$\delta^{\mathrm{virt}}_{\mathrm{fact}} = \delta_{Wdu}(M_\mathrm{W}^2)|_{\Gamma_\mathrm{W}=0} + \delta_{W\nu_l l}(M_\mathrm{W}^2)|_{\Gamma_\mathrm{W}=0}$$

$$\begin{aligned}\delta^{\mathrm{virt}}_{\mathrm{nonfact}}(\hat{s},\hat{t}) &= \delta^{\mathrm{virt}}|_{\hat{s}\to M_\mathrm{W}^2,\Gamma_\mathrm{W}\to 0} - \delta^{\mathrm{virt}}_{\mathrm{fact}} \\ &= -\frac{\alpha}{2\pi}\Bigg\{-2 + Q_d\,\mathrm{Li}_2\left(1+\frac{M_\mathrm{W}^2}{\hat{t}_\mathrm{res}}\right) - Q_u\,\mathrm{Li}_2\left(1+\frac{M_\mathrm{W}^2}{\hat{u}_\mathrm{res}}\right) \\ &\quad + 2\ln\left(\frac{M_\mathrm{W}^2 - \mathrm{i}M_\mathrm{W}\Gamma_\mathrm{W} - \hat{s}}{m_\gamma M_\mathrm{W}}\right)\left[1 + Q_d\ln\left(-\frac{M_\mathrm{W}^2}{\hat{t}_\mathrm{res}}\right) - Q_u\ln\left(-\frac{M_\mathrm{W}^2}{\hat{u}_\mathrm{res}}\right)\right]\Bigg\}\end{aligned}$$

PA versus full $\mathcal{O}(\alpha)$ correction:

$\sqrt{\hat{s}}/\mathrm{GeV}$	40	80	120	200	500	1000	2000
$\hat{\sigma}_0/\mathrm{pb}$	2.646	7991.4	8.906	1.388	0.165	0.0396	0.00979
$\delta/\%$	0.7	2.42	-12.9	-3.3	12	19	23
$\delta_\mathrm{PA}/\%$	0.0	2.40	-12.3	-0.7	18	31	43

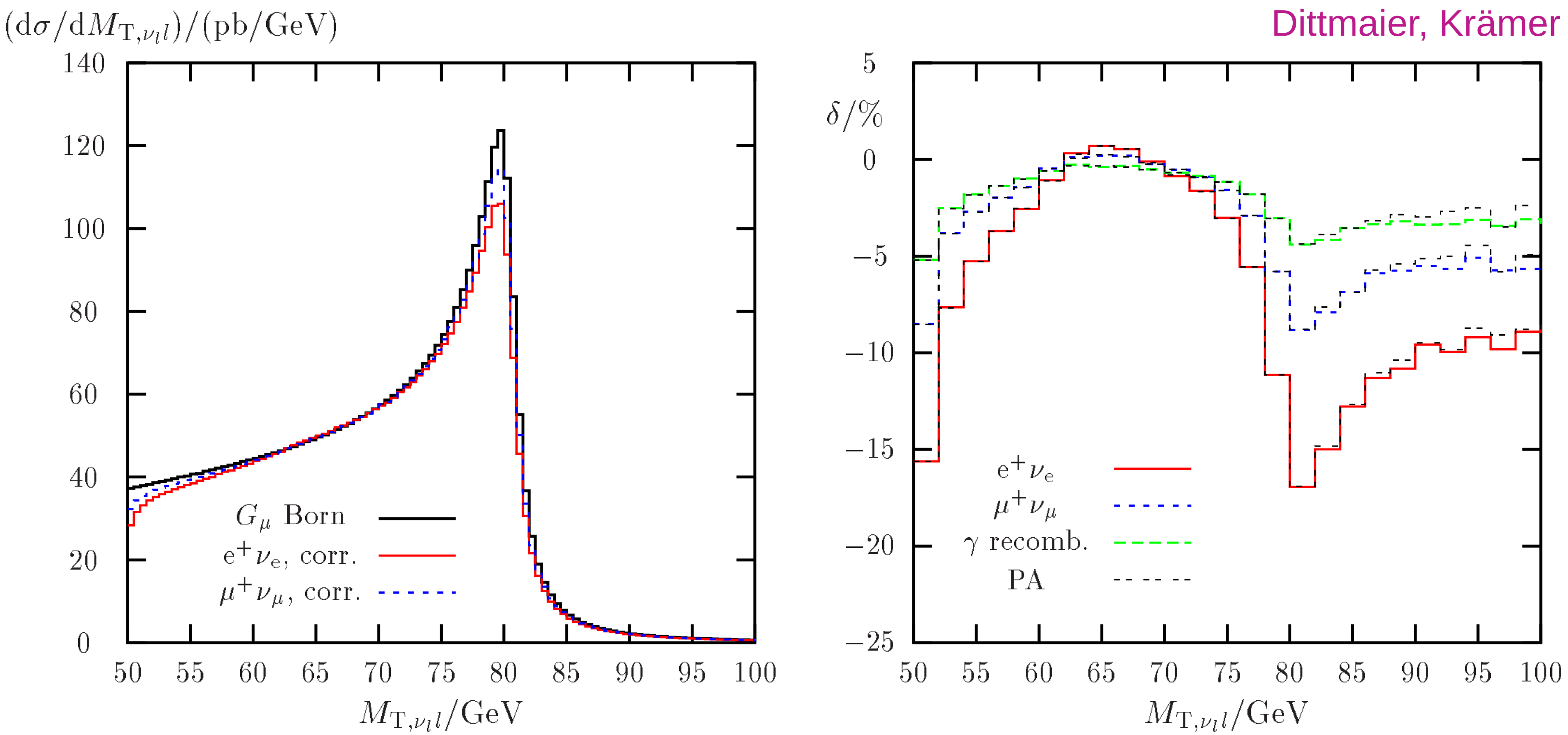
error estimate: $|\delta^{\mathrm{virt}}_{\mathrm{A}} - \delta^{\mathrm{virt}}| \sim \frac{\alpha}{\pi}\max\left\{\frac{\Gamma_\mathrm{W}}{M_\mathrm{W}}, \ln\left(\frac{\hat{s}}{M_\mathrm{W}^2}\right), \ln^2\left(\frac{\hat{s}}{M_\mathrm{W}^2}\right)\right\} \times \mathrm{const.}$

Hadronic pp cross section and Jacobian peak:

Note: ν_l not detectable $\rightarrow$ e.g. study "transverse W mass":

$$M^2_{\mathrm{T},\nu_l l} = (E_{\mathrm{T,miss}} + E_{\mathrm{T},l})^2 - (\mathbf{p}_{\mathrm{T,miss}} + \mathbf{p}_{\mathrm{T},l})^2$$

Dittmaier, Krämer '02



- pole approximation (PA) for W resonance
  sufficient near Jacobian peak, but not for large $M_{\mathrm{T},\nu_l l}$
- EW corrections sensitively depend on treatment of photon radiation
  $\hookrightarrow$ issue of inclusiveness / KLN violation causes large effects

13 $e^+e^-\to WW\to 4f$: double-pole approximation vs. complex-mass scheme

13.1 Double-pole approximation (DPA)

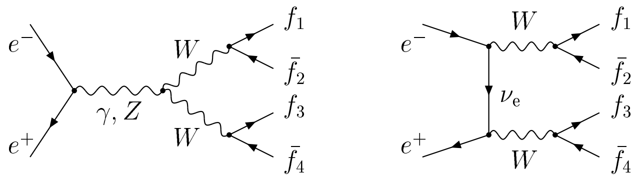
Structure of Monte Carlo generators with EW corrections used at LEP2:

RacoonWW (Denner, Dittmaier,Roth,Wackeroth) and

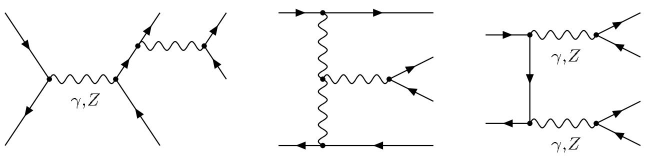
KoralW $\oplus$ *YFSWW* (Jadach,Płaczek,Skrzypek,Ward) include

- full lowest-order matrix elements for $\mathrm{e}^+\mathrm{e}^- \to 4f(+\gamma)$

signal diagrams

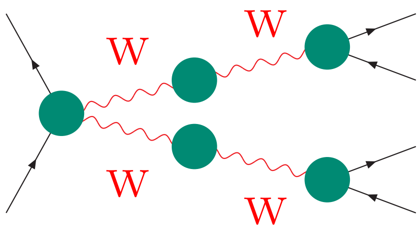


background diagrams



etc.

- non-universal electroweak corrections DPA



leading term in expansion about W resonances

$\hookrightarrow$ contributions:

– corrections to $\mathrm{ee} \to \mathrm{WW}$ — Böhm et al. '88; Fleischer, Jegerlehner, Zralek '89

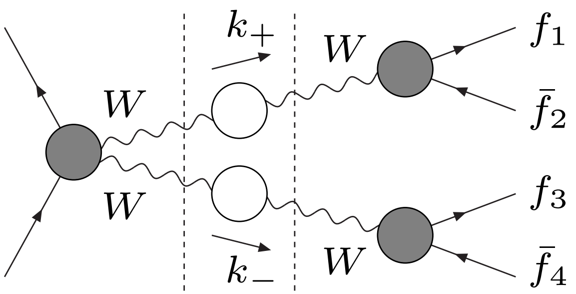
– corrections to $\mathrm{W} \to f\bar{f}'$ — Bardin, S. Riemann, T. Riemann '86; Jegerlehner '86; Denner, Sack '90

– non-factorizable photonic corrections — Melnikov, Yakovlev '96; Beenakker, Berends, Chapovsky '97; Denner, Dittmaier, Roth '97

- improvements by leading higher-order corrections

Virtual corrections in DPA:

- Factorizable corrections:



on-shell production   on-shell decays

$$\mathcal{M}^{e^+e^-\to WW\to 4f}_{\mathrm{virt,fact,DPA}} = \frac{R(M_W^2, M_W^2)}{(k_+^2 - M_W^2 + iM_W\Gamma_W)(k_-^2 - M_W^2 + iM_W\Gamma_W)}$$

with the gauge-independent residue

$$R(M_W^2, M_W^2) = \sum_{\text{W-pols}} \Big(\delta\mathcal{M}^{e^+e^-\to W^+W^-} \mathcal{M}^{W^+\to f_1\bar f_2}_{\mathrm{Born}} \mathcal{M}^{W^-\to f_3\bar f_4}_{\mathrm{Born}} + \mathcal{M}^{e^+e^-\to W^+W^-}_{\mathrm{Born}} \delta\mathcal{M}^{W^+\to f_1\bar f_2} \mathcal{M}^{W^-\to f_3\bar f_4}_{\mathrm{Born}} + \mathcal{M}^{e^+e^-\to W^+W^-}_{\mathrm{Born}} \mathcal{M}^{W^+\to f_1\bar f_2}_{\mathrm{Born}} \delta\mathcal{M}^{W^-\to f_3\bar f_4} \Big)$$

containing the corrections to on-shell production and decay

- Non-factorizable corrections:

$$\mathcal{M}^{\mathrm{e^+e^-\to WW\to 4}f}_{\mathrm{virt,nonfact,DPA}} = \delta\mathcal{M}^{\mathrm{e^+e^-\to 4}f}\Big|_{\text{doubly-resonant part}} - \mathcal{M}^{\mathrm{e^+e^-\to WW\to 4}f}_{\mathrm{virt,fact,DPA}}$$

$$= \mathcal{M}^{\mathrm{e^+e^-\to WW\to 4}f}_{\mathrm{Born,DPA}}\ \delta_{\mathrm{virt,nonfact,DPA}}$$

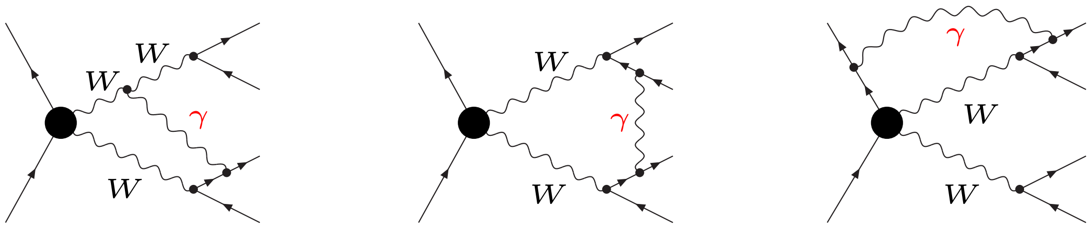
Features of $\delta_{\mathrm{virt,nonfact,DPA}}$ analogous to single-resonance case:

◇ gauge independent, no mass singularities

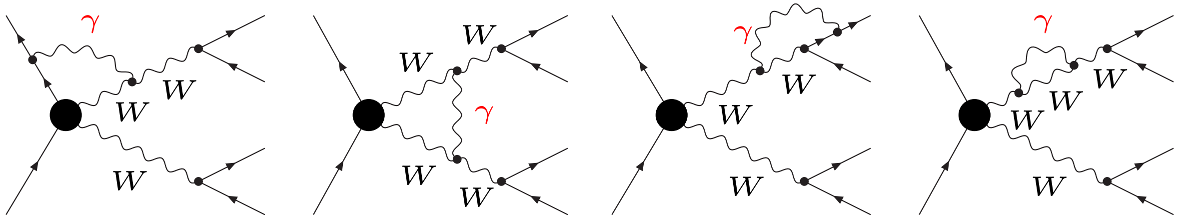
◇ compensates IR singularities of W bosons in $\mathcal{M}^{\mathrm{e^+e^-\to WW\to 4}f}_{\mathrm{virt,fact,DPA}}$

◇ no factorization of Breit–Wigner-type resonances
(complicated dependence on off-shellness $k_\pm^2$ of W bosons)

Manifestly non-factorizable diagrams:



Diagrams contributing to factorizable and non-factorizable RCs:



Combination of contributions: (as implemented in RacoonWW)

$$\int \mathrm{d}\sigma = \frac{1}{2s}\Bigg\{ \int \mathrm{d}\Phi_{4f} \Bigg[|\mathcal{M}_{\mathrm{Born}}^{\mathrm{e^+e^-}\to 4f}|^2 + 2\,\mathrm{Re}\Bigg((\mathcal{M}_{\mathrm{Born,DPA}}^{\mathrm{e^+e^-\to WW}\to 4f})^* \, \delta\mathcal{M}_{\mathrm{virt,fact,DPA}}^{\mathrm{e^+e^-\to WW}\to 4f} + |\mathcal{M}_{\mathrm{Born,DPA}}^{\mathrm{e^+e^-\to WW}\to 4f}|^2 \, \delta_{\mathrm{virt,nonfact,DPA}} \Bigg) \Bigg] + \int \mathrm{d}\Phi_{4f\gamma} \; |\mathcal{M}_{\mathrm{Born}}^{\mathrm{e^+e^-}\to 4f\gamma}|^2 \Bigg\}$$

Note: virtual corrections in DPA $\oplus$ real from full amplitudes

Combination of contributions: (as implemented in RacoonWW)

$$\int \mathrm{d}\sigma = \frac{1}{2s}\Bigg\{ \int \mathrm{d}\Phi_{4f} \Bigg[|\mathcal{M}_{\mathrm{Born}}^{\mathrm{e^+e^-}\to 4f}|^2$$

$$+ 2\,\mathrm{Re}\Bigg((\mathcal{M}_{\mathrm{Born,DPA}}^{\mathrm{e^+e^-}\to\mathrm{WW}\to 4f})^* \, \delta\mathcal{M}_{\mathrm{virt,fact,DPA}}^{\mathrm{e^+e^-}\to\mathrm{WW}\to 4f} + |\mathcal{M}_{\mathrm{Born,DPA}}^{\mathrm{e^+e^-}\to\mathrm{WW}\to 4f}|^2 \, \delta_{\mathrm{virt,nonfact,DPA}} \Bigg)$$

$$+ |\mathcal{M}_{\mathrm{Born,DPA}}^{\mathrm{e^+e^-}\to\mathrm{WW}\to 4f}|^2 \, \delta_{\mathrm{sub,1}}^{4f} \quad \Bigg\} \text{ non-singular}$$

explicit mass singularities $\rightarrow$

$$+ |\mathcal{M}_{\mathrm{Born}}^{\mathrm{e^+e^-}\to 4f}|^2 \otimes \delta_{\mathrm{sub,2}}^{4f} \Bigg]$$

$$+ \int \mathrm{d}\Phi_{4f\gamma} \Bigg[\underbrace{|\mathcal{M}_{\mathrm{Born}}^{\mathrm{e^+e^-}\to 4f\gamma}|^2 - |\mathcal{M}_{\mathrm{Born}}^{\mathrm{e^+e^-}\to 4f}|^2 \, \delta_{\mathrm{sub}}^{4f\gamma}}_{\text{non-singular}} \Bigg] \Bigg\}$$

Note: virtual corrections in DPA $\oplus$ real from full amplitudes

$\hookrightarrow$ redistribution of singular contributions to avoid mismatch in cancellations

From LEP2 to the ILC:

Experimental vs. theoretical uncertainties for some observables:

Observable	$\Delta_{\text{exp}}(\text{LEP2})$	$\Delta_{\text{exp}}(\text{ILC})$	$\Delta_{\text{th}}(\text{DPA}/\text{IBA})$
σ_{WW}	$\sim 1\%$	$\lesssim 0.5\%$	2% for $\sqrt{s} < 170\,\text{GeV}$ (IBA range) 0.7% for $170\,\text{GeV} < \sqrt{s} < 180\,\text{GeV}$ 0.5% for $180\,\text{GeV} < \sqrt{s} < 500\,\text{GeV}$
$M_{\text{W}}(\text{threshold})$	$\sim 200\,\text{MeV}$	$\sim 7\,\text{MeV}$	? but $> 50\,\text{MeV}$
$M_{\text{W}}(\text{reconstr.})$	$\sim 30\,\text{MeV}$	$\sim 10\,\text{MeV}$	$5{-}10\,\text{MeV}$
TGC	some $\%$	$\sim 0.1\%$	$\lesssim 1\%$ at LEP2 ? at $\sqrt{s} \gg 200\,\text{GeV}$

Exceptional case: threshold region and below ($\sqrt{s} < 170\,\text{GeV}$)

error estimate of DPA not reliable

$\to$ description at LEP2 via IBA = “Improved Born Approximation”
(off-shell Born calculation dressed with universal corrections such as ISR)

$\Rightarrow$ DPA/IBA approach sufficiently accurate at LEP2

but precision beyond DPA needed at ILC

$\hookrightarrow$ recent treatment beyond DPA in complex-mass scheme

13.1 The complex-mass scheme at one loop and application to $e^+e^- \longrightarrow 4f$

The complex-mass scheme at one loop
Denner, Dittmaier, Roth, Wieders '05

mass^2 = location of propagator pole in complex p^2 plane

$\hookrightarrow$ complex mass renormalization: $\underbrace{M^2_{\mathrm{W},0}}_{\text{bare mass}} = \mu^2_{\mathrm{W}} + \underbrace{\delta\mu^2_{\mathrm{W}}}_{\text{ren. constant}},$ etc.

$\hookrightarrow$ Feynman rules with complex masses and counterterms

Virtues and drawbacks:

- perturbative calculations as usual
- no double counting of contributions (bare Lagrangian unchanged !)
- spurios terms are of $\mathcal{O}(\alpha^2)$, but spoil unitarity
- complex gauge-boson masses also in loop integrals

Convenient choice:

complex field renormalization $\underbrace{W_0^\pm}_{\text{bare field}} = \left(1 + \frac{1}{2}\underbrace{\delta\mathcal{Z}_W}_{\text{ren. constant}}\right)W^\pm,$ etc.

- complex $\delta\mathcal{Z}_W$ applies to W^+ and W^- $\Rightarrow$ $(W^\pm)^\dagger \neq W^\pm$
- $\delta\mathcal{Z}_W$ drops out in S-matrix elements without external W bosons

Complex renormalization for W bosons explicitly:

On-shell renormalization conditions for renormalized (transverse) self-energy

$$\hat{\Sigma}_{\mathrm{T}}^{W}(\mu_{\mathrm{W}}^2) \;=\; 0, \qquad \hat{\Sigma}_{\mathrm{T}}^{\prime W}(\mu_{\mathrm{W}}^2) \;=\; 0$$

$\hookrightarrow$ μ_{W}^2 is location of propagator pole, and residue = 1

Solution of renormalization conditions:

$$\delta\mu_{\mathrm{W}}^2 \;=\; \Sigma_{\mathrm{T}}^{W}(\mu_{\mathrm{W}}^2), \qquad \delta\mathcal{Z}_W \;=\; -\Sigma_{\mathrm{T}}^{\prime W}(\mu_{\mathrm{W}}^2)$$

Note: evaluation of $\Sigma_{\mathrm{T}}^{W}(p^2)$ at complex p^2 can be avoided

$$\Sigma_{\mathrm{T}}^{W}(\mu_{\mathrm{W}}^2) \;=\; \Sigma_{\mathrm{T}}^{W}(M_{\mathrm{W}}^2) \;+\; (\mu_{\mathrm{W}}^2 - M_{\mathrm{W}}^2)\Sigma_{\mathrm{T}}^{\prime W}(M_{\mathrm{W}}^2) \;+\; \underbrace{\mathcal{O}(\alpha^3)}_{\text{beyond one loop and finite}}$$

$\Rightarrow$ Renormalized W self-energy:

$$\hat{\Sigma}_{\mathrm{T}}^{W}(p^2) \;=\; \Sigma_{\mathrm{T}}^{W}(p^2) - \delta M_{\mathrm{W}}^2 + (p^2 - M_{\mathrm{W}}^2)\delta Z_W$$

with $\quad \delta M_{\mathrm{W}}^2 \;=\; \Sigma_{\mathrm{T}}^{W}(M_{\mathrm{W}}^2), \qquad \delta Z_W \;=\; -\Sigma_{\mathrm{T}}^{\prime W}(M_{\mathrm{W}}^2)$

Differences to the usual on-shell scheme:

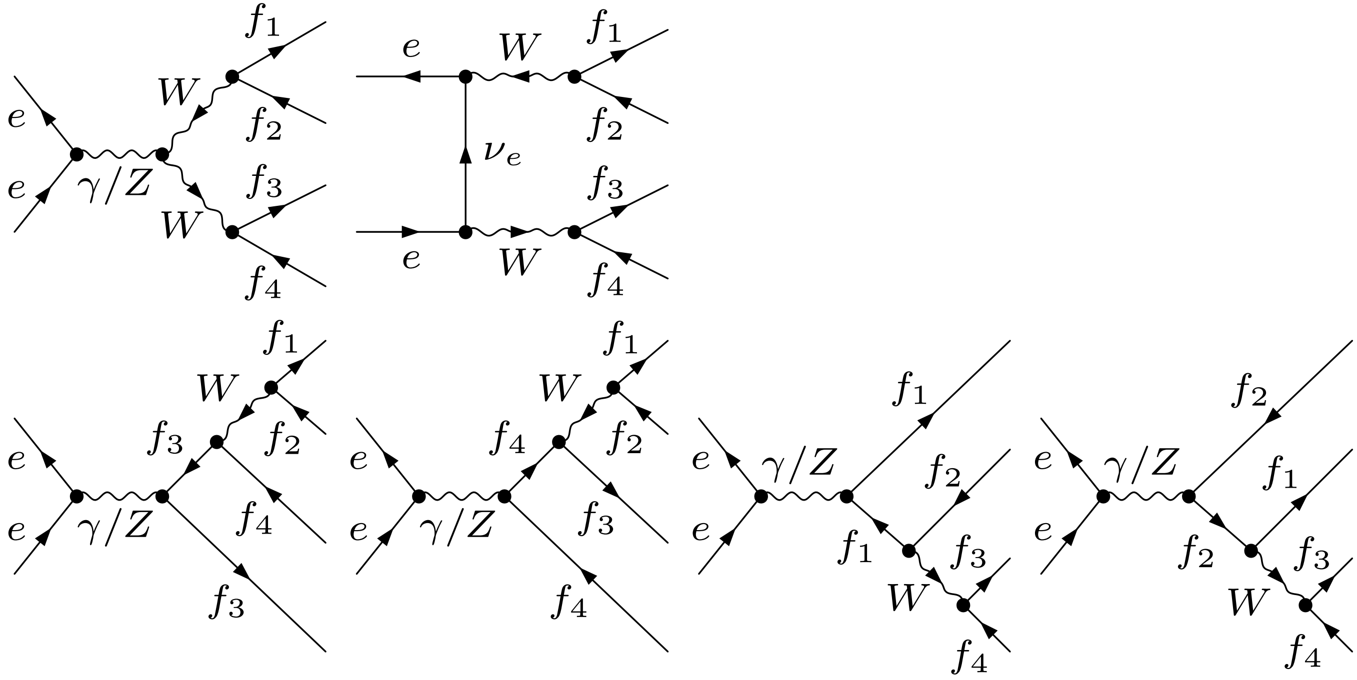
- no real parts taken from Σ_{T}^{W}
- Σ_{T}^{W} evaluated with complex masses and couplings

Full $\mathcal{O}(\alpha)$ corrections to (charged-current) $\mathrm{e}^+\mathrm{e}^- \to 4f$

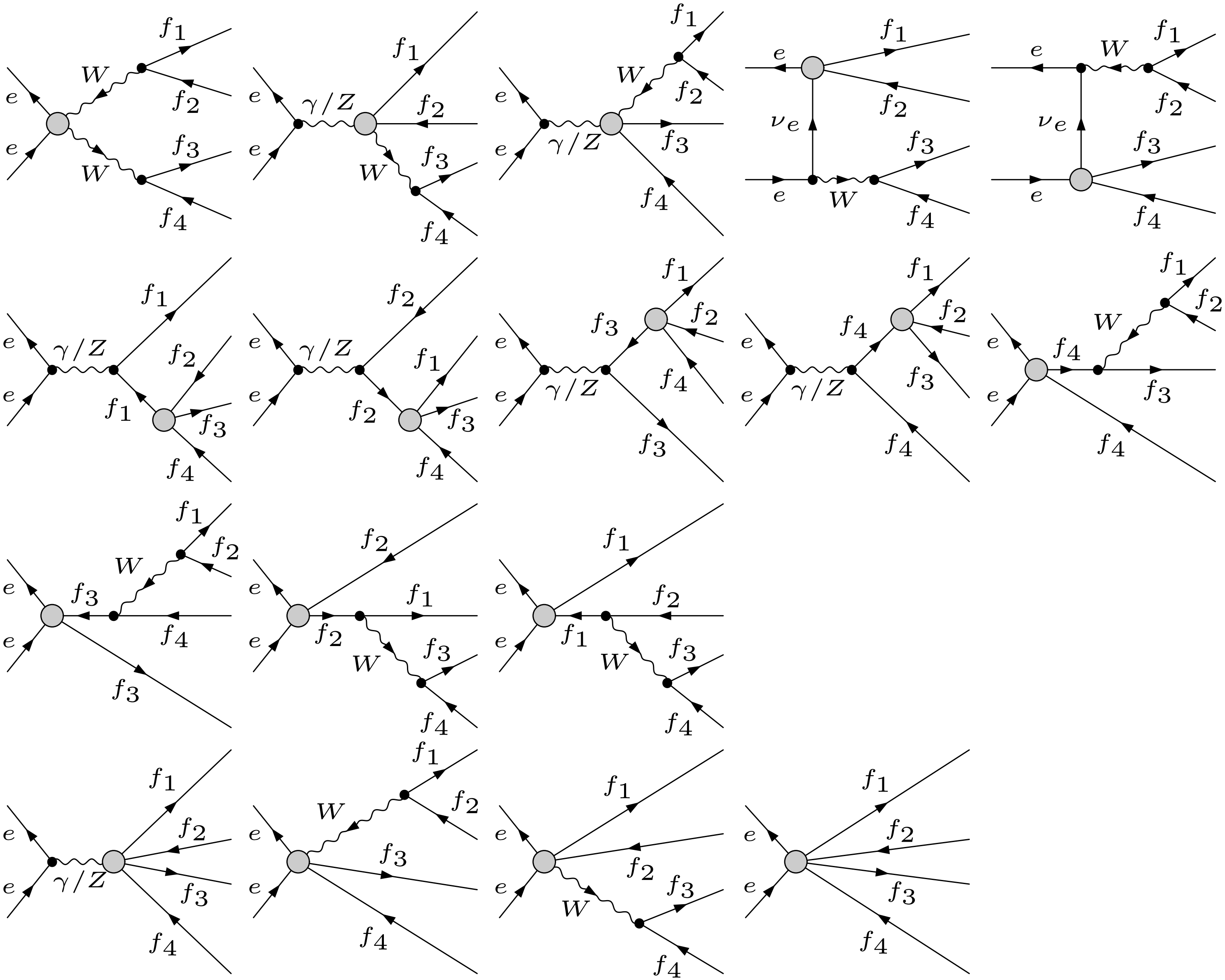
Features of the calculation:

- # 1-loop diagrams $\sim$ 1200, loops up to 6-point integrals
- W resonances treated in the complex-mass scheme
- all loop integrals with complex W/Z masses
- new tensor reduction methods for stability in exceptional phase-space points
- real-photonic corrections taken from RACOONWW

11 lowest-order diagrams: ("CC11 class")

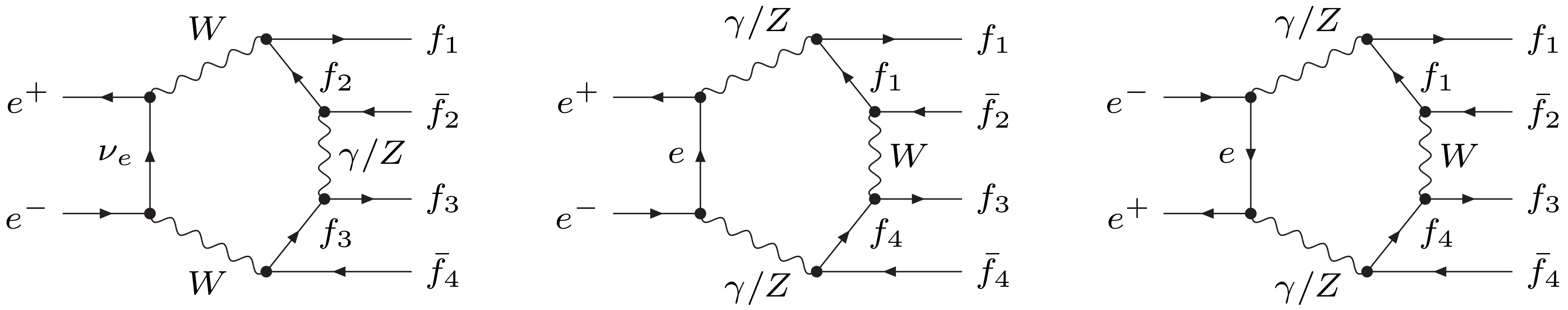


Generic diagrams for loop insertions (4-, 5-, 6-point functions)



$\mathcal{O}(1200)$ one-loop diagrams per channel:

- 40 hexagons

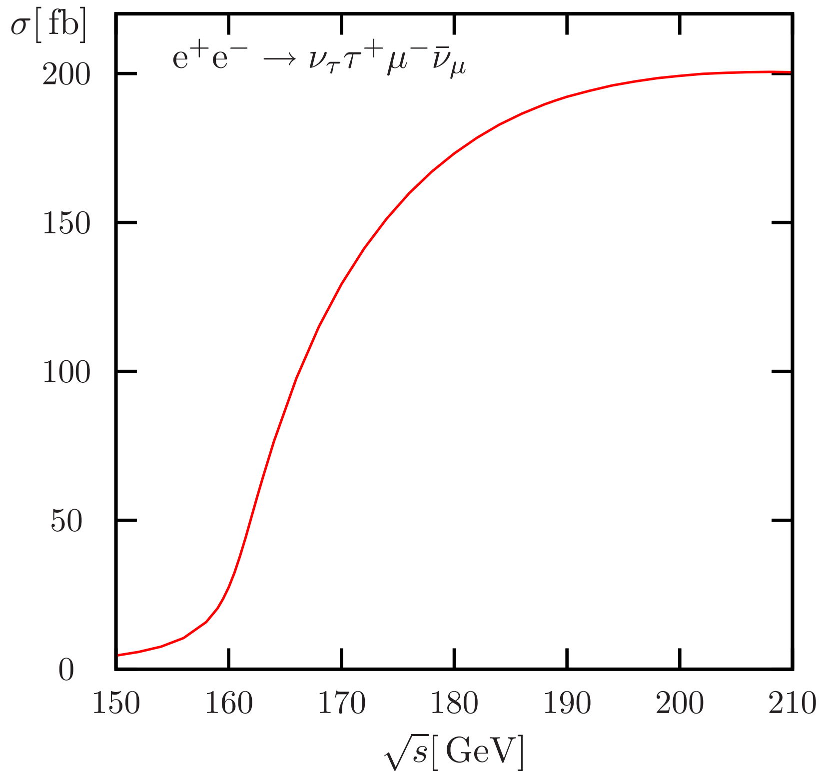


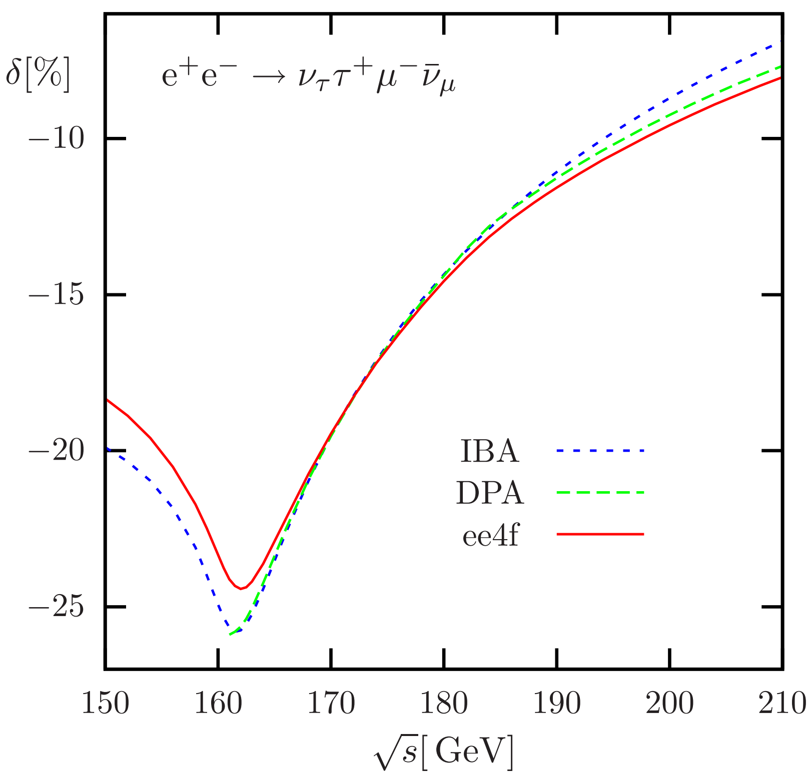
+ graphs with reversed fermion-number flow in final state

- 112 pentagons
- 227 boxes ('t Hooft–Feynman gauge)
- many vertex corrections and self-energy diagrams

Complete $\mathcal{O}(\alpha)$ corrections to the total cross section – LEP2 energies

Denner, Dittmaier, Roth, Wieders '05





- $|\mathrm{ee4f} - \mathrm{DPA}| \quad \sim \quad 0.5\%$ for $170\,\mathrm{GeV} \lesssim \sqrt{s} \lesssim 210\,\mathrm{GeV}$
- $|\mathrm{ee4f} - \mathrm{IBA}| \quad \sim \quad 2\%$ for $\sqrt{s} \lesssim 170\,\mathrm{GeV}$

$\hookrightarrow$ agreement with error estimates of DPA and IBA

Remaining theoretical uncertainty from higher-order EW effects $\sim$ a few 0.1%

Complete $\mathcal{O}(\alpha)$ corrections to the total cross section – ILC energies

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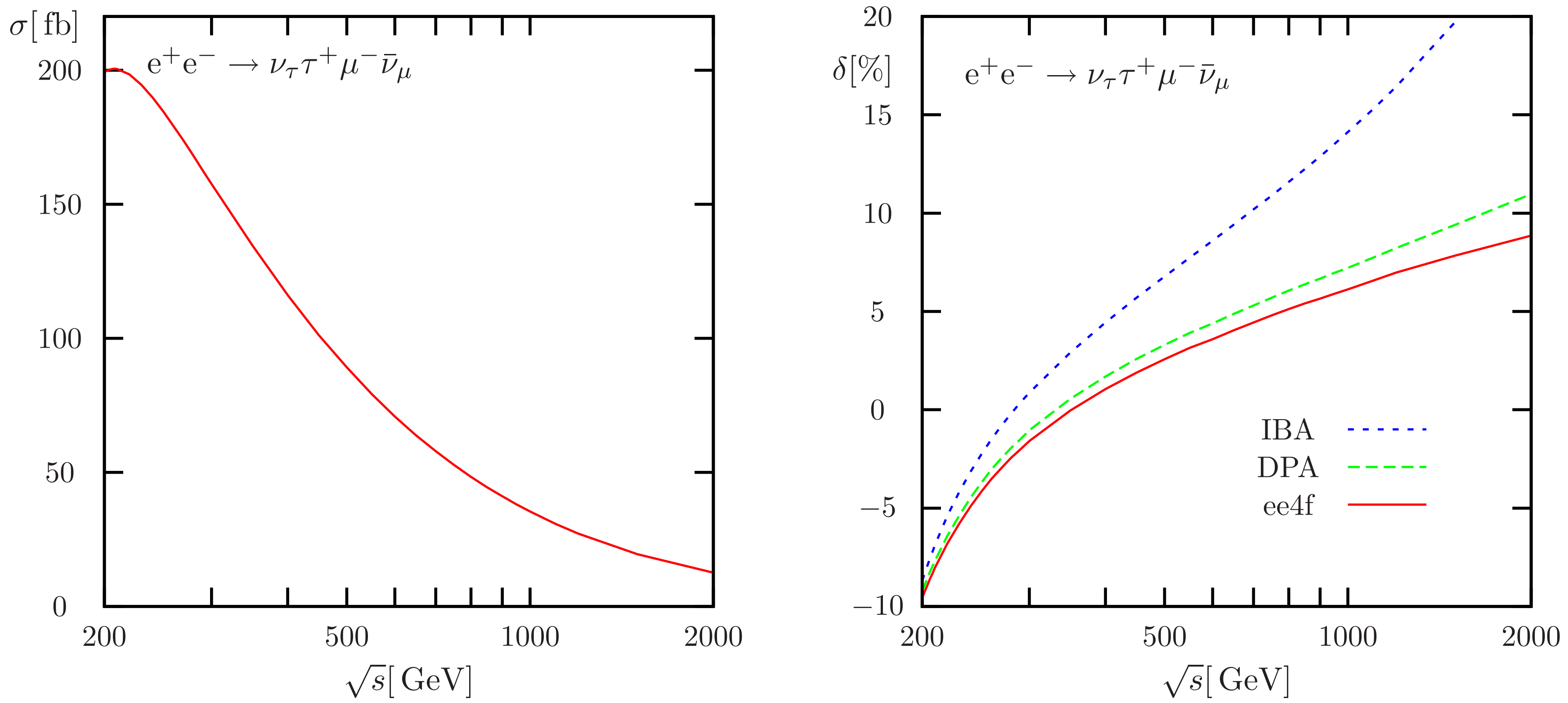

- $|\mathrm{ee4f} - \mathrm{DPA}| \quad \sim \ 0.7\%$ for $200\,\mathrm{GeV} \lesssim \sqrt{s} \lesssim 500\,\mathrm{GeV}$

  $\hookrightarrow$ agreement with error estimate of DPA

- $|\mathrm{ee4f} - \mathrm{DPA}| \quad \sim \ 1{-}2\%$ for $500\,\mathrm{GeV} \lesssim \sqrt{s} \lesssim 1{-}2\,\mathrm{TeV}$

A (not exhaustive) selection of literature

- Radiative corrections to resonance processes (see also references therein)
  - expansion about resonance poles ("pole scheme"):
    R. G. Stuart, Phys. Lett. B 262 (1991) 113;
    A. Aeppli, G. J. van Oldenborgh and D. Wyler, Nucl. Phys. B 428 (1994) 126 [hep-ph/9312212];
    H. G. J. Veltman, Z. Phys. C 62 (1994) 35.
  - electroweak corrections to Drell–Yan-like W production:
    U. Baur, S. Keller and D. Wackeroth, Phys. Rev. D 59 (1999) 013002 [hep-ph/9807417];
    S. Dittmaier and M. Krämer, Phys. Rev. D 65 (2002) 073007 [hep-ph/0109062];
    U. Baur and D. Wackeroth, Phys. Rev. D 70 (2004) 073015 [hep-ph/0405191].
  - $\mathrm{e^+e^-} \to \mathrm{WW} \to 4f$ in DPA:
    W. Beenakker, F. A. Berends and A. P. Chapovsky, Nucl. Phys. B 548 (1999) 3 [hep-ph/9811481];
    S. Jadach *et al.*, Phys. Rev. D 61 (2000) 113010 [hep-ph/9907436];
    A. Denner, S. Dittmaier, M. Roth and D. Wackeroth, Nucl. Phys. B 587 (2000) 67 [hep-ph/0006307].
  - $\mathrm{e^+e^-} \to 4f$ and complex-mass scheme at one loop:
    A. Denner, S. Dittmaier, M. Roth and L. H. Wieders, Phys. Lett. B 612 (2005) 223 [hep-ph/0502063] and Nucl. Phys. B 724 (2005) 247 [hep-ph/0505042].