The Cosmological Standard Model

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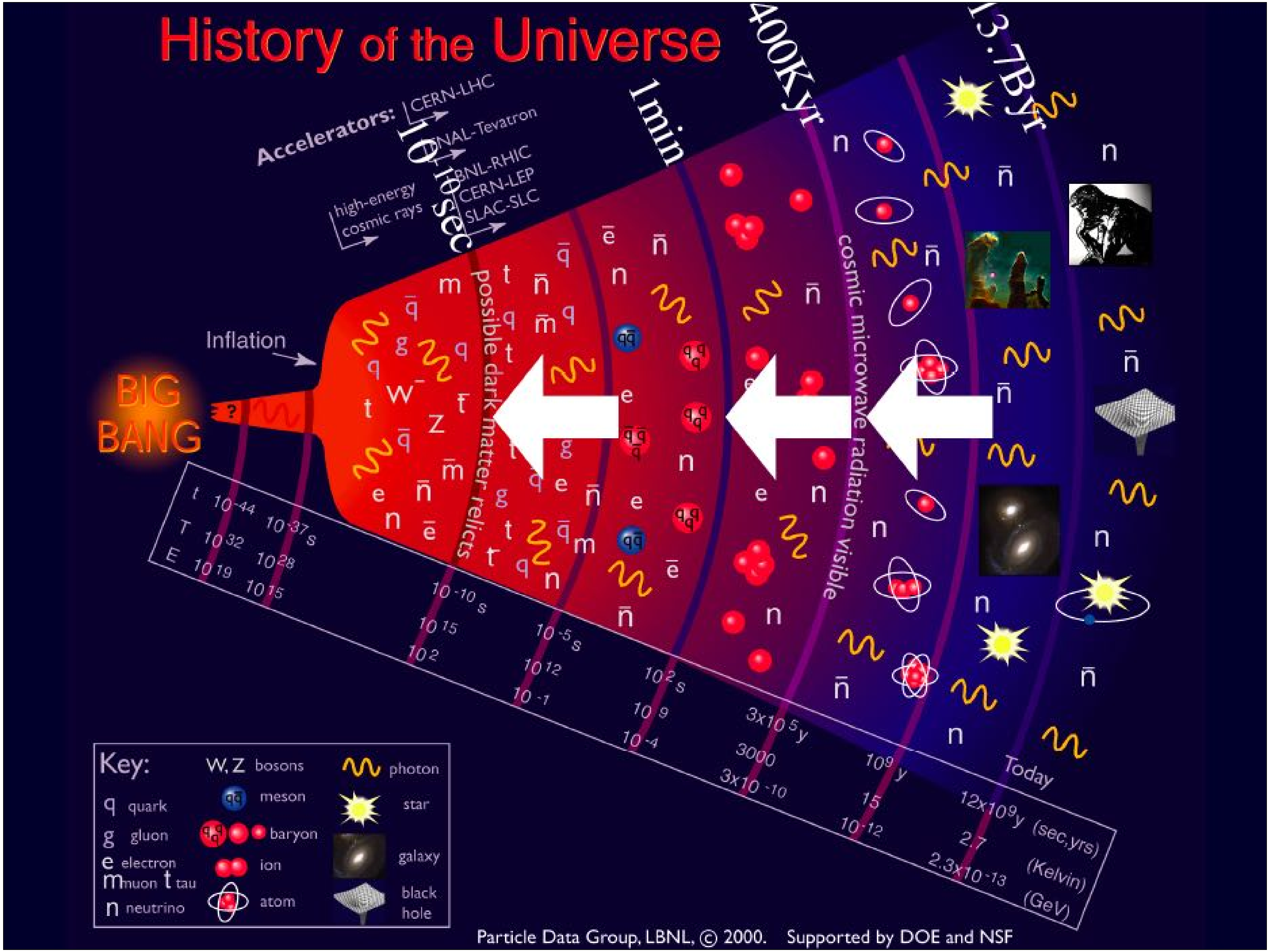
INFN Padova & CERN


History of the Universe
Accelerators:
CERN-LHC
FNAL-Tevatron
BNL-RHIC
CERN-LEP
SLAC-SLC
high-energy cosmic rays
10-10 sec
1min
400Kyr
13.7Byr
Inflation
BIG BANG
possible dark matter relicts
cosmic microwave radiation visible
t 10-44 10-37 s
T 1032 1028
E 1019 1015
10-10 s
1015
102
10-5 s
1012
10-1
102 s
109
10-4
3x105 y
3000
3x10-10
109 y
15
10-12
Today
12x109 y (sec,yrs)
2.7 (Kelvin)
2.3x10-13 (GeV)
Key:
q quark
g gluon
e electron
m muon
t tau
n neutrino
W, Z bosons
meson
baryon
ion
atom
photon
star
galaxy
black hole
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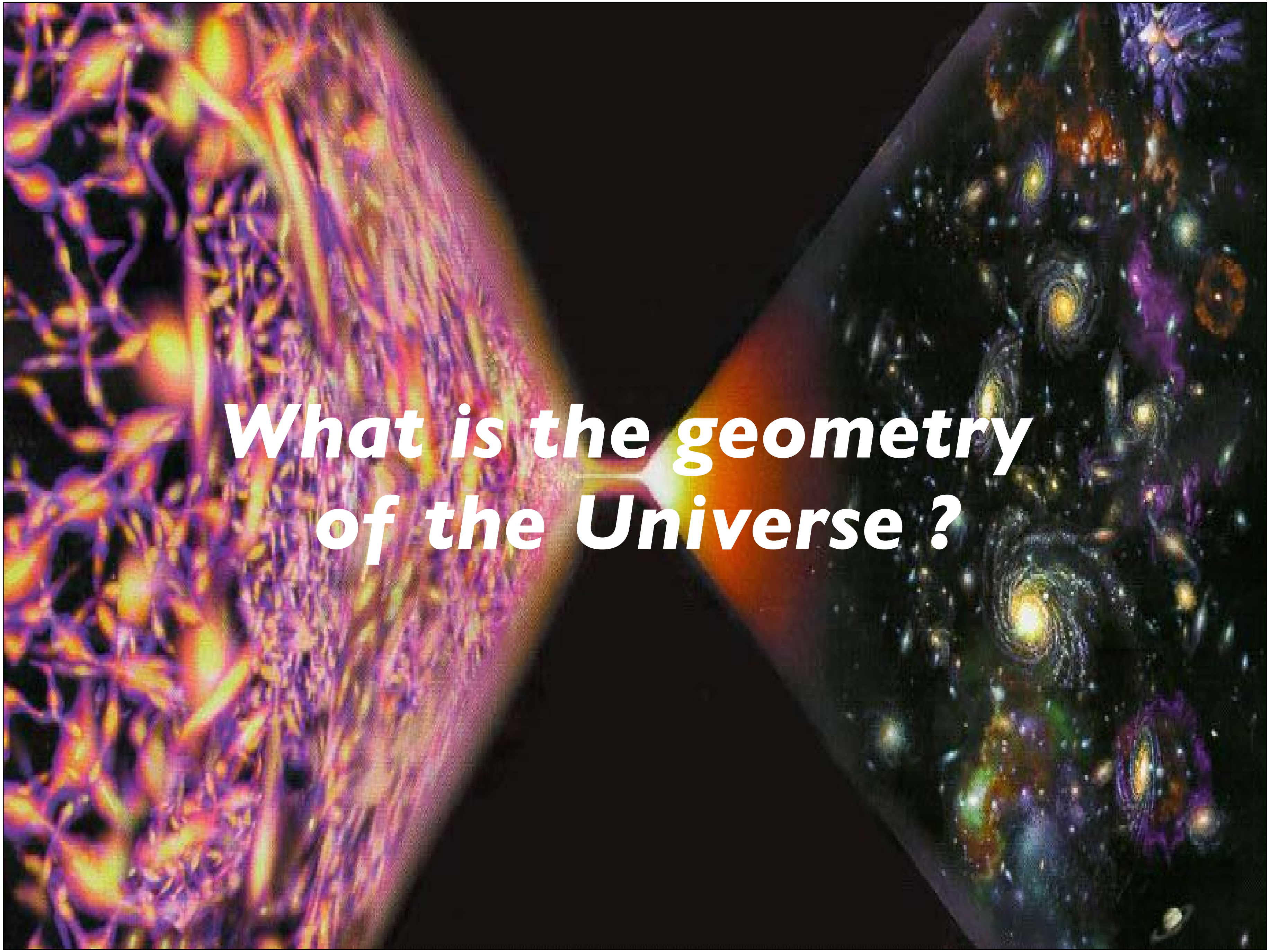
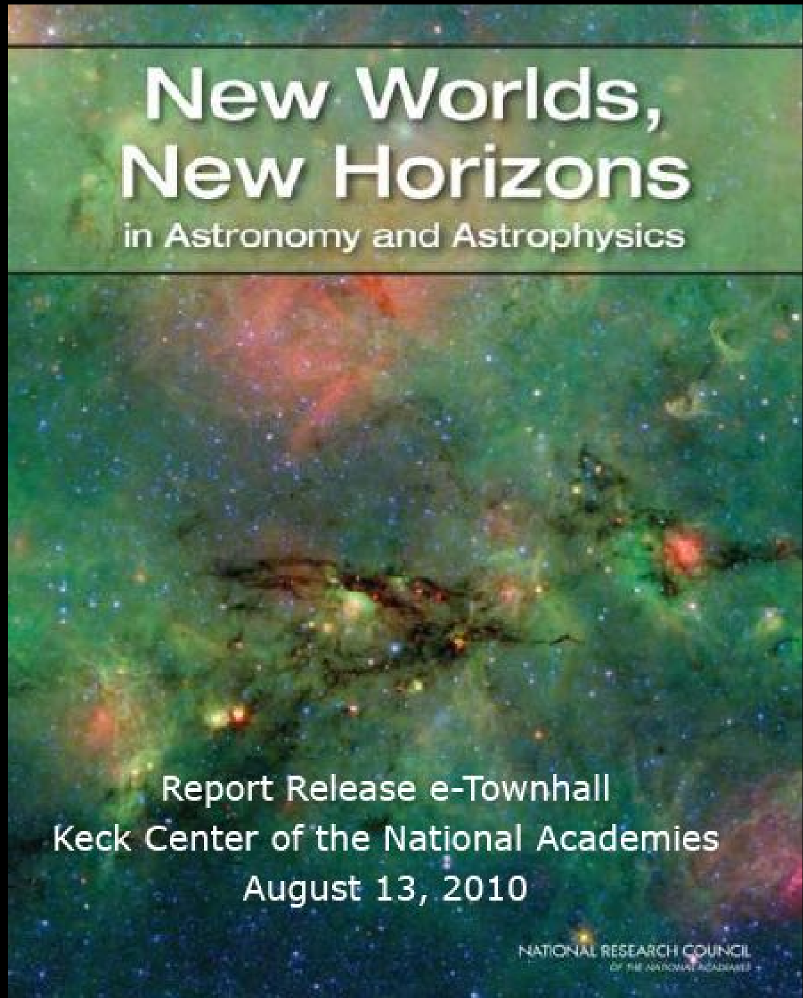
Where is our Universe
coming from ?


What is the fate
of the Universe ?


What is the geometry
of the Universe ?


What is our Universe made of?

US decadal Survey (Astro 2010)



The Science Frontier
discovery areas and principal questions

Discovery areas:

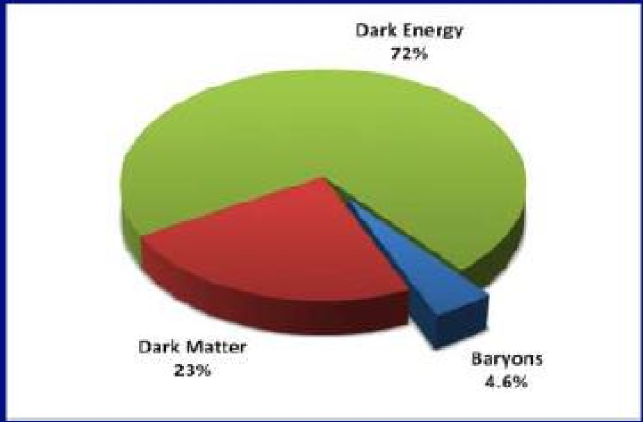
- Identification and characterization of nearby habitable exoplanets
- Gravitational wave astronomy
- Time-domain astronomy
- Astrometry
- The epoch of reionization

Questions:

- How did the universe begin?
- What were the first objects to light up the universe and when did they do it?
- How do cosmic structures form and evolve?
- What are the connections between dark and luminous matter?
- What is the fossil record of galaxy assembly and evolution from the first stars to the present?
- How do stars and black holes form?
- How do circumstellar disks evolve and form planetary systems?
- How do baryons cycle in and out of galaxies and what do they do while they are there?
- What are the flows of matter and energy in the circumgalactic medium?
- What controls the mass-energy-chemical cycles within galaxies?
- How do black holes work and influence their surroundings?
- How do rotation and magnetic fields affect stars?
- How do massive stars end their lives?
- What are the progenitors of Type Ia supernovae and how do they explode?
- How diverse are planetary systems and can we identify the telltale signs of life on an exoplanet?
- Why is the universe accelerating?
- What is dark matter?
- What are the properties of the neutrinos?
- What controls the masses, spins and radii of compact stellar remnants?

Physics of the Universe

Understanding Scientific Principles



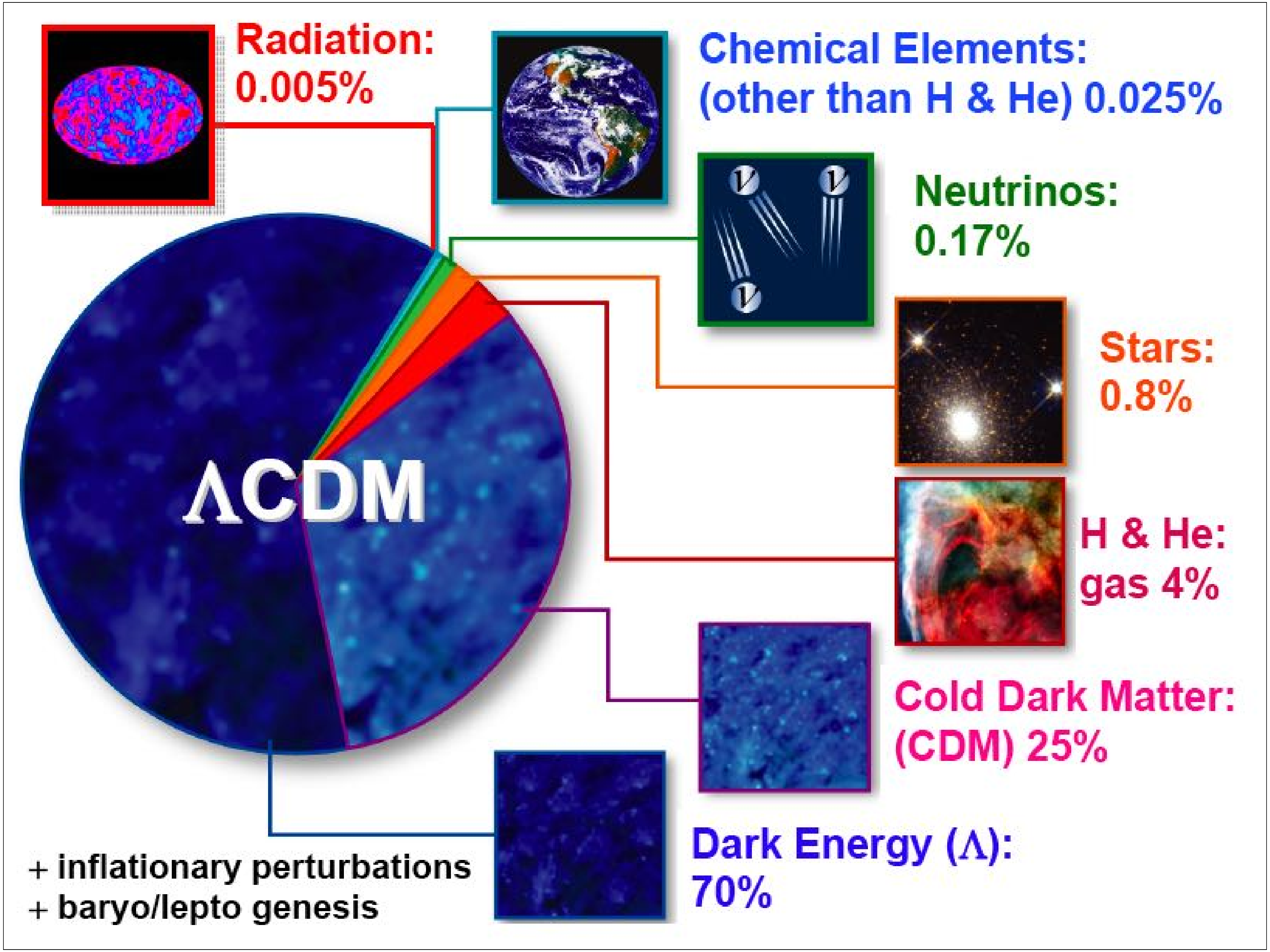
- Determine properties of dark energy, responsible for perplexing acceleration of present-day universe
- Reveal nature of mysterious dark matter, likely composed of new types of elementary particles
- Explore epoch of inflation, earliest instants when seeds of structure in the universe were sown
- Test Einstein's general theory of relativity in new important ways by observing black hole systems and detecting mergers

Collisions at CERN

Plan of the lectures

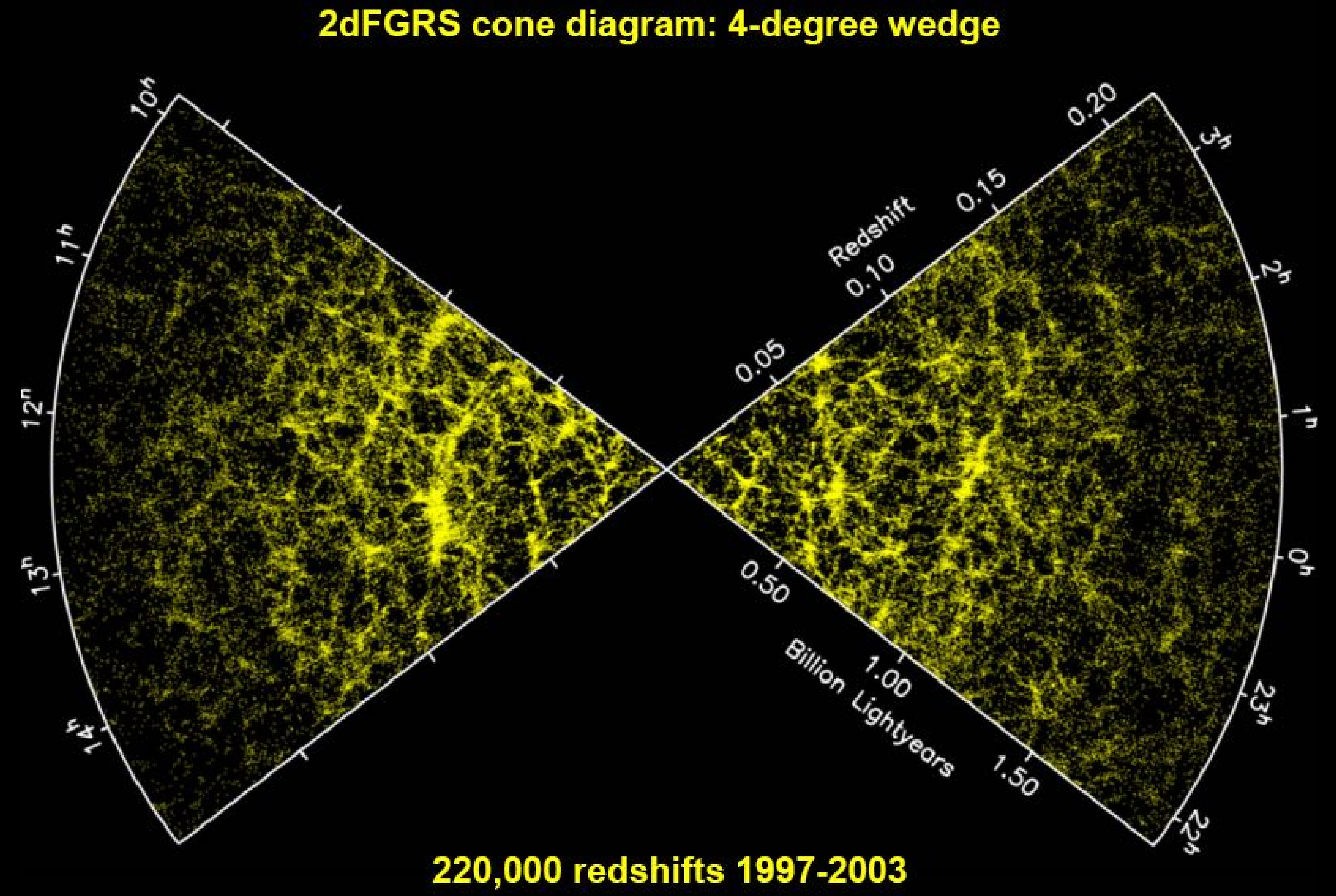
- Introduction to the Standard Big-Bang cosmology and to Inflationary cosmology
- The cosmological perturbations and the CMB anisotropy
- The DE and DM puzzles

Lecture one:
the standard Big-Bang cosmology
and
the inflationary cosmology



Radiation: 0.005%
Chemical Elements: (other than H & He) 0.025%
Neutrinos: 0.17%
Stars: 0.8%
H & He: gas 4%
Cold Dark Matter: (CDM) 25%
Dark Energy (Λ): 70%
ΛCDM
+ inflationary perturbations
+ baryo/lepto genesis

The Universe has structure



The structure in the Universe

Perturbing around the average energy density we may define the density contrast

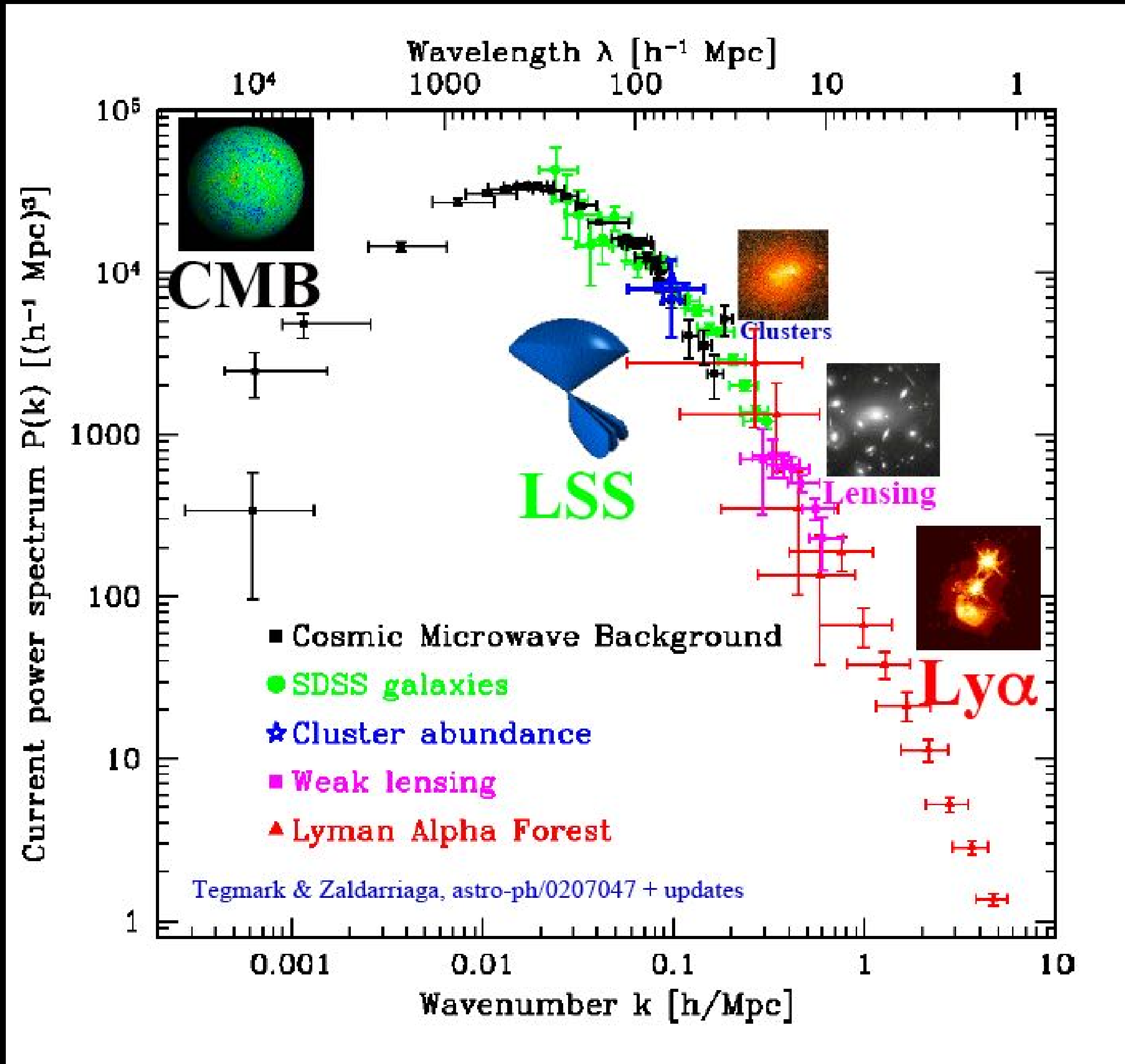
$$\delta(\mathbf{x},t) \equiv \frac{\rho(\mathbf{x},t)-\overline{\rho}}{\overline{\rho}} = \int \frac{d^3k}{(2\pi)^3}\,\delta_{\mathbf{k}}(t)\,e^{-i\,\mathbf{k}\cdot\mathbf{x}}$$

The power spectrum is defined by

$$\langle \delta_{\mathbf{k}}\,\delta_{\mathbf{k}'}\rangle = (2\pi)^3\,P_\delta(k)\,\delta(\mathbf{k}-\mathbf{k}')$$

$$\boxed{\Delta_\delta(k) = \frac{k^3\,P_\delta(k),}{2\pi^2},\;\; P_\delta = A\,k^n\,T(k)}$$

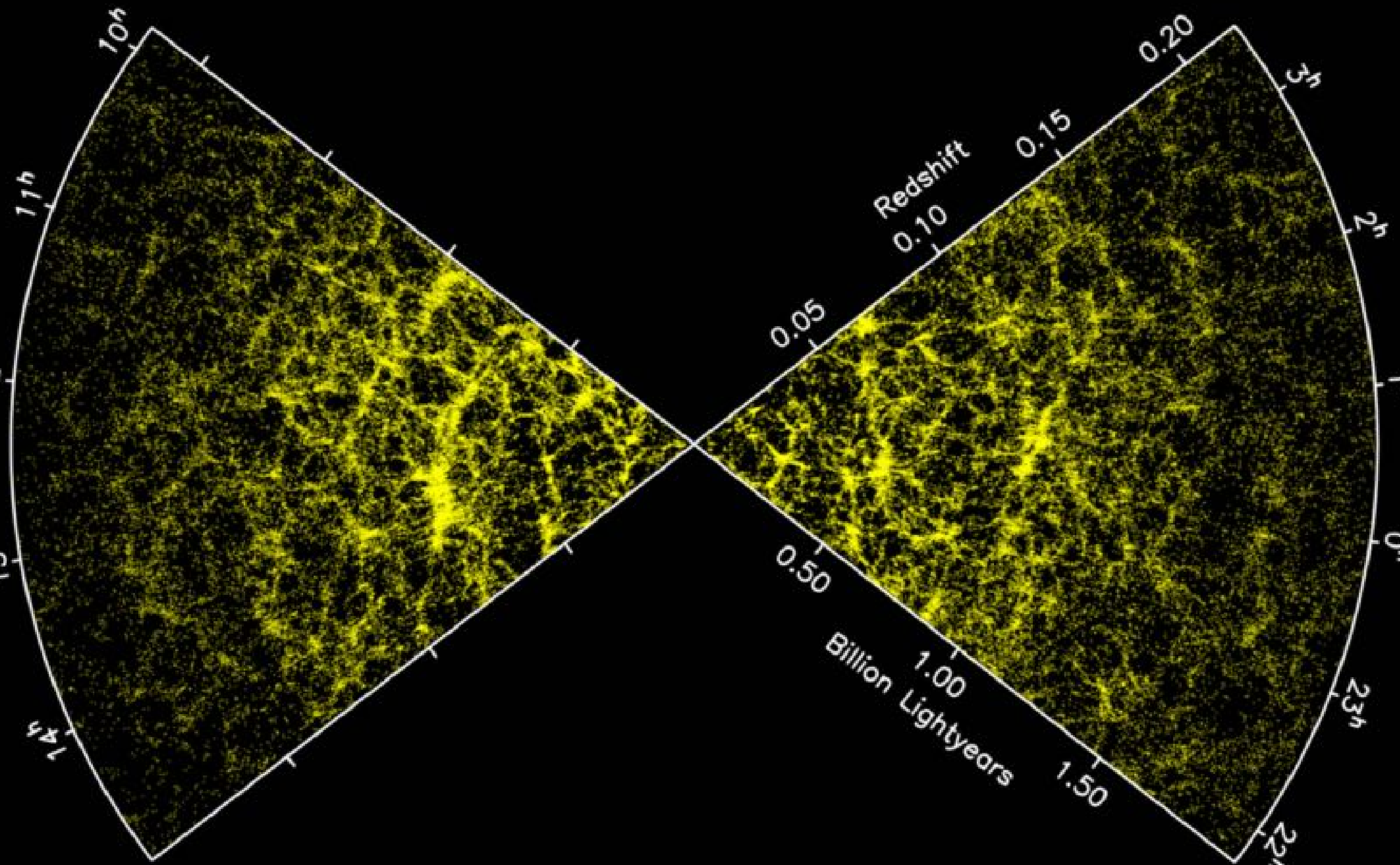
$$n \simeq 1,\; T(k) = \text{transfer function}$$



Wavelength λ [h⁻¹ Mpc]
10⁴
1000
100
10
1
Current power spectrum P(k) [(h⁻¹ Mpc)³]
10⁵
10⁴
1000
100
10
1
CMB
LSS
Clusters
Lensing
Lyα
Cosmic Microwave Background
SDSS galaxies
Cluster abundance
Weak lensing
Lyman Alpha Forest
Tegmark & Zaldarriaga, astro-ph/0207047 + updates
0.001
0.01
0.1
1
10
Wavenumber k [h/Mpc]

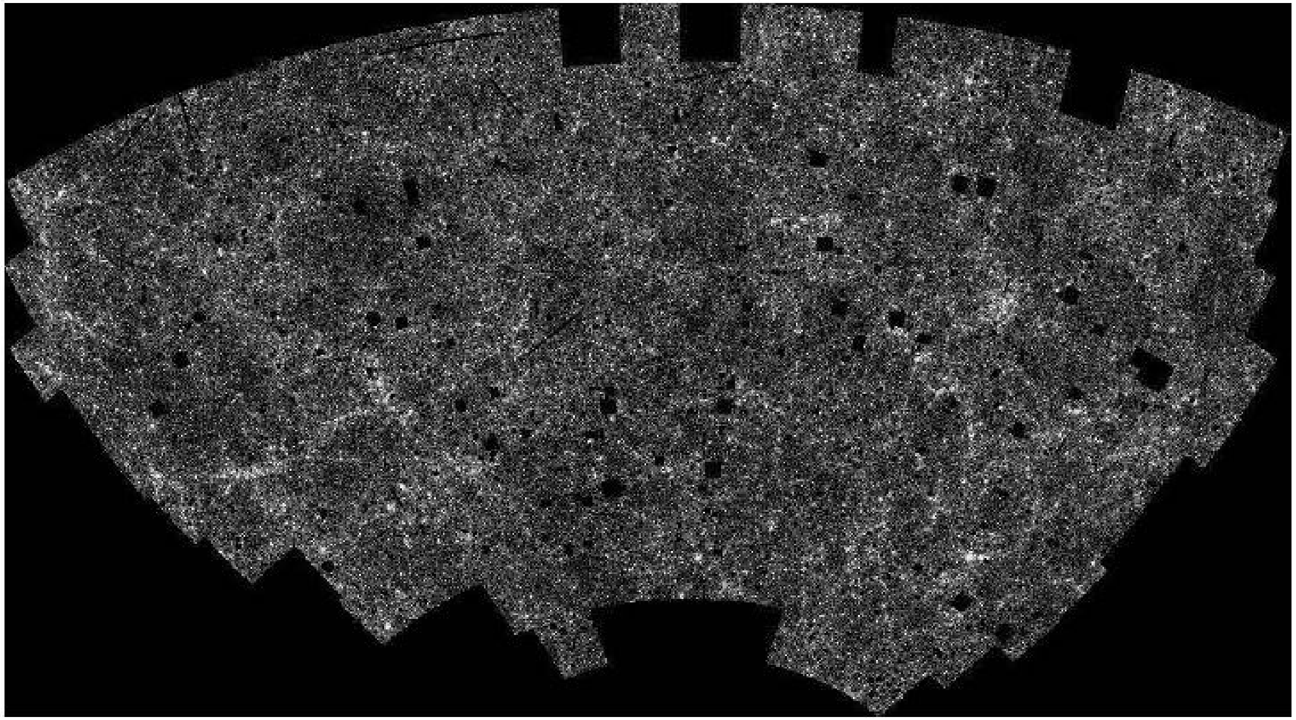
The Universe is homogeneous and isotropic on sufficiently large scales

2dFGRS cone diagram: 4-degree wedge



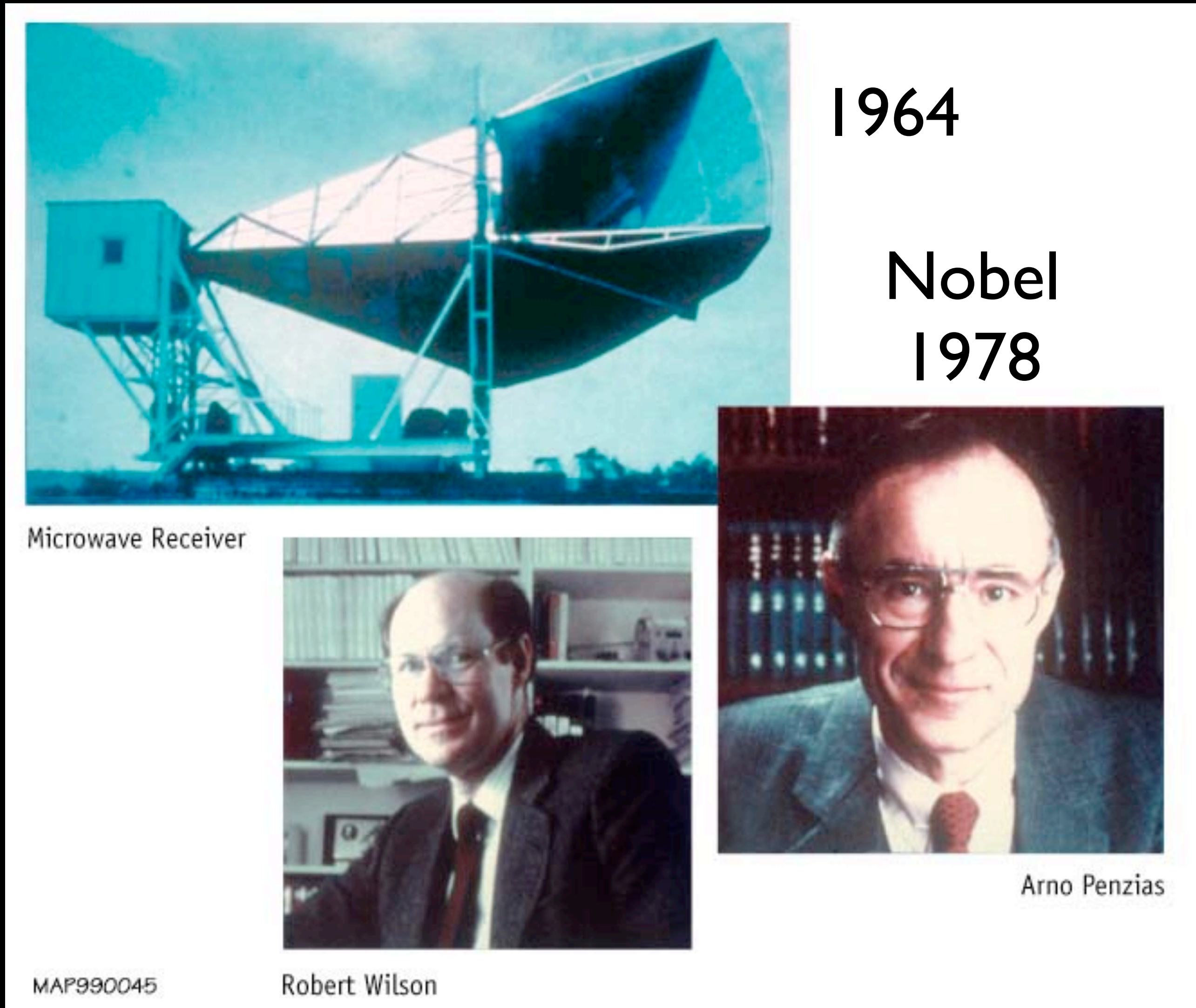
220,000 redshifts 1997-2003

The Universe is homogeneous and isotropic on sufficiently large scales

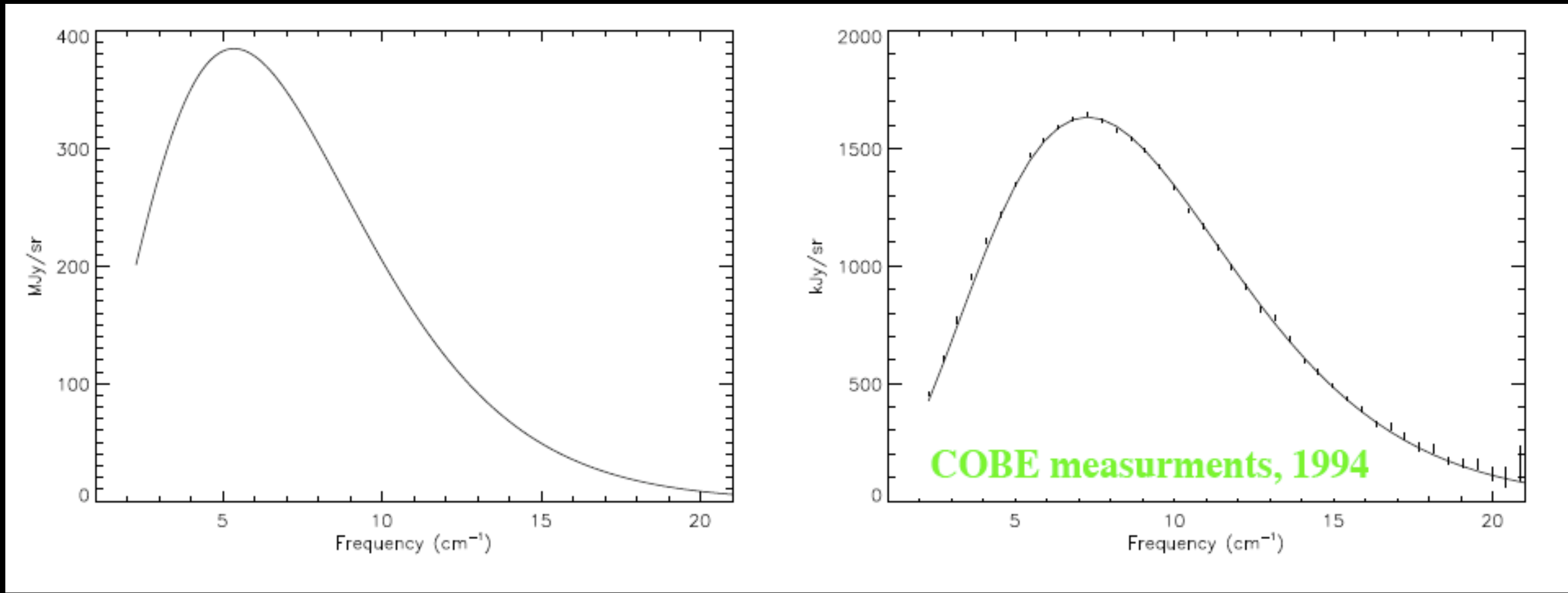


APM survey. This image covers $100^\circ \times 50^\circ$ around south pole. Contains about 2 million galaxies. Intensity of each pixel is scaled to the number of galaxies in a pixel.

Cosmic Microwave Background



The Cosmic Microwave Background Radiation

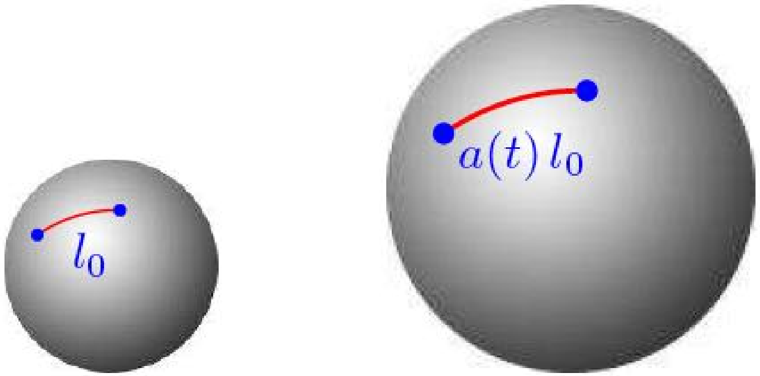


- 2.725 K above absolute zero
- mm-cm wavelength
- 410.4 photons per cubic cm
- Perfect black-body spectrum
- Nobel prize 1978: Penzias & Wilson
- Nobel Prize 2006: Mather & Smoot

The Cosmological Principle:

The Universe is
homogeneous and isotropic
(ON LARGE SCALES)

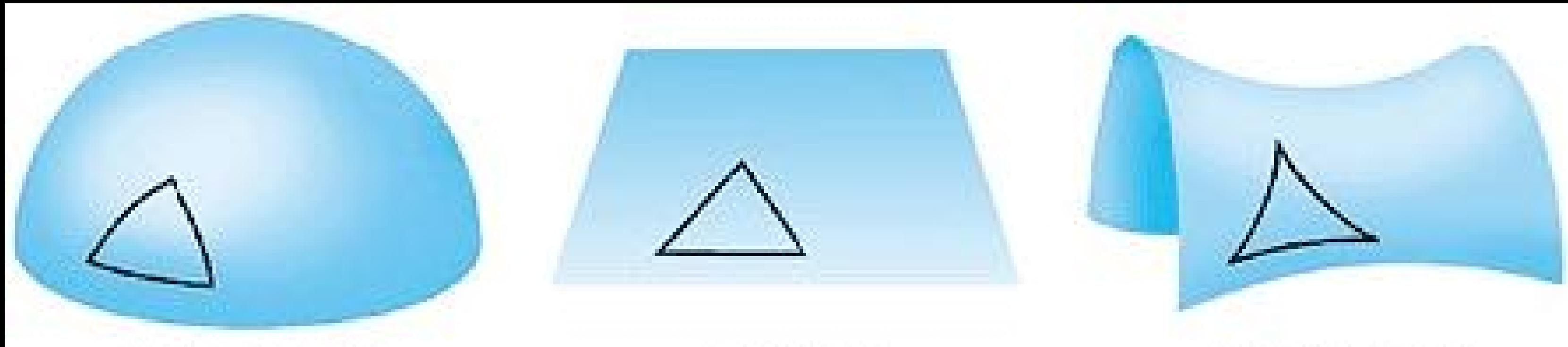
The Universe is homogeneous and isotropic:
Friedmann-Robertson-Walker metric



$$ds^2 = dt^2 - a^2(t)\, dl^2$$

$$dl^2 = \frac{dr^2}{1 - kr^2} + r^2 \left(d\theta^2 + \sin^2\theta d\phi^2 \right)$$

The geometry of space

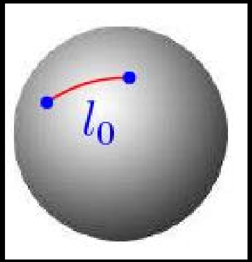


$k = 1$ Sphere

$k = 0$ Plane

$k = -1$ Hyperboloid

Example: geometry of a sphere

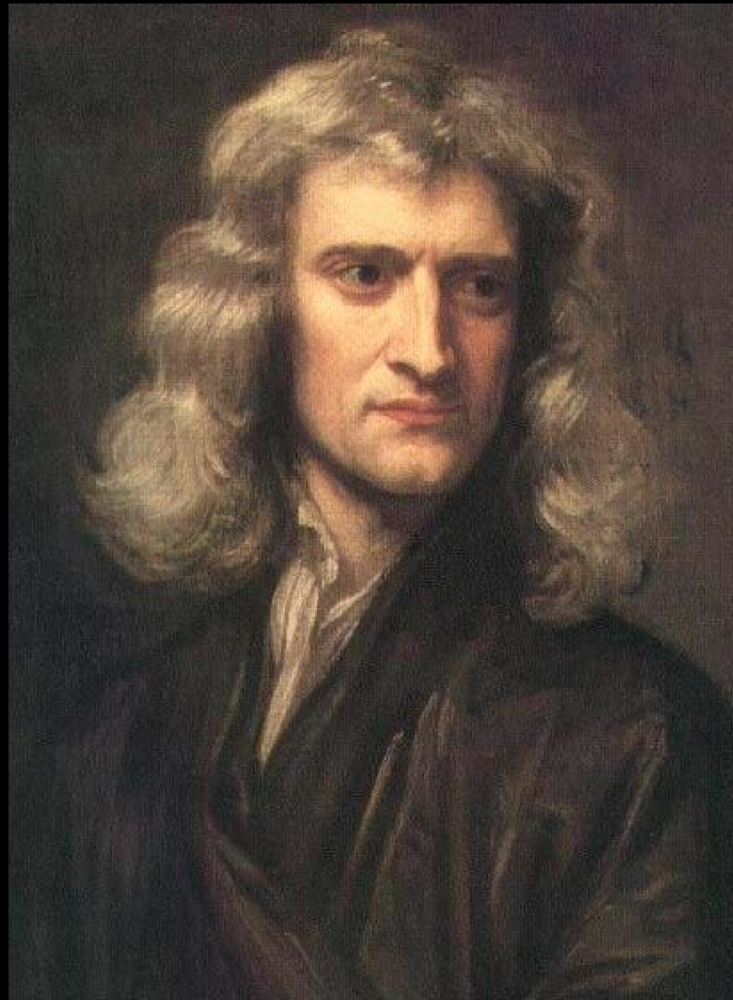


$$x^2 + y^2 + z^2 = 1 \Rightarrow z^2 = 1 - x^2 - y^2$$

$$x = r\cos\theta,\ y = r\sin\theta \Rightarrow \ dl^2 = \frac{dr^2}{1-r^2} + r^2 d\theta^2$$

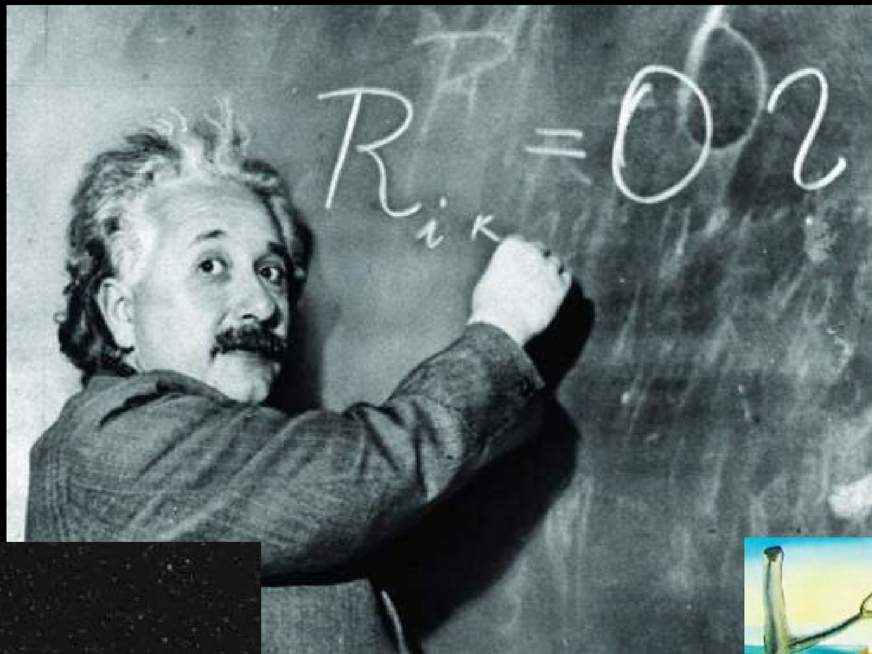
Absolute space,
in its own nature,
without relation
to anything external,
remains always similar
and immovable.

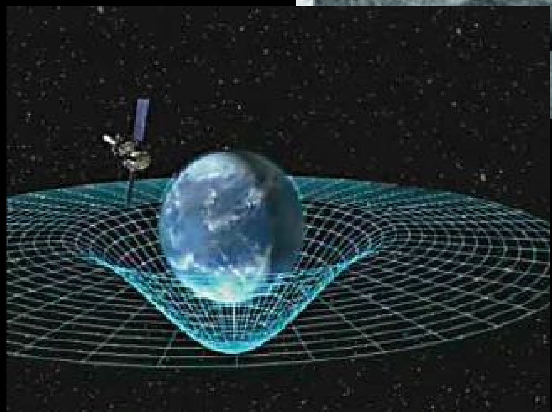
Isaac Newton
1686
Principia



Space and time are linked (1905)

Space and time are dynamical (1915)

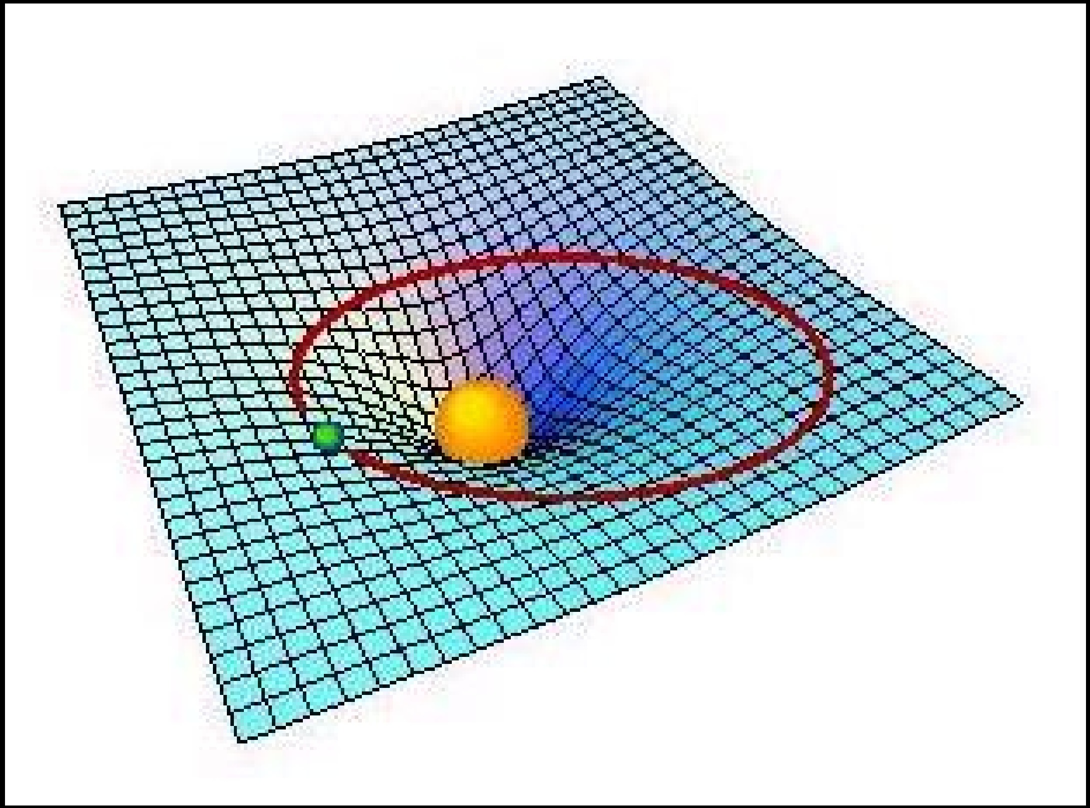


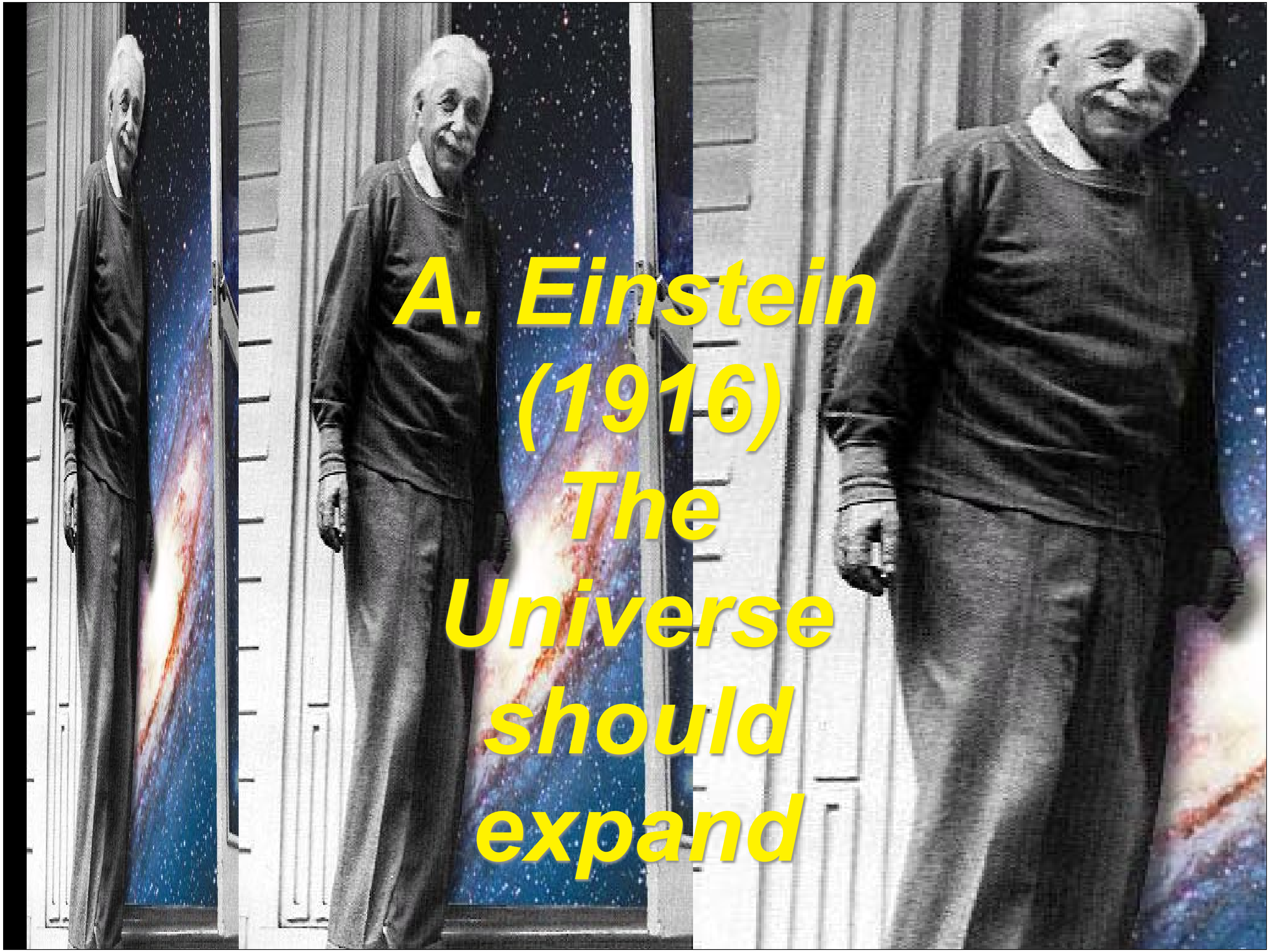


Spacetime Geometry

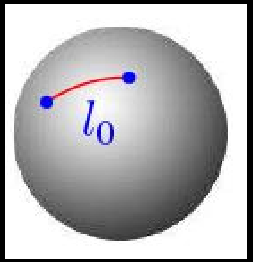
=

Distribution of Energy (and Pressure) density




A. Einstein
(1916)
The
Universe
should
expand

Implication: Hubble's Law (1929)

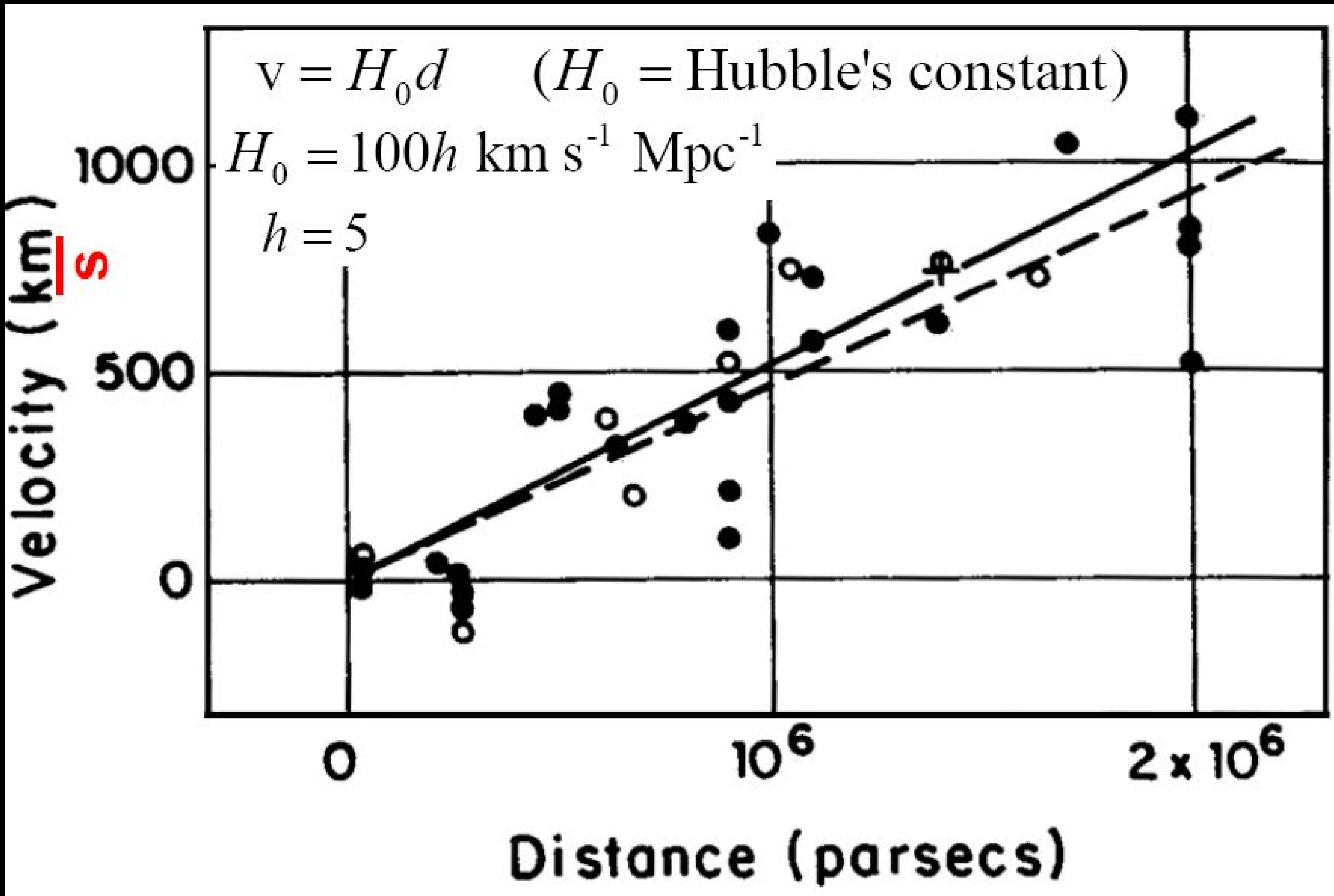


$$\vec{x} = a(t)\vec{l}_0 \Rightarrow \vec{v} = \frac{d\vec{x}}{dt} = H(t)\vec{x}$$

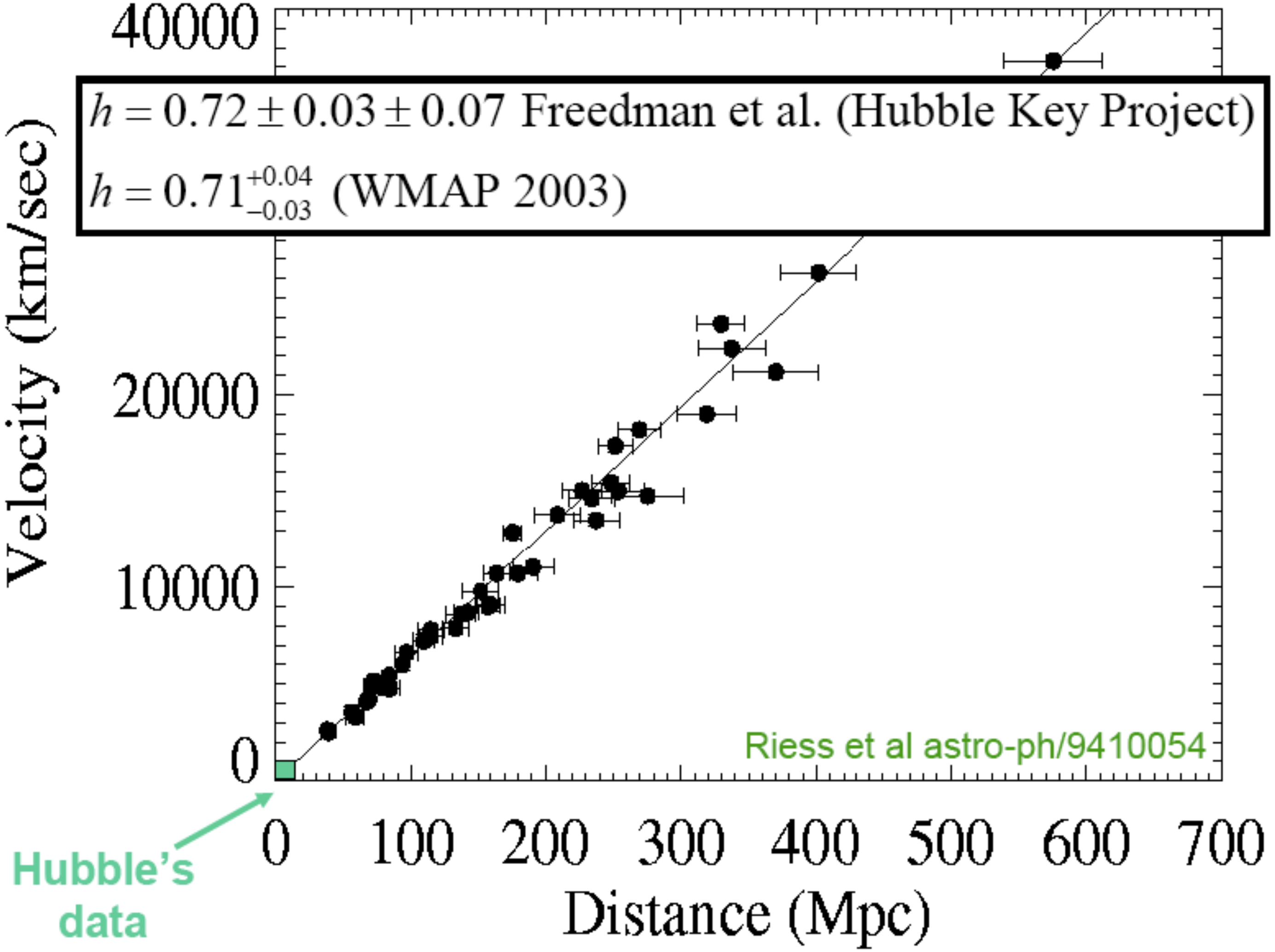
Recession velocities are proportional to distance

$$H(t) \equiv \frac{\dot{a}}{a}$$

Hubble's law

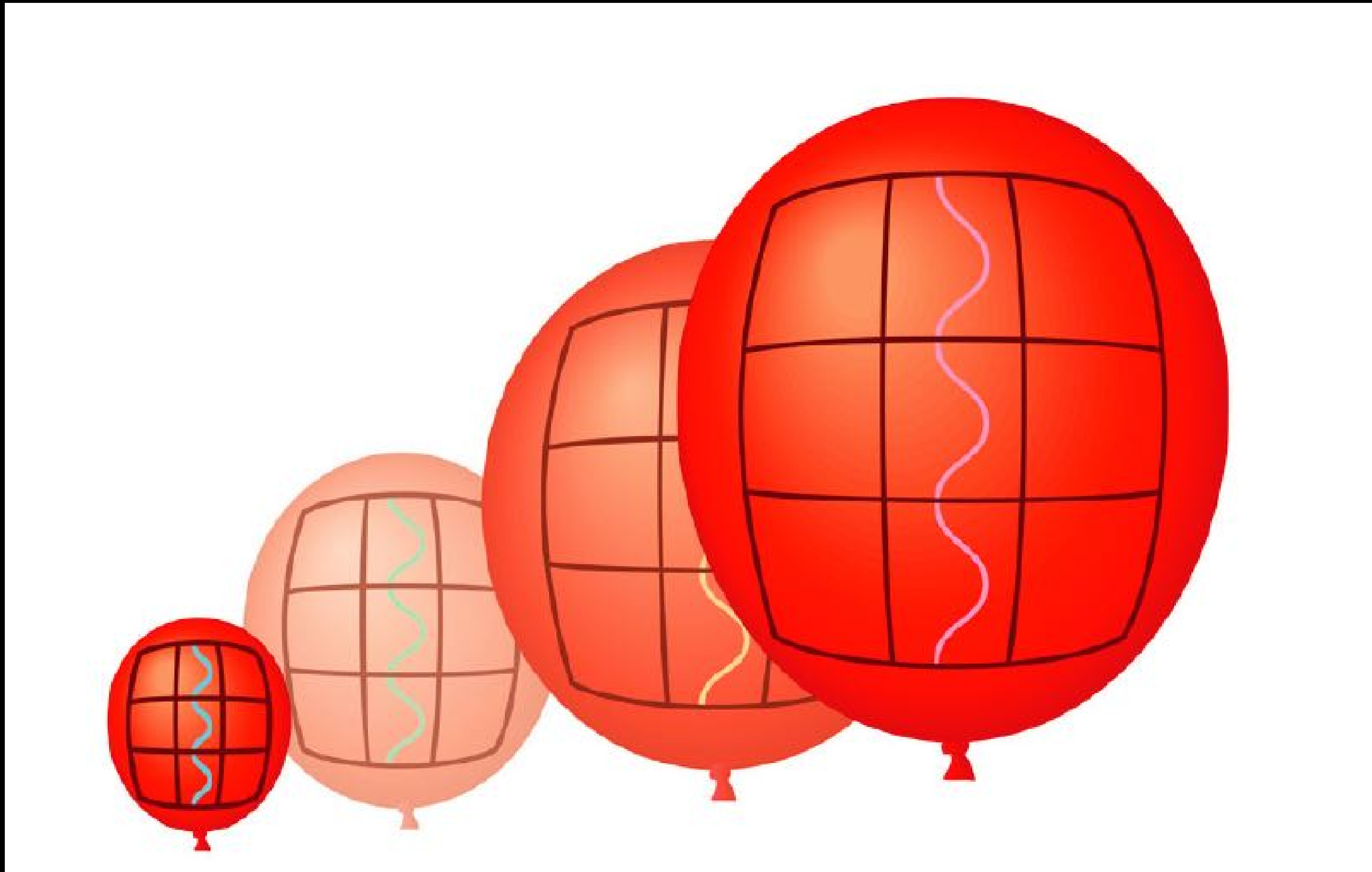


v = H₀d (H₀ = Hubble's constant)
H₀ = 100h km s⁻¹ Mpc⁻¹
h = 5
1000
500
0
Velocity (km/s)
0
10⁶
2 x 10⁶
Distance (parsecs)



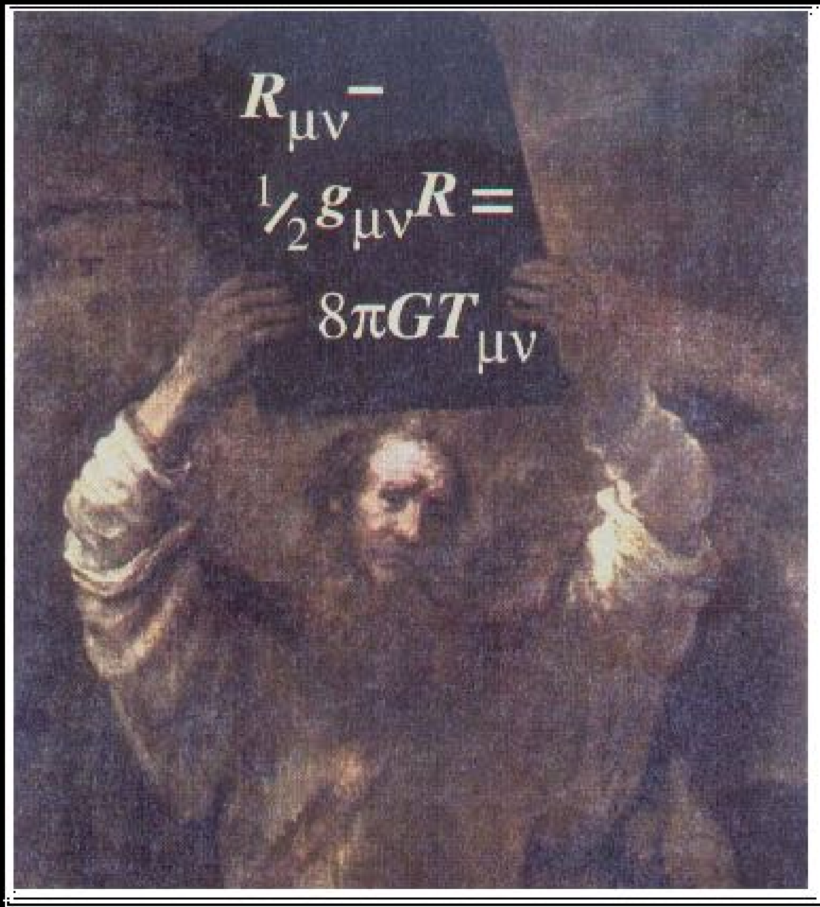
40000
20000
10000
0
Velocity (km/sec)
$h = 0.72 \pm 0.03 \pm 0.07$ Freedman et al. (Hubble Key Project)
$h = 0.71^{+0.04}_{-0.03}$ (WMAP 2003)
Riess et al astro-ph/9410054
0
100
200
300
400
500
600
700
Distance (Mpc)
Hubble's data

It is space which is expanding



How is the Universe expanding?

The scale factor in the Friedman-Robertson-Walker metric satisfies Einstein equations



Space-time geometry = energy

The Cosmological Principle imposes that the energy momentum tensor is of the form

$$T^{\mu}{}_{\nu} = \mathrm{Diag}\,(\rho, -P, -P, -P)$$

$\rho =$ Energy density $\qquad P =$ Pressure

Einstein equations take the form

$$\begin{aligned} H^2 &= \frac{8\pi G_N}{3}\rho - \frac{k}{a^2} \\ \frac{\ddot{a}}{a} &= -\frac{4\pi G_N}{3}(\rho + 3P) \end{aligned}$$

Energy momentum conservation takes the form

$$\dot{\rho} + 3H(\rho + P) = 0$$

Physics behind:

Take a test particle of unit mass immersed in a pressureless fluid of given energy density

$$r = a r_0, \;\; M = \frac{4\pi}{3}\rho r^3$$

$$\frac{1}{2}\dot{r}^2 - \frac{G_N M}{r} = -\frac{k r_0^2}{2}$$

Energy conservation of a test particle:
the value of the binding energy tells if the Universe will recollapse or expand for ever

The Golden Rule of the expansion

$$H^2 = \frac{8\pi G_N}{3}\rho - \frac{k}{a^2}$$

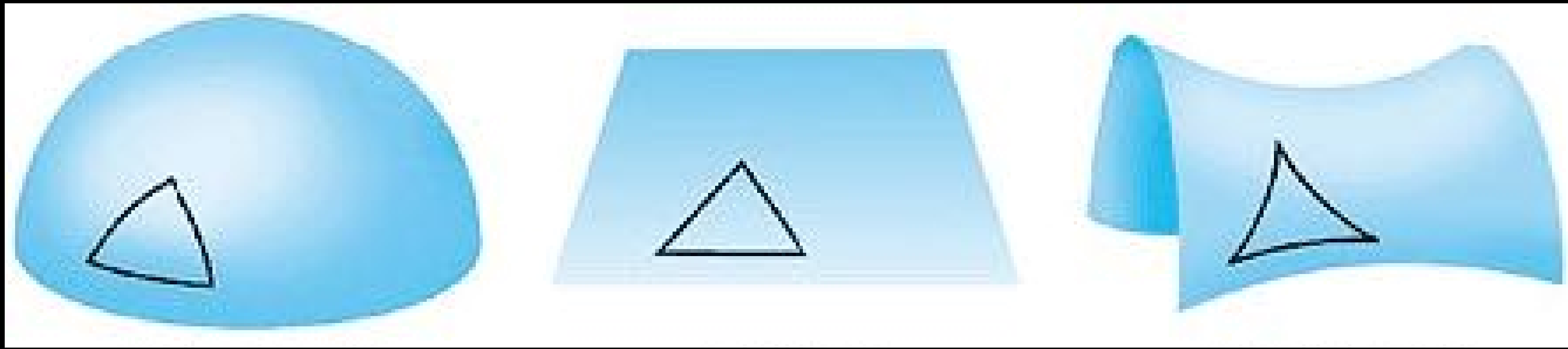
is equivalent to

$$\begin{aligned} \Omega - 1 &= \frac{k}{a^2 H^2} \\ \Omega &= \frac{\rho}{\rho_c}, \;\; \rho_c = \frac{3H^2}{8\pi G_N} \end{aligned}$$

Today $\rho_c \simeq 10^4 \, \text{eV cm}^{-3}$

$$^{(3)}R = \frac{6k}{a^2} \Rightarrow R_{\text{curv}} \sim \frac{H^{-1}}{|\Omega - 1|}$$

The Geometry of space



$\Omega > 1$ $\Omega = 1$ $\Omega < 1$

A measurement of the total energy density of the Universe implies a measurement of the geometry of space

Various types of fluids:

$$\text{Suppose } P = w\rho \quad \Rightarrow \rho \propto a^{-3(1+w)}$$

Relativistic $\quad w = 1/3 \Rightarrow \rho_R \propto a^{-4} = a^{-3} \times a^{-1}$

Nonrelativistic $\quad w \simeq 0 \Rightarrow \rho_{NR} \propto a^{-3}$

Cosmological constant $\quad w \simeq -1 \Rightarrow \rho \propto a^{0}$

Curvature term $\quad w = -1/3 \Rightarrow \rho \propto a^{-2}$

Dynamics is determined by energy content

$$H^2 + \frac{k}{a^2} = \frac{8\pi G_N}{3}\rho, \quad \rho = \sum_i \rho_i(a)$$

$a(t)$ and $H(t)$ depend on energy content

$a(t)$ measurable by redshift

$1 + z = a_0/a$ is a proxy for the scale factor

$$H^2(z) = H_0^2\left[\Omega_R(1+z)^4 + \Omega_{NR}(1+z)^3 + \Omega_w(1+z)^{3(1+w)} + (1-\Omega_{\text{total}})(1+z)^2\right]$$

In HEP units:

$$
\begin{aligned}
H_0^{-1} &\sim 10^{28}\,\mathrm{cm} \sim 10^{42}\,\mathrm{GeV}^{-1} \\
h_0 &\equiv (H_0/\mathrm{Km/sec/Mpc}) = 0.75 \\
h_0^2 &= \frac{1}{2} \\
T_0 &\sim 10^{-4}\,\mathrm{eV} \\
\rho_c &\simeq 10^{-66}\,\mathrm{GeV}^4
\end{aligned}
$$

Time evolution

$$a \propto t^{\frac{2}{3}(1+w)}$$
$$H = \frac{2}{3}(1+w)\frac{1}{t}$$

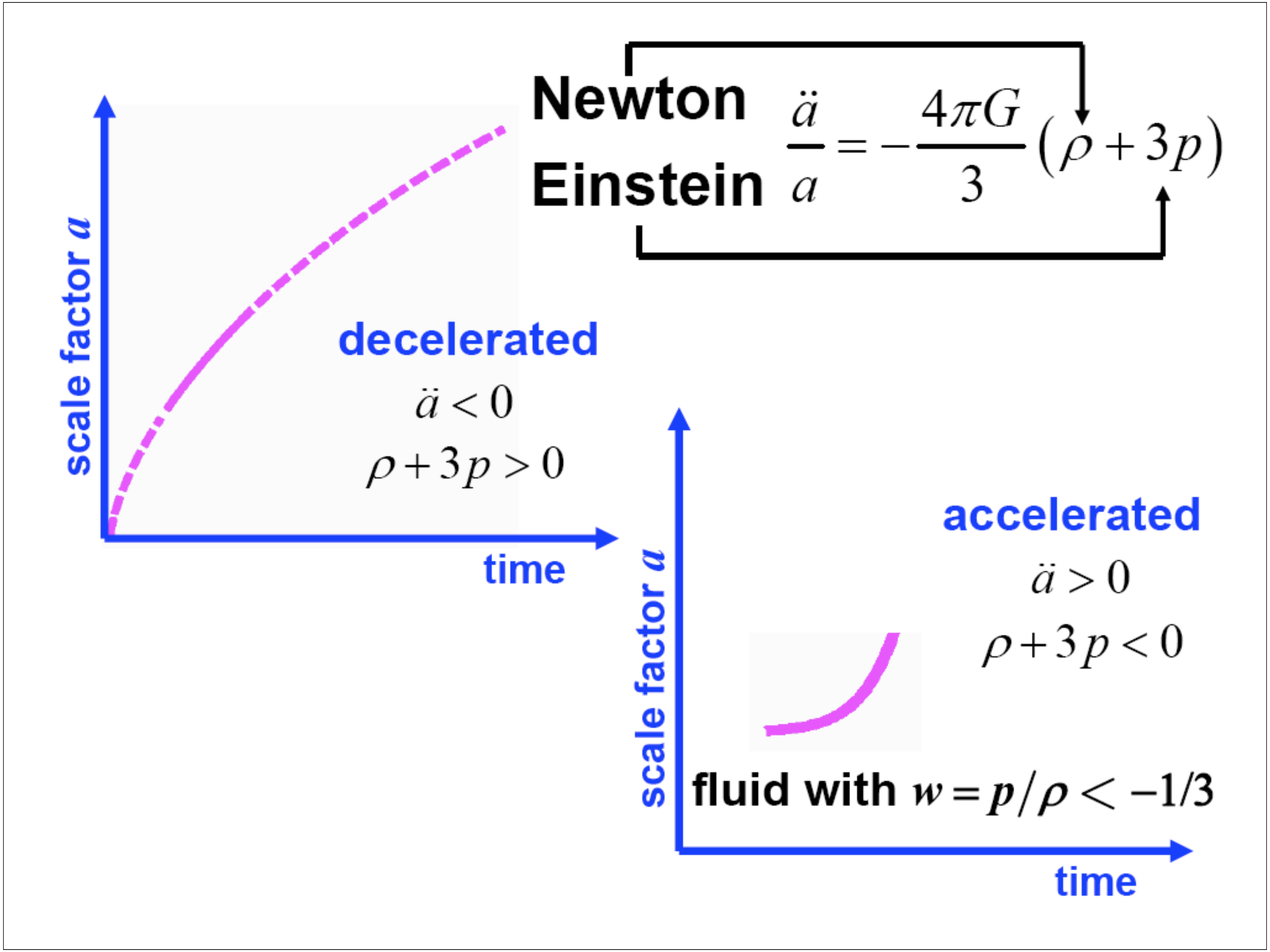
RD

$$a \propto t^{\frac{1}{2}}$$

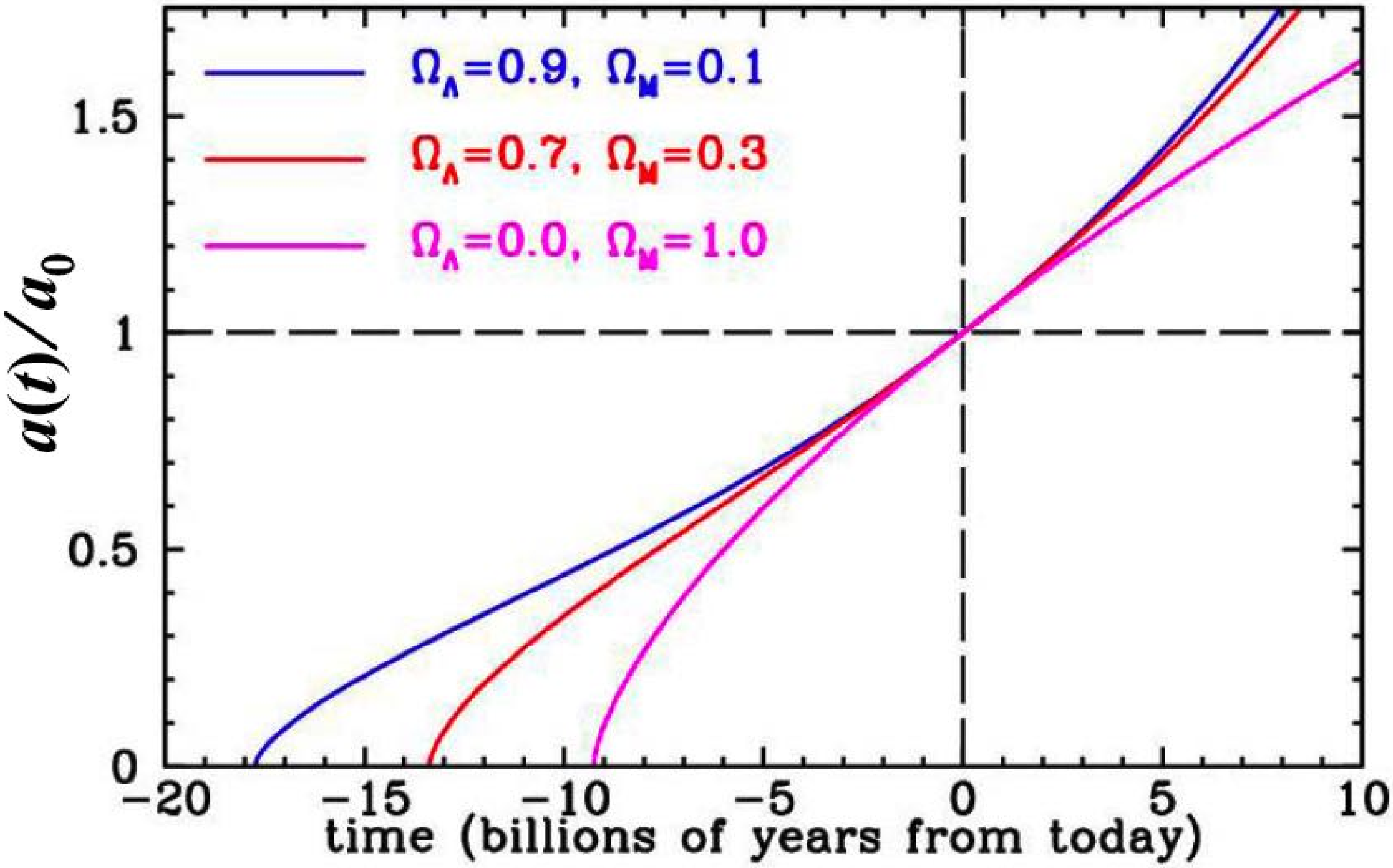
MD

$$a \propto t^{\frac{2}{3}}$$

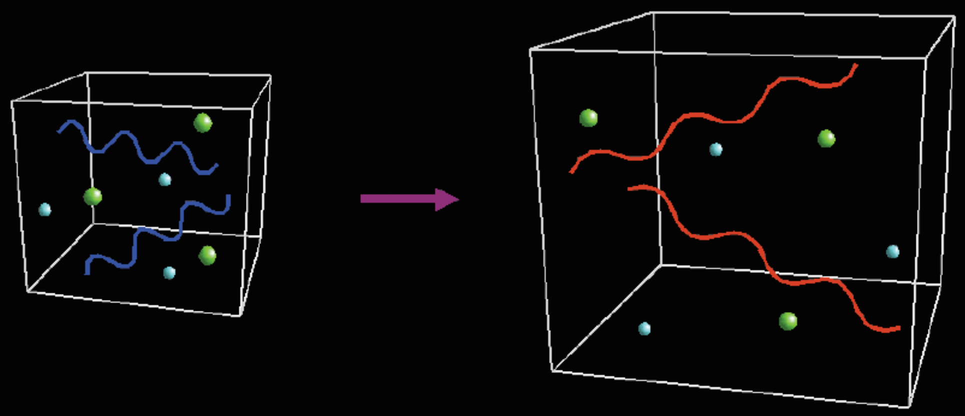
The expansion is decelerated

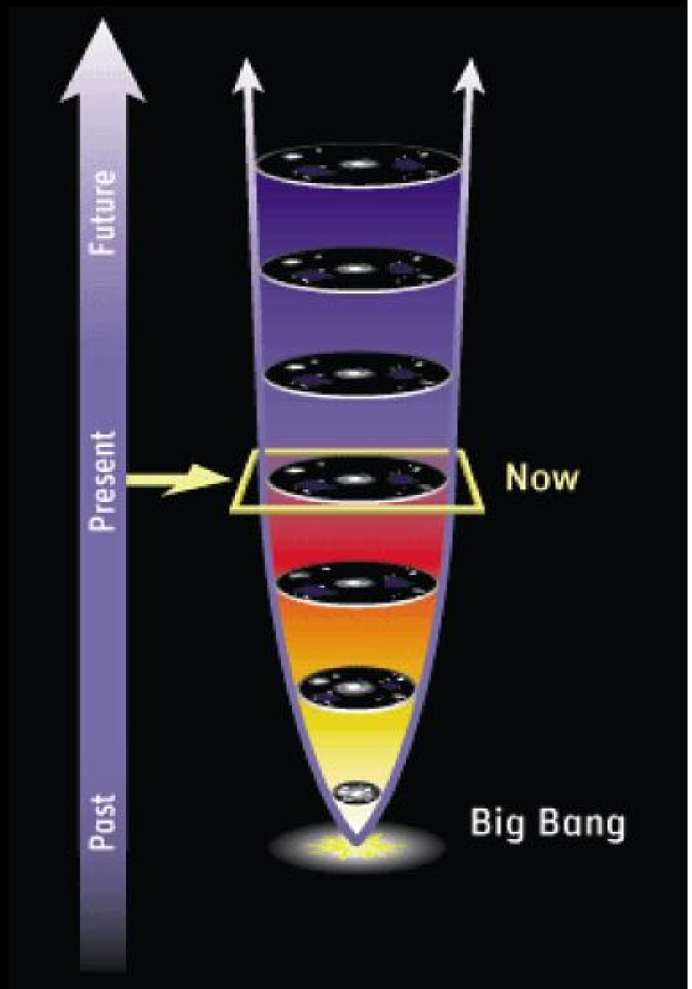

Newton
Einstein
$\frac{\ddot{a}}{a} = -\frac{4\pi G}{3}(\rho + 3p)$
scale factor a
decelerated
$\ddot{a} < 0$
$\rho + 3p > 0$
time
scale factor a
accelerated
$\ddot{a} > 0$
$\rho + 3p < 0$
fluid with $w = p/\rho < -1/3$
time

$$\left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G}{3}\left[\rho_{M0}\left(\frac{a_0}{a}\right)^3 + \rho_\Lambda\right]$$

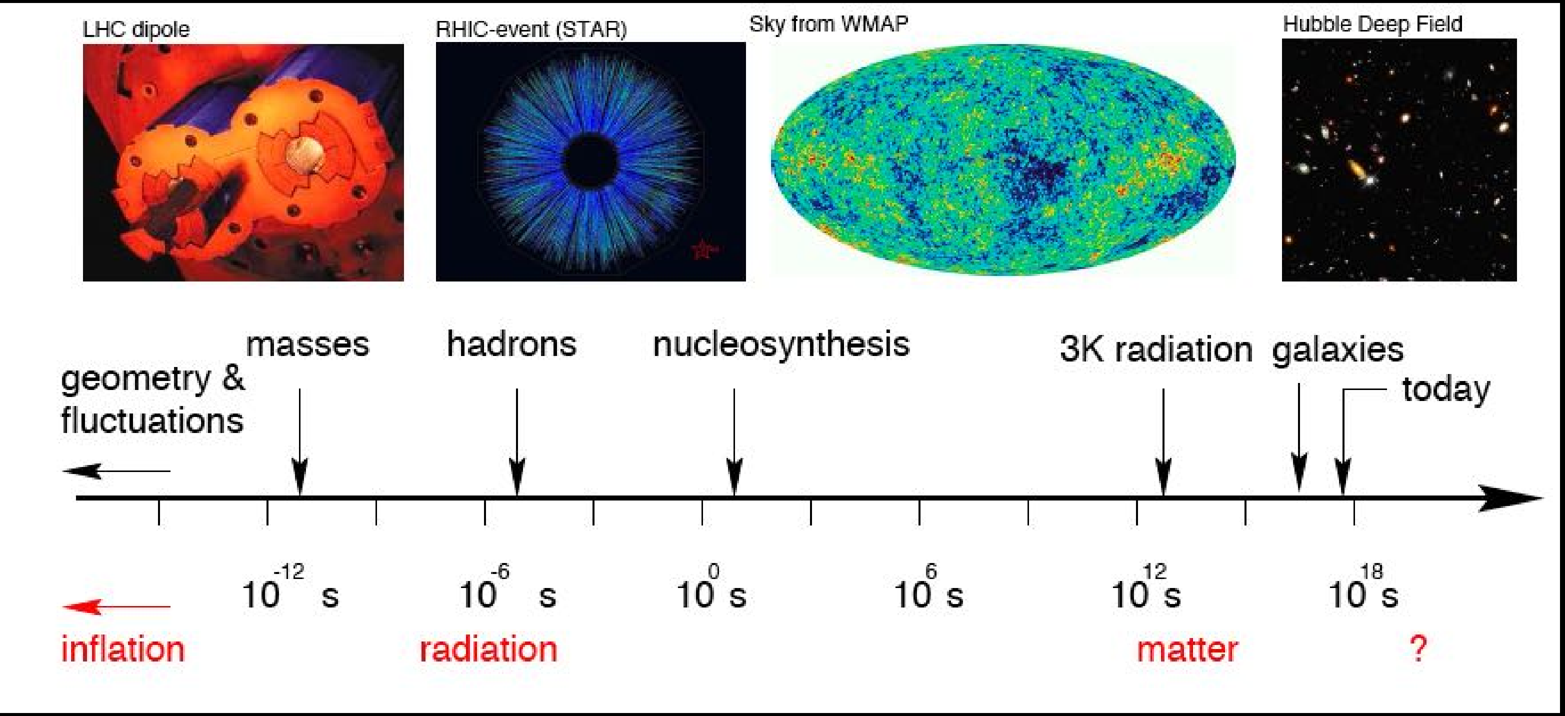


The past




Future
Present
Past
Now
Big Bang

Brief History of the Universe



At high temperatures, the Universe is expected to be Radiation Dominated

IF equilibrium holds, then

$$\rho_R = \frac{\pi^2}{30} g_* T^4 \quad (T \gg m)$$

$$g_* = \sum_{\text{bosons}} g_{\text{b}} + \frac{7}{8} \sum_{\text{fermions}} g_{\text{f}}$$

$$8\pi G_N = \frac{1}{M_p^2}$$

$$H \simeq 1.66\, g_*^{1/2} \frac{T^2}{M_p}, \;\; M_p \simeq 1.2 \times 10^{19}\,\text{GeV}$$

$$\frac{t}{\text{sec}} \sim \left(\frac{\text{MeV}}{T}\right)^2$$

$$t_{\text{LHC}} \sim 10^{-14}\,\text{sec}$$

No Big Bang at the LHC

Entropy Density

$$s = \frac{\rho_R + P_R}{T} = \frac{4}{3}\frac{\rho_R}{T} = \frac{2\pi^2}{45} g_* T^3$$

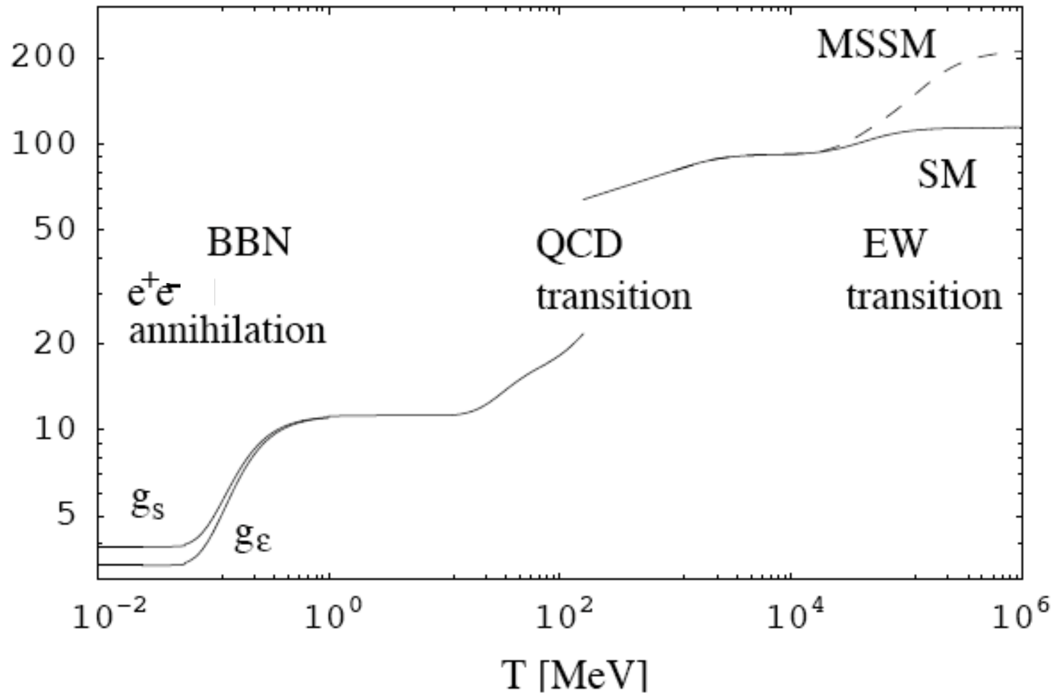
If expansion is adiabatic:

$$S \equiv s \times V = \text{constant} \Rightarrow g_* (Ta)^3 = \text{constant}$$

$$T \propto \frac{1}{g_*^{1/3} a}$$

Only particles with $m \ll T$ should be counted,

i. e. g_* is a function of temperature



Equilibrium holds only if the time-scale for interaction is smaller than the time of the Universe

$$\tau_{\rm int} \simeq (1/n\sigma v) \gg t_U \sim H^{-1} \sim t \sim (M_p/g_*^{1/2}T^2)$$

$$n \sim T^3,\ \sigma \sim \alpha^2/T^2,\ v \sim 1 \Rightarrow T \ll (\alpha^2/g_*^{1/2})M_p$$

$$T \ll 10^{14}\ {\rm GeV}$$

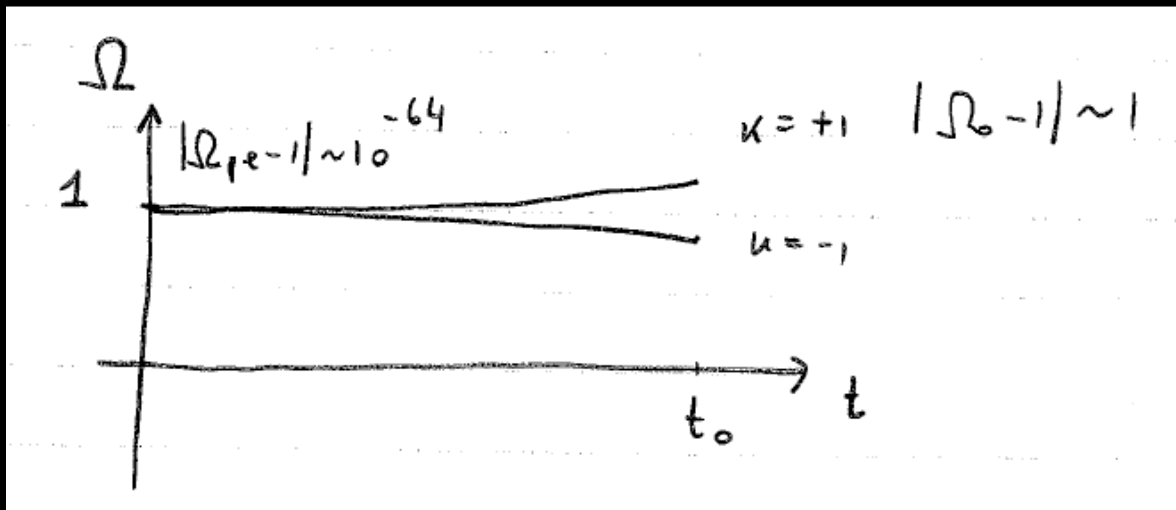
Shortcomings of the standard Big-Bang cosmology

Flatness Problem

Going back in time

$$\Omega - 1 = \frac{k}{a_R^2 H_R^2} \propto \frac{k M_p^2}{a_R^2 \rho_R} \propto \frac{k M_p^2}{a_R^2 T^4} \propto k\, a_R^2$$

$$\frac{|\Omega - 1|_{T=M_p}}{|\Omega - 1|_{T=T_0}} \simeq \left(\frac{T_0}{M_p}\right)^2 \simeq 10^{-64}$$



Flatness Problem = Entropy Problem

$$\Omega - 1 = \frac{kM_p^2}{a_R^2 T^4} = \frac{kM_p^2}{(a_R T)^2 T^2} \sim \frac{kM_p^2}{S^{2/3} T^2}$$

IF entropy is conserved

$$S = S_0 \sim (T_0 H_0^{-1})^3 \sim 10^{90} \Rightarrow |\Omega - 1|_{T=M_p} \sim 10^{-64}$$

The flatness problem is equivalent to ask why there is so much entropy in our visible Universe

Educated guess: break adiabaticity

The flateness problem is more a
fine-tuning problem
about the initial conditions

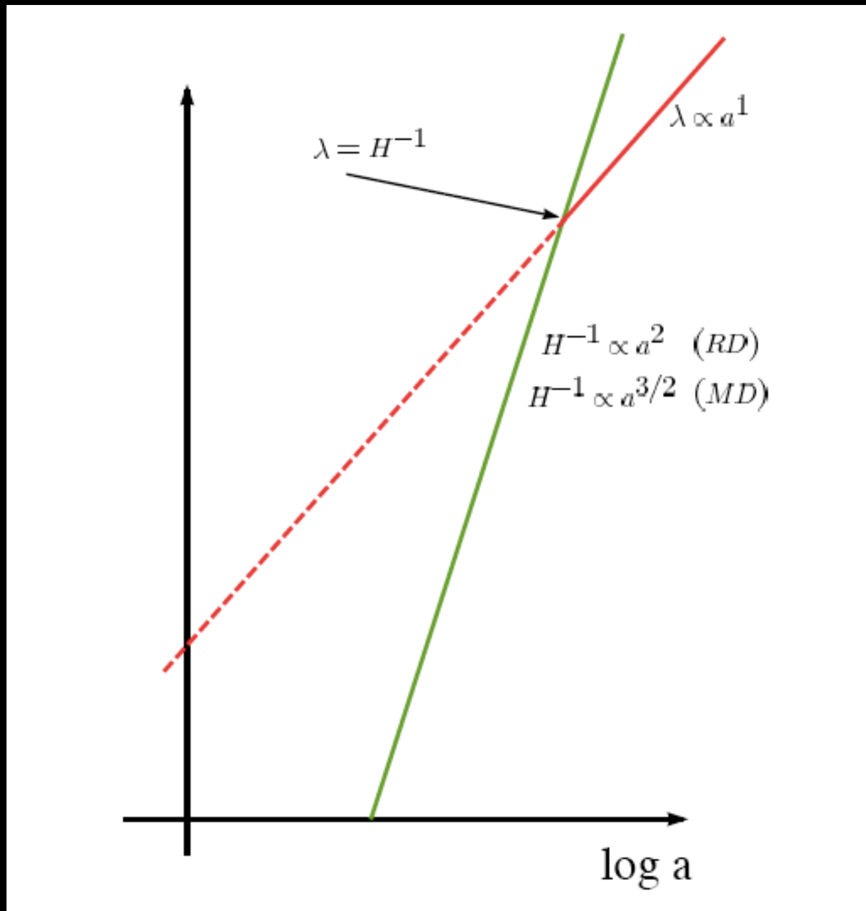
The Particle Horizon

It is the maximum distance travelled by light in an expanding Universe within a given time t

$$ds = 0 \Rightarrow dl = \frac{dt}{a} \qquad R_H(t) = a(t) \int_0^t \frac{dt'}{a(t')}$$

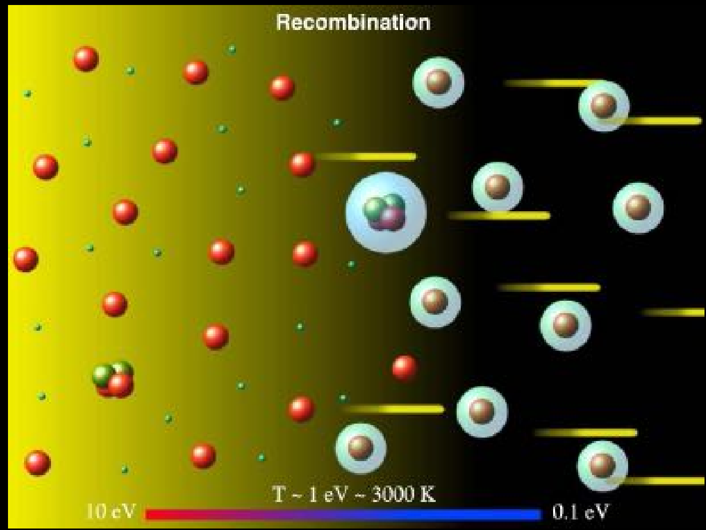
$$a(t) \propto t^n \Rightarrow R_H(t) \simeq \frac{1}{1-n} t^{-1} \sim H^{-1}(t)$$

Standard Cosmology and the Horizon Problem



$$R_H(t) = a(t)\int_0^t \frac{dt'}{a(t')} \simeq \frac{a(t)}{\dot{a}(t)} = H^{-1}(t)$$

Hydrogen Recombination & Last Scattering Surface

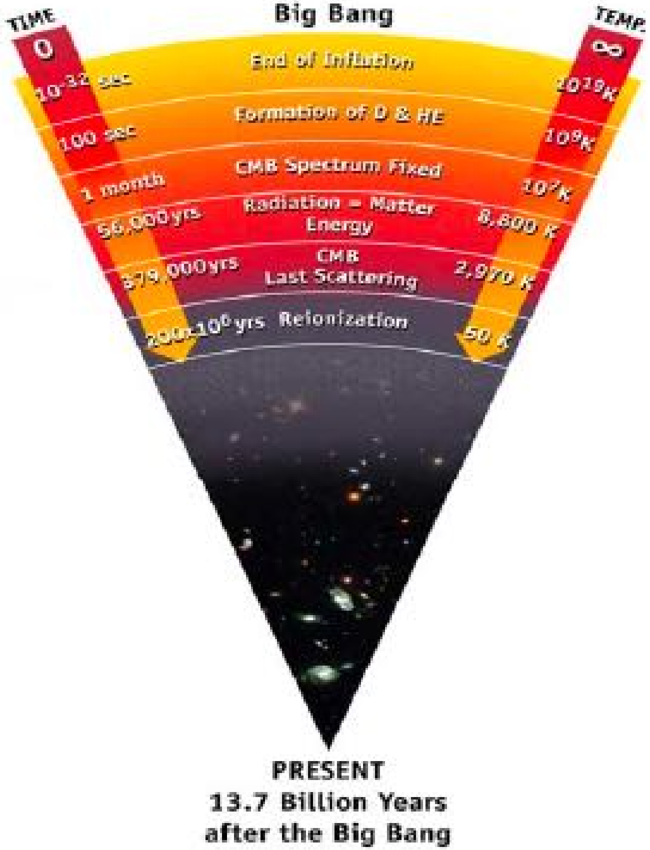


Matter is ionized at temperatures higher than the hydrogen ionization energy of 13.6 eV

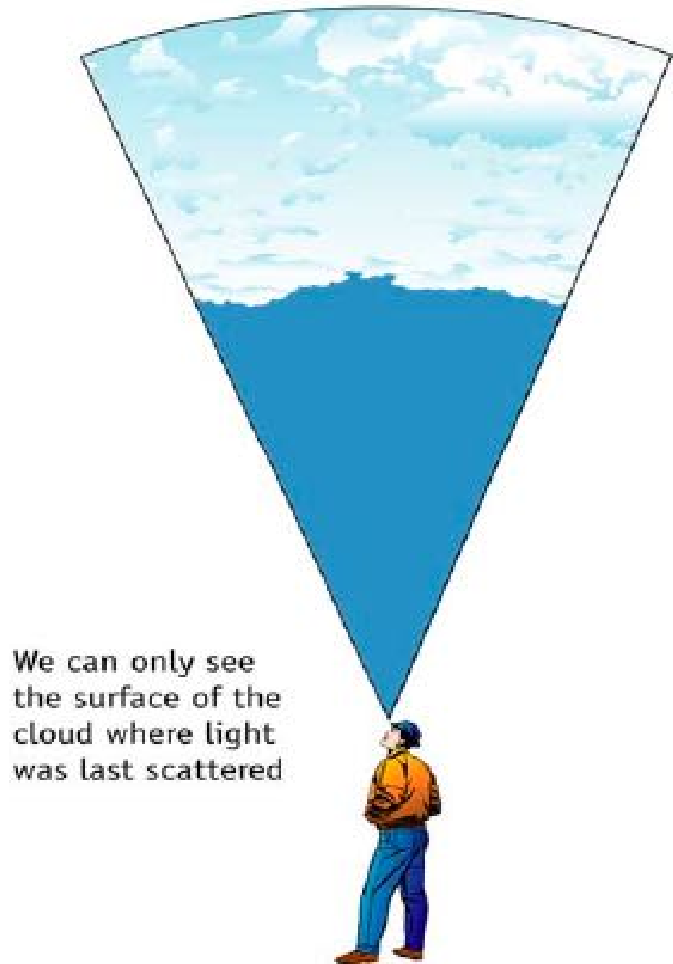
$$\frac{n_e n_p}{n_H} = \left(\frac{m_e T}{2\pi}\right)^{3/2} e^{-E_{\rm ion}/T}$$

The Universe becomes transparent to photons when

$$(\sigma_{e\gamma} n_e)^{-1} \sim t,\ \sigma_{e\gamma} = 8\pi\alpha^2/3m_e^2,\ T_{\rm LS} \simeq 0.26\ {\rm eV}$$

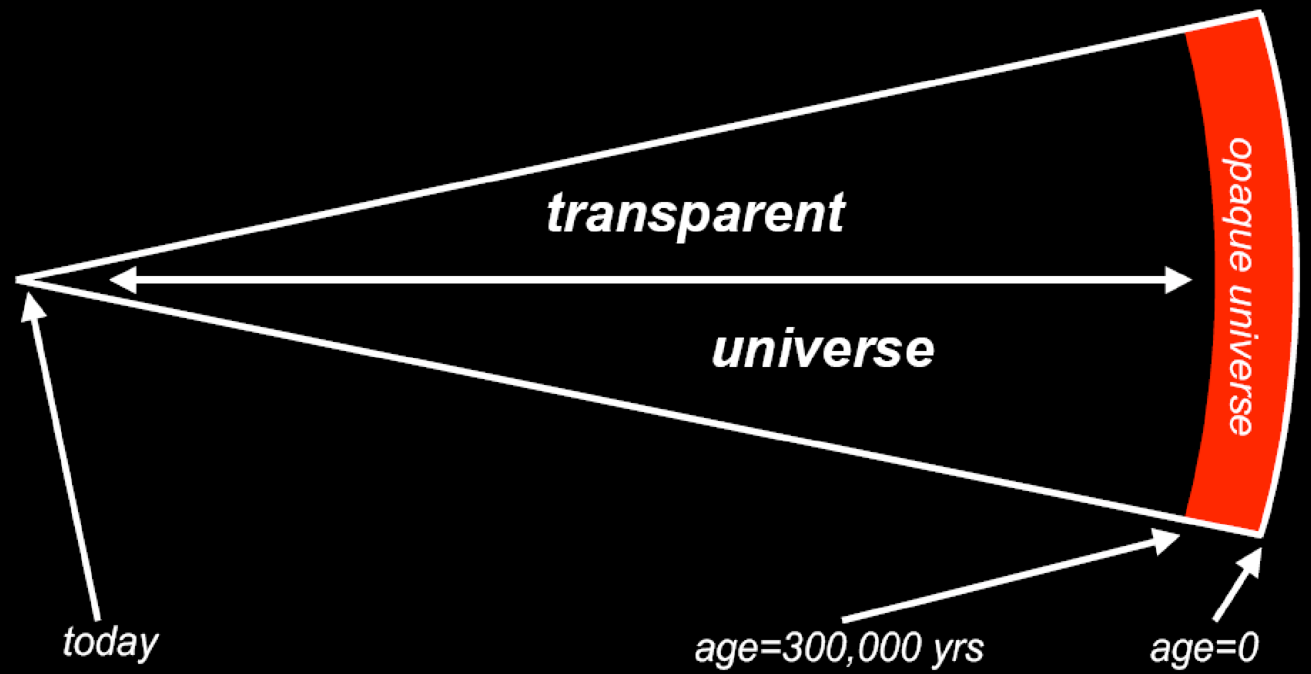


The cosmic microwave background Radiation's "surface of last scatter" is analogous to the light coming through the clouds to our eye on a cloudy day.

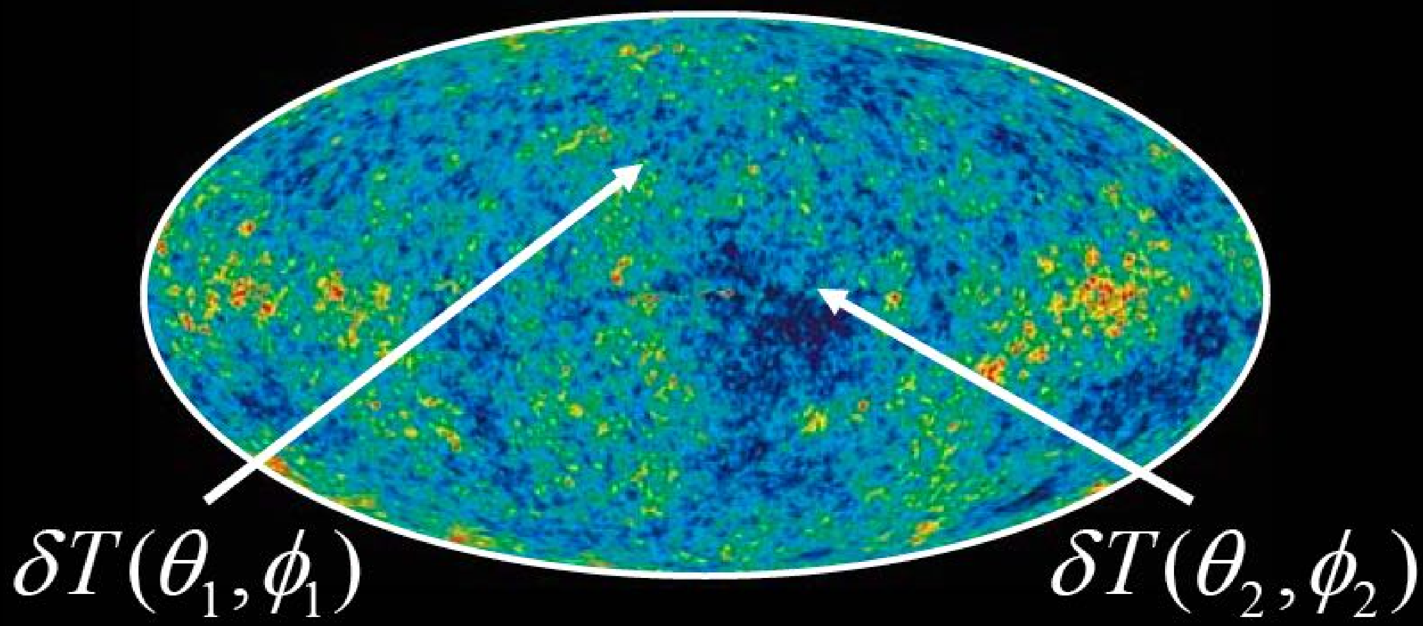


Cosmic background radiation

looking out in space is looking back in time



CMB anisotropy



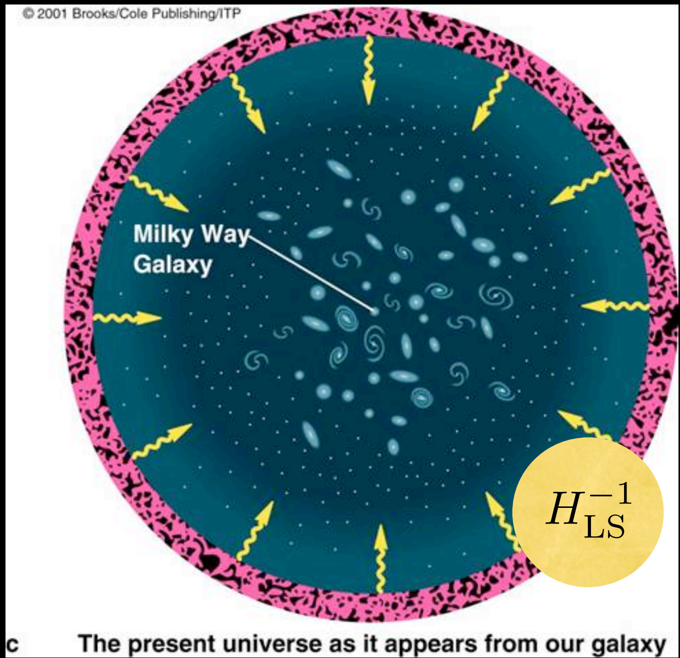
$$\frac{\Delta T}{T}(x_0,\tau_0,\mathbf{n}) = \sum_{\ell m} a_{\ell m}(x_0) Y_{\ell m}(\mathbf{n})$$

$$\langle a_{\ell m} a_{\ell' m'} \rangle = \delta_{\ell\ell'}\delta_{mm'} C_\ell$$

$$\langle \frac{\Delta T}{T}(\mathbf{n}) \frac{\Delta T}{T}(\mathbf{n}') \rangle = \sum_{\ell} \frac{(2\ell+1)}{4\pi} C_\ell P_\ell(\mathbf{n}\cdot\mathbf{n}')$$

(ensemble averages)

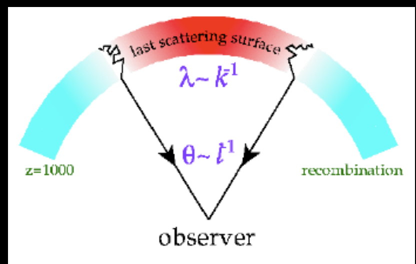
Horizon at Last Scattering



Comoving distance between us and the last scattering surface

$$d\tau = dt/a$$

$$\int_{t_{\rm LS}}^{t_0} \frac{dt}{a} = \int_{\tau_{\rm LS}}^{\tau_0} d\tau = (\tau_0 - \tau_{\rm LS})$$



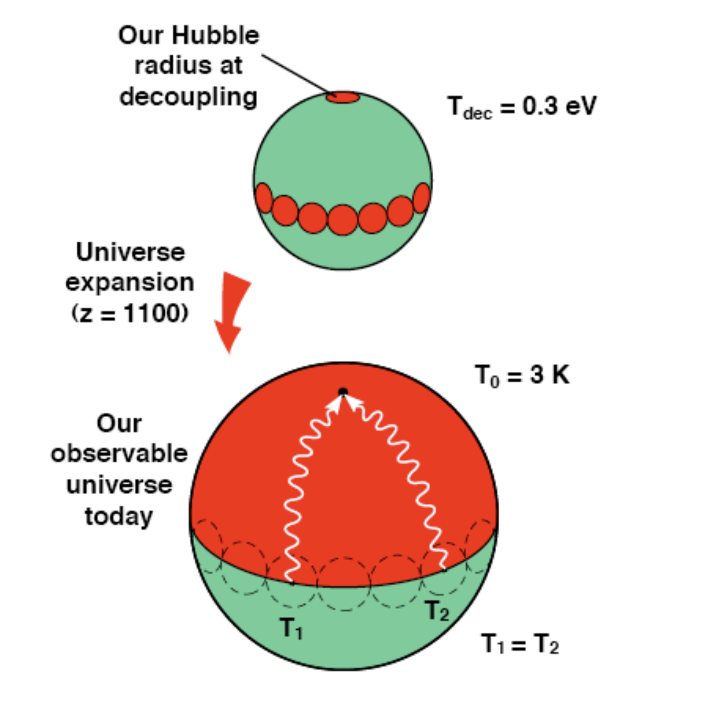
Angle subtended by a given comoving length scale

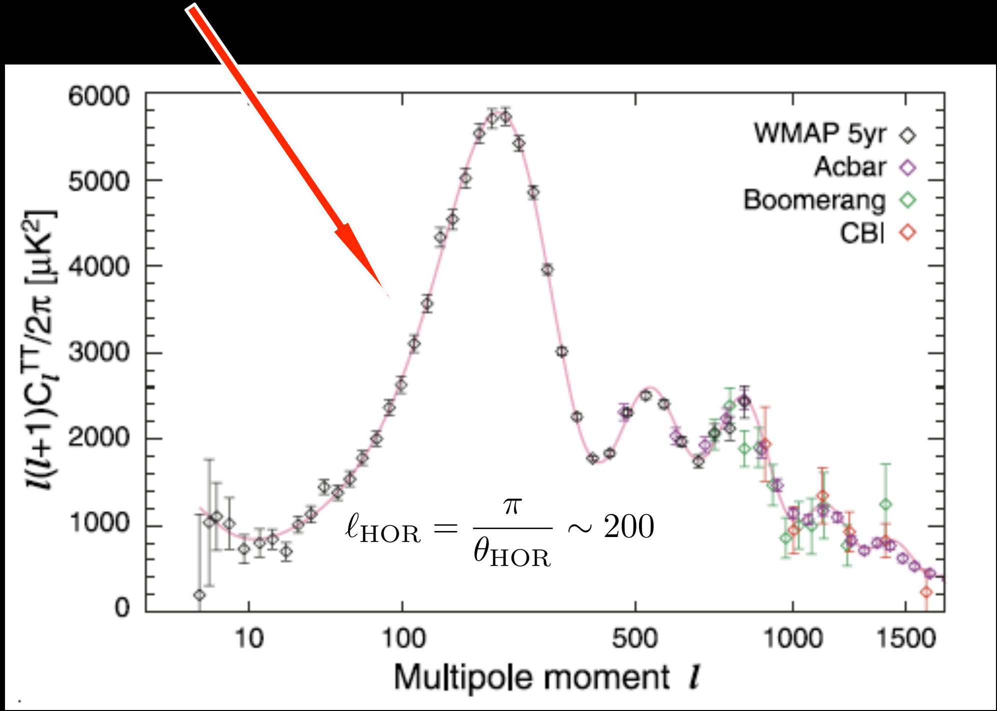
$$\theta \simeq \frac{\lambda}{(\tau_0 - \tau_{\rm LS})}$$

Sound Horizon

$$\theta_{\rm HOR} \simeq c_s \frac{\tau_{\rm LS}}{(\tau_0 - \tau_{\rm LS})} \simeq c_s \frac{\tau_{\rm LS}}{\tau_0} \simeq c_s \left(\frac{T_0}{T_{\rm LS}}\right)^{1/2} \simeq 1^{\rm o}$$

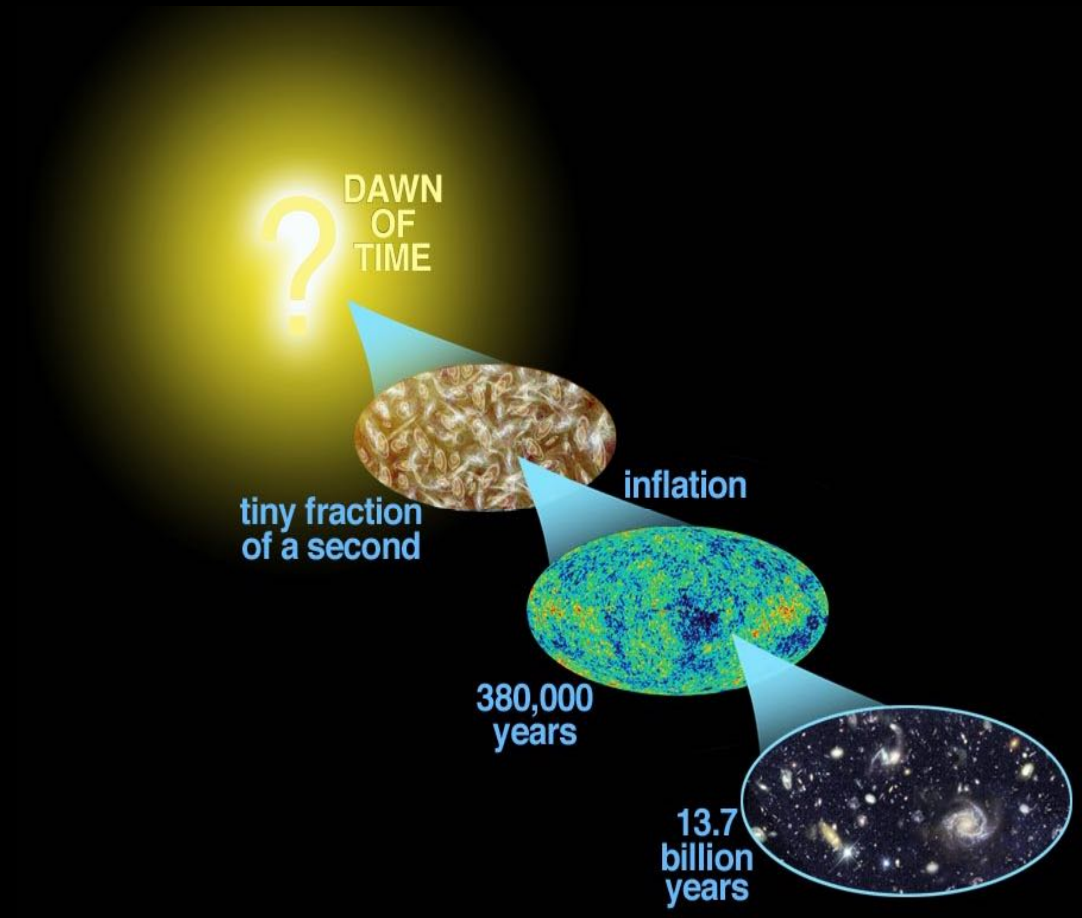
Super-Horizon mode detected in the CMB anisotropy



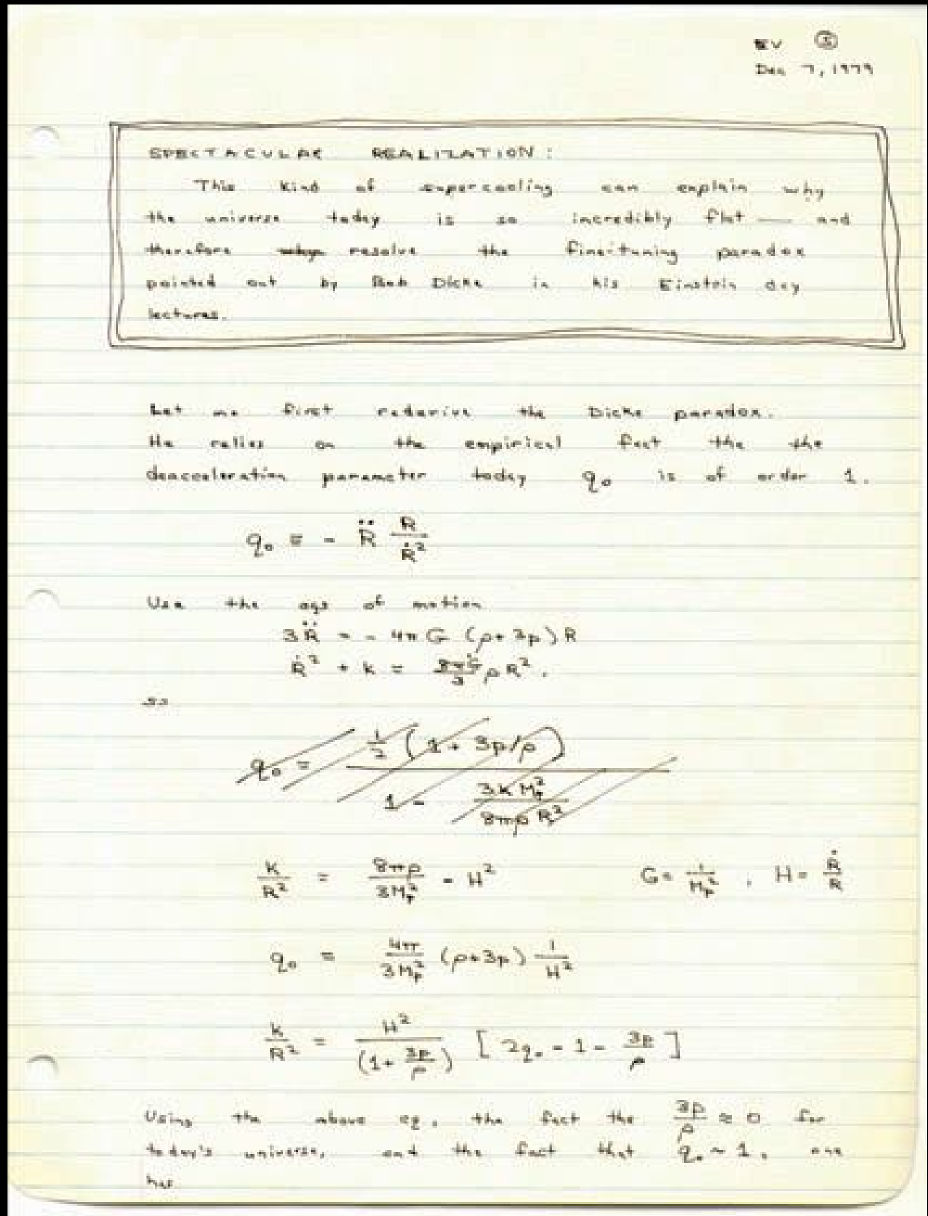


Why is the Universe so homogeneous and isotropic if, back in time, it was a collection of separated Universes?

The Inflationary Cosmology



Alan Guth



SV ⑤
Dec 7, 1979

SPECTACULAR REALIZATION:
This kind of supercooling can explain why the universe today is so incredibly flat — and therefore ~~also~~ resolve the fine-tuning paradox pointed out by Bob Dicke in his Einstein day lectures.

Let me first rederive the Dicke paradox.
He relies on the empirical fact the the deaccoleration parameter today q_0 is of order 1.

$$q_0 \equiv - \ddot{R} \frac{R}{\dot{R}^2}$$

Use the eqs of motion

$$3\ddot{R} = -4\pi G (\rho + 3p) R$$

$$\dot{R}^2 + k = \frac{8\pi G}{3} \rho R^2 .$$

so

~~$q_0 = \frac{\frac{1}{2}(1 + 3p/\rho)}{1 - \frac{3 k M_P^2}{8\pi\rho R^2}}$~~

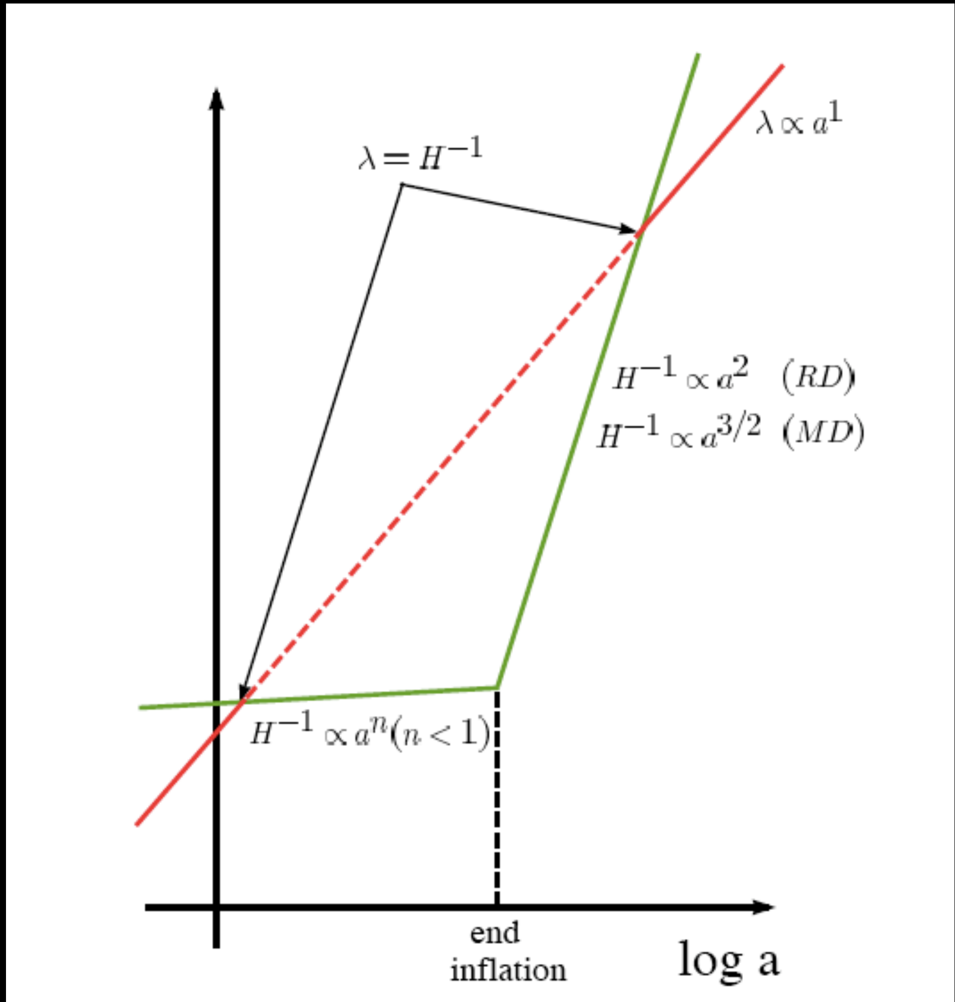
$$\frac{k}{R^2} = \frac{8\pi\rho}{3M_P^2} - H^2 \qquad G = \frac{1}{M_P^2} , \quad H = \frac{\dot{R}}{R}$$

$$q_0 = \frac{4\pi}{3M_P^2} (\rho + 3p) \frac{1}{H^2}$$

$$\frac{k}{R^2} = \frac{H^2}{(1 + \frac{3p}{\rho})} \left[2q_0 - 1 - \frac{3p}{\rho} \right]$$

Using the above eq., the fact the $\frac{3p}{\rho} \approx 0$ for today's universe, and the fact that $q_0 \sim 1$, one has

Inflationary Cosmology



$$\left(\frac{\lambda}{H^{-1}}\right)^{\cdot} = \ddot{a} > 0 \Leftrightarrow \text{Inflation}$$

Suppose there is a period during which the Hubble rate is constant (pure de Sitter epoch)

$$H = \text{constant} = \frac{\dot{a}}{a} \Rightarrow a = a_i e^{H_*(t-t_i)} \equiv a_i e^N$$

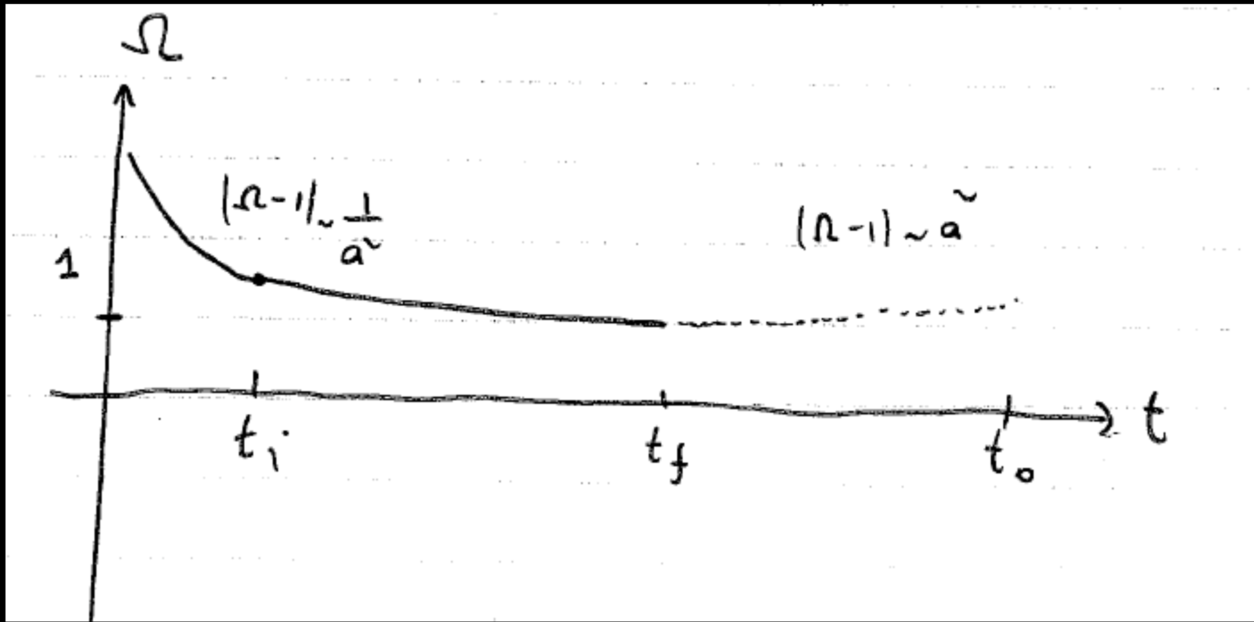
$$N = \text{number of efolds}$$

$$\text{In conformal time}\, a(\tau) = -\frac{1}{H\tau}\, (\tau < 0)$$

Flatness Problem

$$\Omega - 1 = \frac{k}{a^2 H^2} \sim \frac{1}{a^2}$$

$$\frac{|\Omega - 1|_{\rm end}}{|\Omega - 1|_{\rm in}} = \left(\frac{a_{\rm in}}{a_{\rm end}}\right)^2 = e^{-2N} \simeq 10^{-64} \Rightarrow N > \mathcal{O}(60)$$



Flatness Problem = Entropy Problem

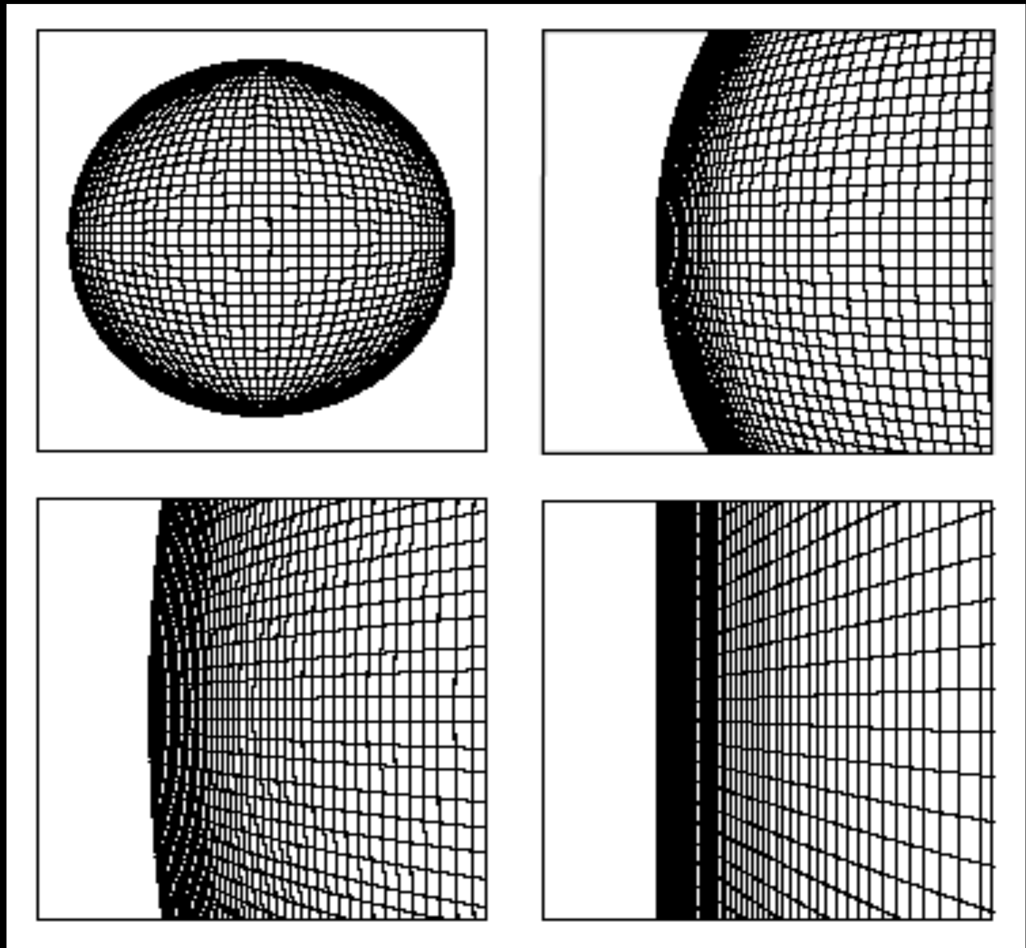
Adiabaticity is broken when the inflation energy density is released under the form of relativistic degrees of freedom

=

phase transition

$$\frac{S_{\rm end}}{S_{\rm in}} \sim \left(\frac{a_{\rm end}T_{\rm end}}{a_{\rm in}T_{\rm in}}\right)^3 \sim \frac{10^{90}}{1} \sim e^{3N} \Rightarrow N > \mathcal{O}(60)$$

Inflation does NOT change the global structure of space, but LOCALLY it makes it flat

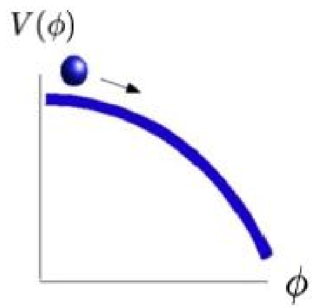


$$\text{IF } N \gg 60 \Rightarrow \Omega_0 = 1 + \mathcal{O}\left(e^{60-N}\right)$$

How to get Inflation

Inflation

For a review,see
D.H. Lyth and A.R.,
Phys. Rept. 314
(1999) 1



Potential → Vacuum Energy → Inflation

Friedmann equation:

slow roll

$$H^2 = \left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi}{3m_{\rm Pl}^2}\left[\frac{1}{2}\dot{\phi}^2 + V(\phi)\right] \simeq \text{const.}$$

Scalar field equation of motion:

$$\ddot{\phi} + 3\left(\frac{\dot{a}}{a}\right)\dot{\phi} + V'(\phi) = 0 \qquad a(t) \propto e^{\int H dt} \equiv e^N$$

How to get Inflation

Slow Roll Parameters

$\epsilon(\phi)$ Parameterizes equation of state:

$$\epsilon \equiv \frac{m_{\rm Pl}^2}{4\pi}\left[\frac{H'(\phi)}{H(\phi)}\right]^2 \simeq \frac{m_{\rm Pl}^2}{16\pi}\left[\frac{V'(\phi)}{V(\phi)}\right]$$

$$p = \rho\left(\frac{2}{3}\epsilon - 1\right)$$

Inflation ⟷ $\epsilon(\phi) < 1$

Second slow roll parameter:

$$\eta \equiv \frac{m_{\rm Pl}^2}{4\pi}\left[\frac{H''(\phi)}{H(\phi)}\right] \simeq \frac{m_{\rm Pl}^2}{8\pi}\left[\frac{V''(\phi)}{V(\phi)}\right] - \frac{m_{\rm Pl}^2}{16\pi}\left[\frac{V'(\phi)}{V(\phi)}\right]$$

Slow-Roll parameters are small and vary slowly with time

$$\begin{aligned}
\epsilon &= -\frac{\dot{H}}{H^2} = 4\pi G \frac{\dot{\phi}^2}{H^2} = \frac{1}{16\pi G}\left(\frac{V'}{V}\right)^2, \\
\eta &= \frac{1}{8\pi G}\left(\frac{V''}{V}\right) = \frac{1}{3}\frac{V''}{H^2}, \\
\delta &= \eta - \epsilon = -\frac{\ddot{\phi}}{H\dot{\phi}}.
\end{aligned}$$

$$\dot{\epsilon} \sim \left(\frac{\dot{\phi}\ddot{\phi}}{H^2} - \frac{\dot{\phi}^2}{H^3}\dot{H}\right)\frac{1}{M_p^2} \sim H(\epsilon\delta - \epsilon^2)$$

The total number of efolds

$$
\begin{aligned}
N &= \int_{t_i}^{t_f} dt\, H(t) \\
&= \int_{\phi_i}^{\phi_f} d\phi \frac{dt}{d\phi}\, H(\phi) \\
&= \int_{\phi_i}^{\phi_f} d\phi \frac{H}{\dot{\phi}} \\
&= (\mathrm{slow-roll}) \\
&= -3 \int_{\phi_i}^{\phi_f} d\phi \frac{H^2}{V'} \\
&= (\mathrm{slow-roll}) \\
&= 8\pi G_N \int_{\phi_f}^{\phi_i} d\phi \frac{V}{V'}
\end{aligned}
$$

Example: $V(\phi) = \frac{m^2}{2}\phi^2$

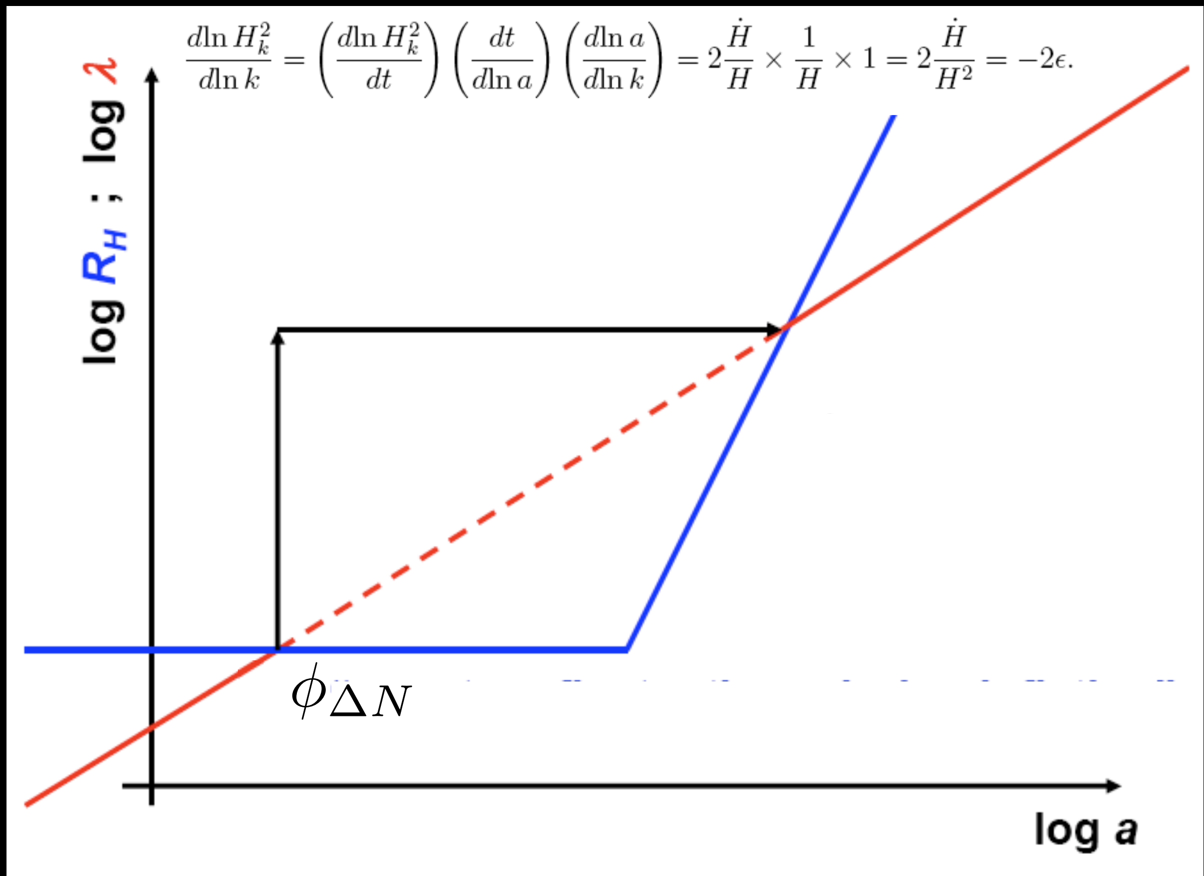
$$V(\phi_i) \sim M_p^4 \Rightarrow \phi_i \sim (M_p^2/m)$$

$$N \sim 4\pi G_N \phi_i^2 \sim (M_p/m)^4$$

In fact it turns out that $(M_p/m) \sim 10^6$

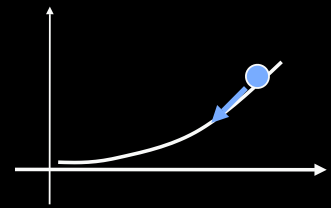
The number of efolds
till the end of inflation

$$\Delta N \simeq 8\pi G_N \int_{\phi_f}^{\phi_{\Delta N}} d\phi \frac{V}{V'}$$



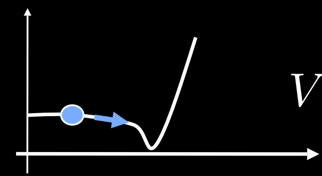
Standard scenario = one-single field (slow-roll) models

1. **large field**
   e.g. chaotic inflation



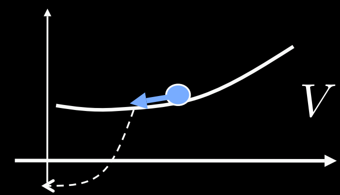
$$V \sim \mu^{4-p}\phi^p$$

2. **small field**
   e.g. new or natural inflation



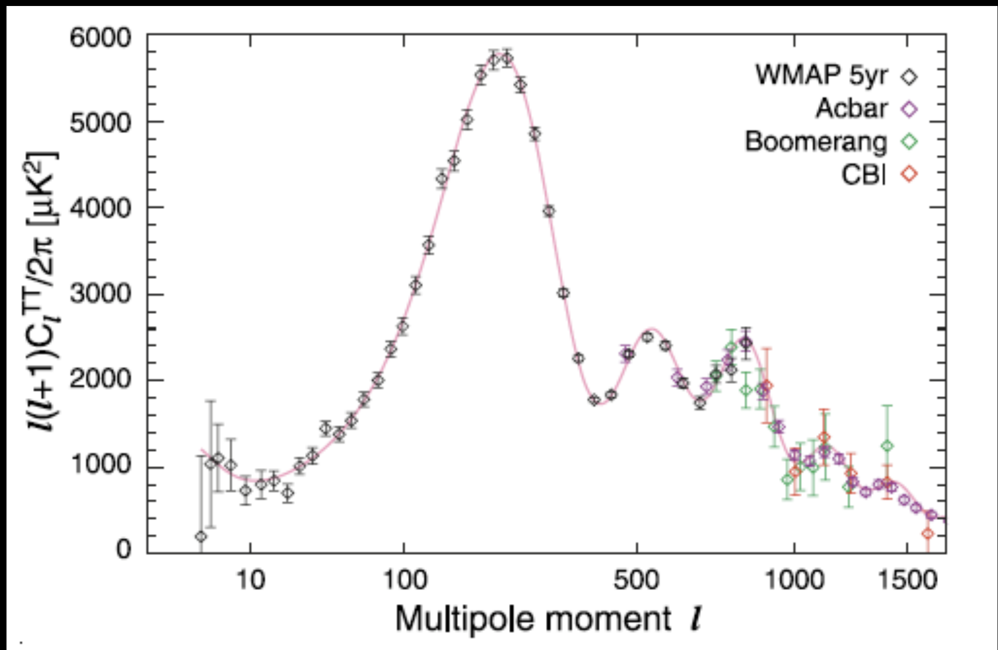
$$V \sim V_0 - \mu^{4-p}\phi^p$$

3. **hybrid inflation**
   e.g., Susy or Sugra models

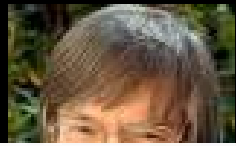


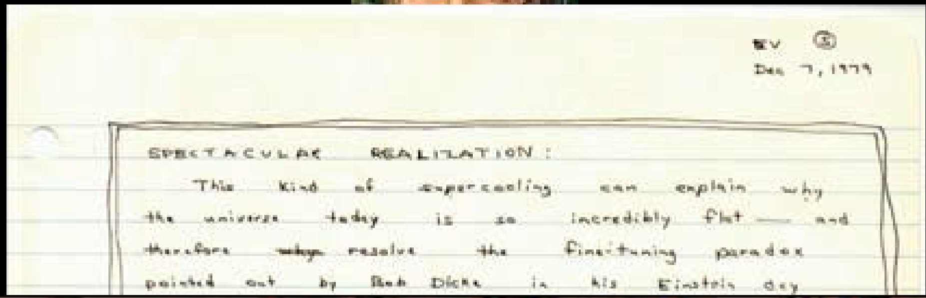
$$V \sim V_0 + \mu^{4-p}\phi^p$$

Lecture two: the cosmological perturbations and CMB anisotropy



The Universe is NOT homogeneous and isotropic

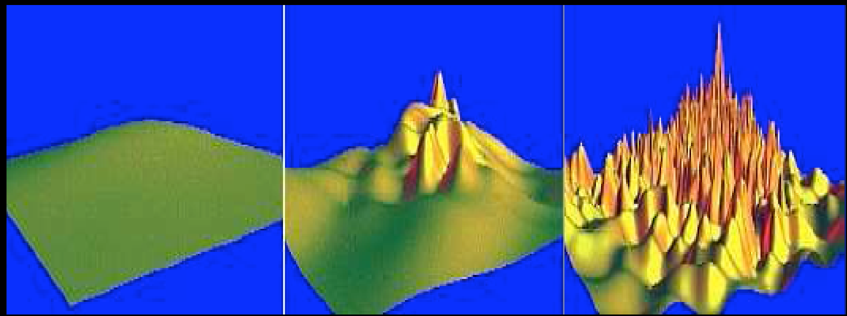




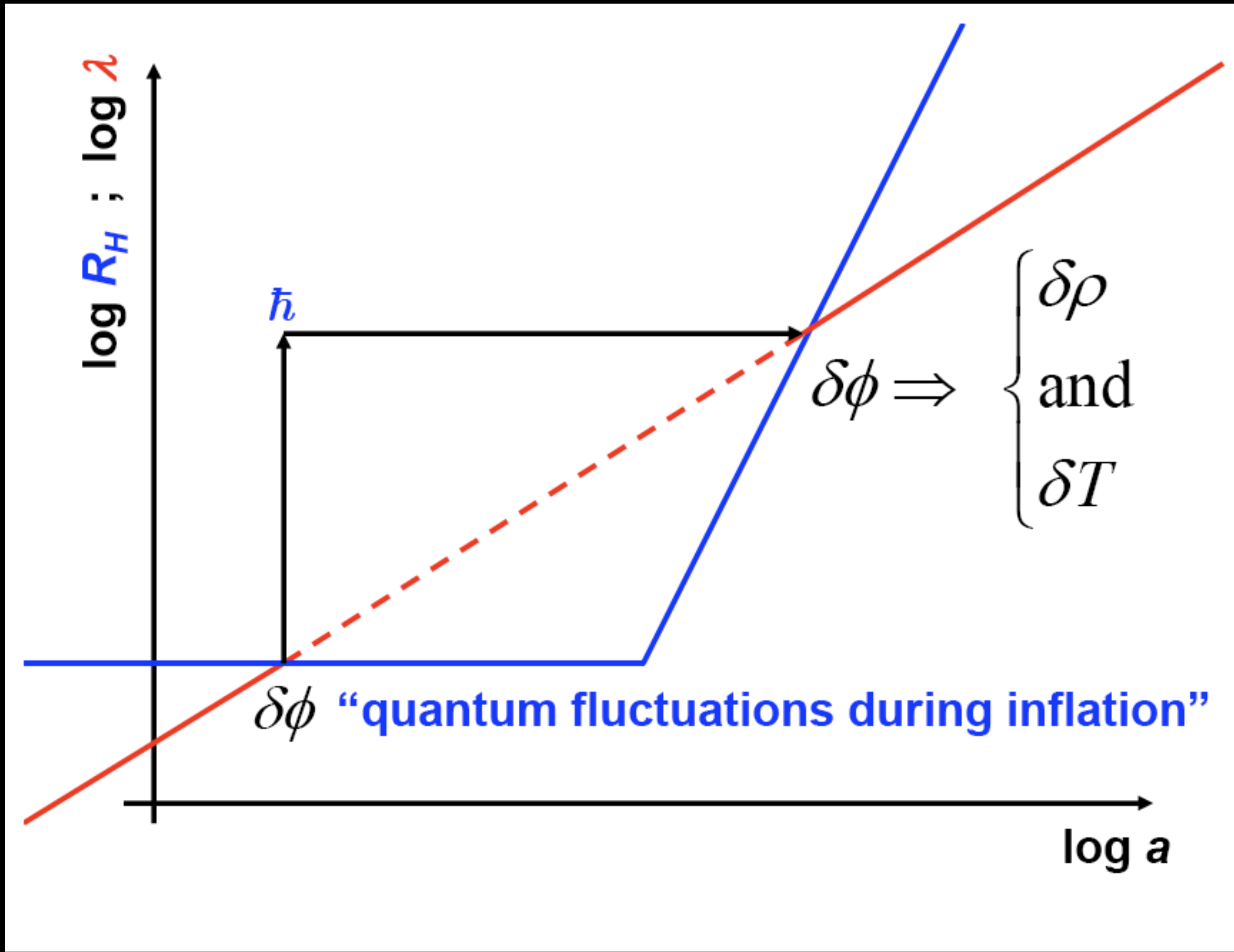
EV ②
Dec 7, 1979

SPECTACULAR REALIZATION:

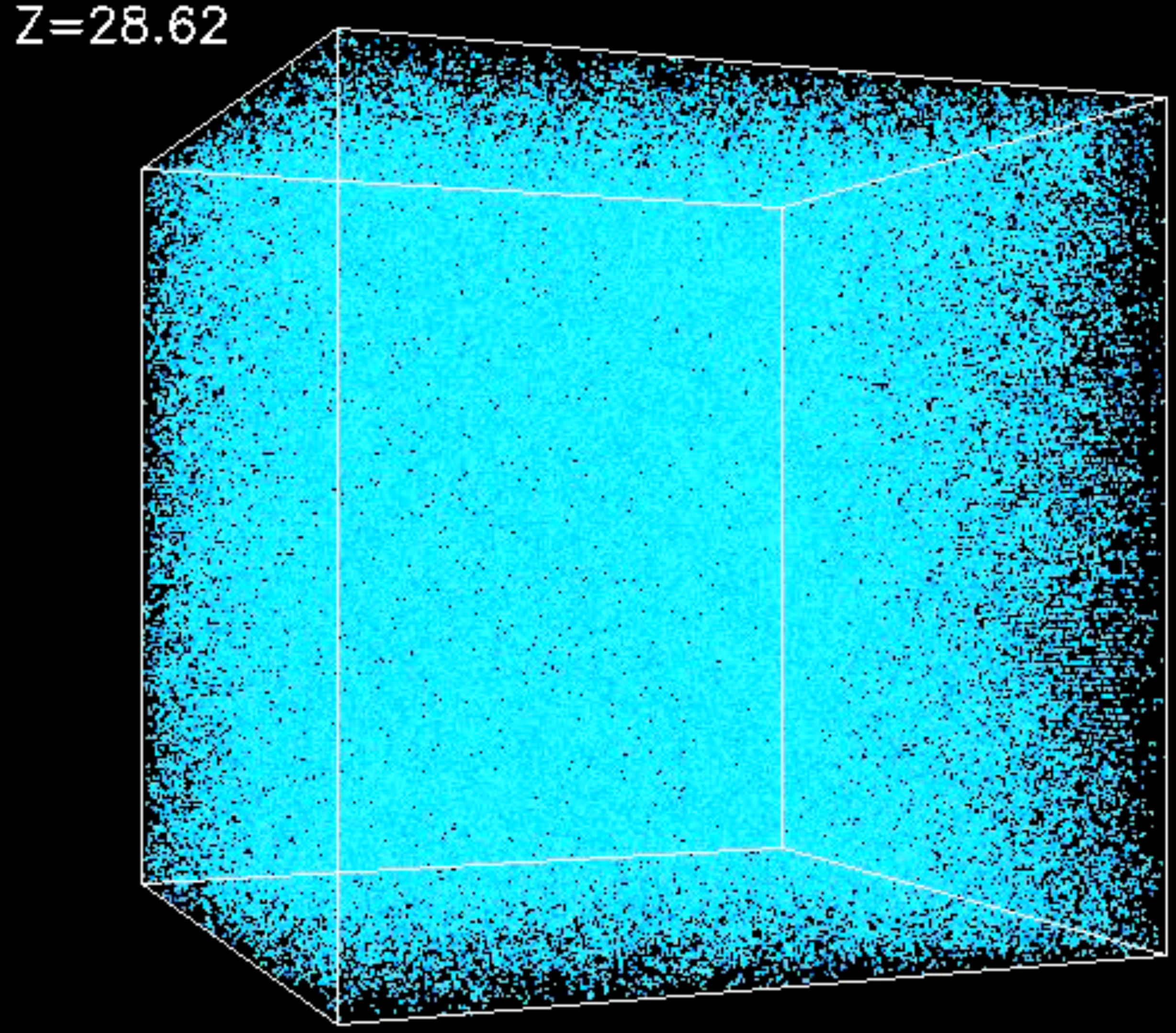
This kind of supercooling can explain why the universe today is so incredibly flat — and therefore resolve the fine-tuning paradox pointed out by Bob Dicke in his Einstein day



From Quantum Fluctuations to the Large Scale Structure

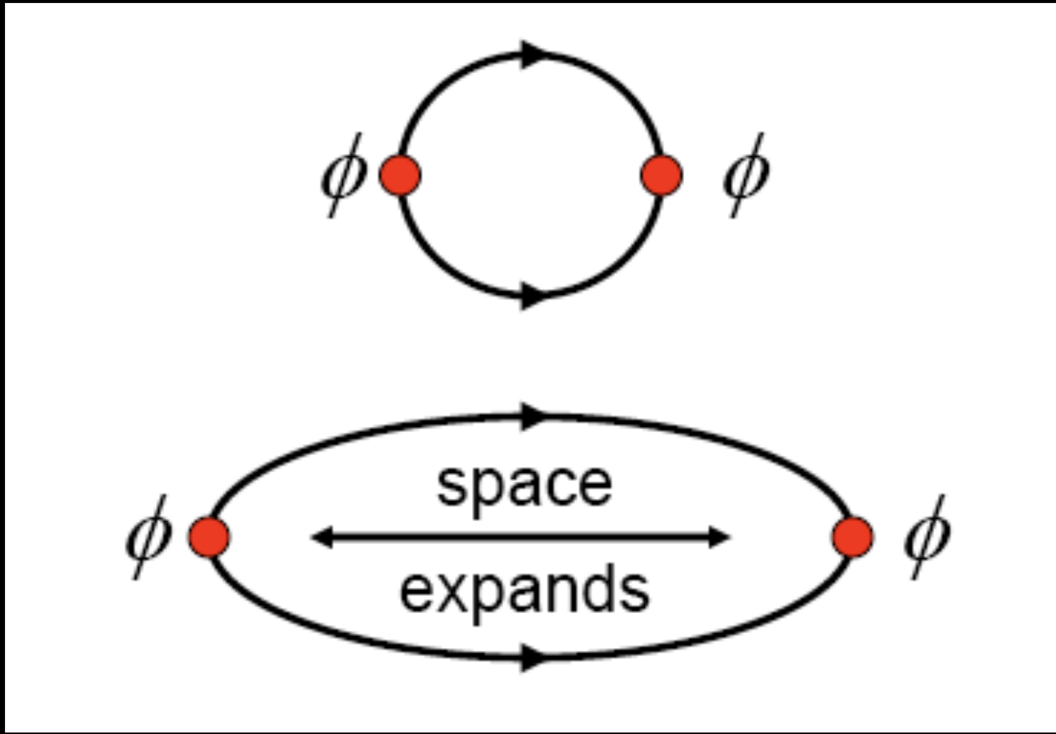


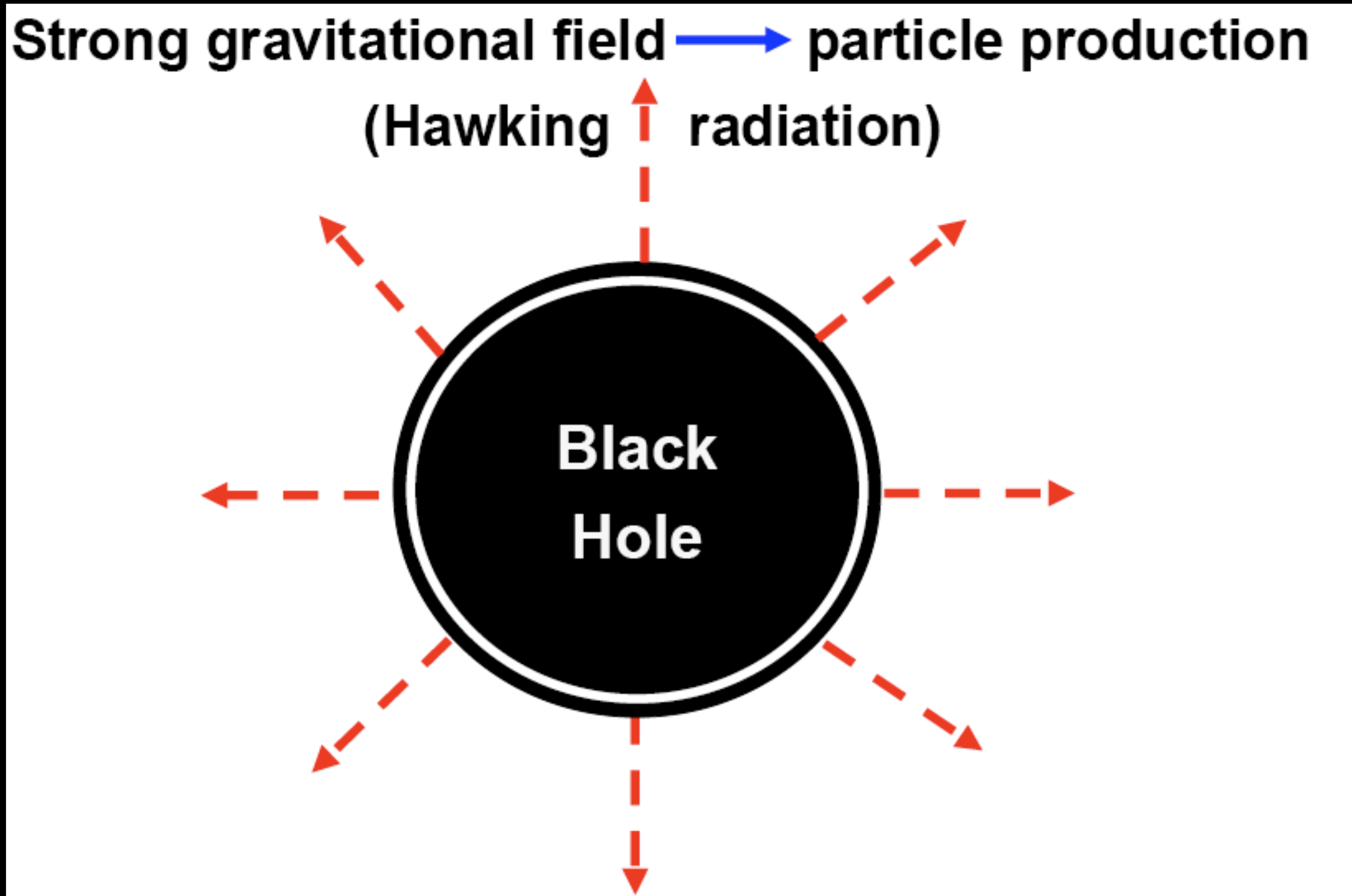
log R_H ; log λ
$\hbar$
$\delta\phi \Rightarrow$
$\delta\rho$
and
δT
$\delta\phi$ "quantum fluctuations during inflation"
log a



The Millenium Simulation Project:
http://www.mpa-garching.mpg.de/galform/virgo/millennium/

Particle production in an expanding Universe




Strong gravitational field
particle production
(Hawking radiation)
Black Hole

Take now perturbations of the inflaton field: heuristic explanation of why the inflaton field is perturbed

$$
\begin{aligned}
\phi(\mathbf{x},t) &= \phi_0(t) + \delta\phi(\mathbf{x},t) \\
\delta\ddot{\phi} + 3H\delta\dot{\phi} &- \cancel{\frac{\nabla^2\delta\phi}{a^2}} + V''\delta\phi = 0 \\
\ddot{\phi}_0 + 3H\dot{\phi}_0 + V'(\phi_0) &= 0 \Rightarrow \dddot{\phi}_0 + 3H\ddot{\phi}_0 + V''\dot{\phi}_0 = 0
\end{aligned}
$$

$$
\begin{aligned}
\delta\phi &= \dot{\phi}_0\tau(\mathbf{x}) \\
\phi(\mathbf{x},t) &= \phi_0(t+\tau(\mathbf{x}))
\end{aligned}
$$

The inflaton field has different classical values at different points in space

All massless scalar fields are excited during Inflation

Linear Theory

$$\begin{aligned} \sigma(\mathbf{x},\tau) &= \sigma_0(\tau) + \delta\sigma(\mathbf{x},\tau), \\ u_k(\tau) &= a(\tau)\delta\sigma_k(\tau), \\ d\tau &= \frac{dt}{a} \end{aligned}$$

$$\boxed{u_k'' + \left(k^2 - \frac{a''}{a}\right) u_k = 0}$$

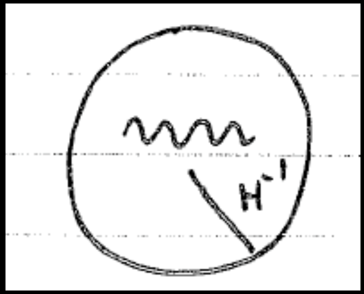
$$\left(\delta\ddot{\sigma}_k + 3H\delta\dot{\sigma}_k + \frac{k^2}{a^2}\delta\sigma_k = 0\right)$$

Oscillator with time-dependent frequency

a) For modes with wavelengths inside the horizon:

$$\lambda_{\rm phys} \ll H^{-1} \Rightarrow k/a \gg H \Rightarrow (-k\tau) \gg 1$$

$$a''/a = 2/\tau^2 \Rightarrow (-k\tau) \gg 1 \Rightarrow k^2 \gg a''/a$$



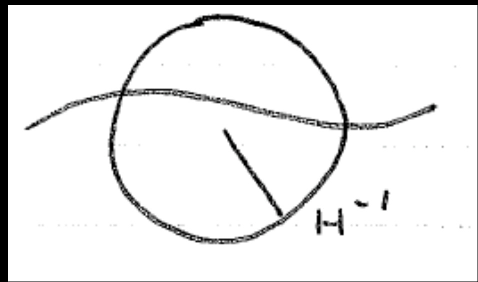
$$u_k'' + k^2 u_k = 0 \Rightarrow u_k = A(k)\frac{e^{-ik\tau}}{\sqrt{2k}} + B(k)\frac{e^{ik\tau}}{\sqrt{2k}}$$

Set of independent plane waves: locally Minkowski, no curvature seen by the waves

b) For modes with wavelengths outside the horizon:

$$\lambda_{\rm phys} \gg H^{-1} \Rightarrow k/a \ll H \Rightarrow (-k\tau) \ll 1$$

$$a''/a = 2/\tau^2 \Rightarrow (-k\tau) \ll 1 \Rightarrow k^2 \ll a''/a$$



$$u_k'' - \frac{a''}{a} u_k = 0 \Rightarrow u_k = C(k)a(\tau) \Rightarrow \delta\sigma_k = C(k)$$

Superhorizon perturbations do not evolve in time

Exact solution exists:

$$u_k(\tau) = A(k)\frac{e^{-ik\tau}}{\sqrt{2k}}\left(1 - \frac{i}{k\tau}\right) + B(k)\frac{e^{ik\tau}}{\sqrt{2k}}\left(1 + \frac{i}{k\tau}\right)$$

Choose the boundary conditions in the far UV such that the solution is a plane wave propagating with positive frequency (Bunch-Davies vacuum)

$$(-k\tau) \gg 1 \quad \Rightarrow \quad A(k) = 1,\ B(k) = 0$$

$$u_k(\tau) = \frac{e^{-ik\tau}}{\sqrt{2k}}\left(1 - \frac{i}{k\tau}\right),$$

$$\delta\sigma_k = \frac{u_k}{a} = (-H\tau)\frac{e^{-ik\tau}}{\sqrt{2k}}\left(1 - \frac{i}{k\tau}\right)$$

Power Spectrum

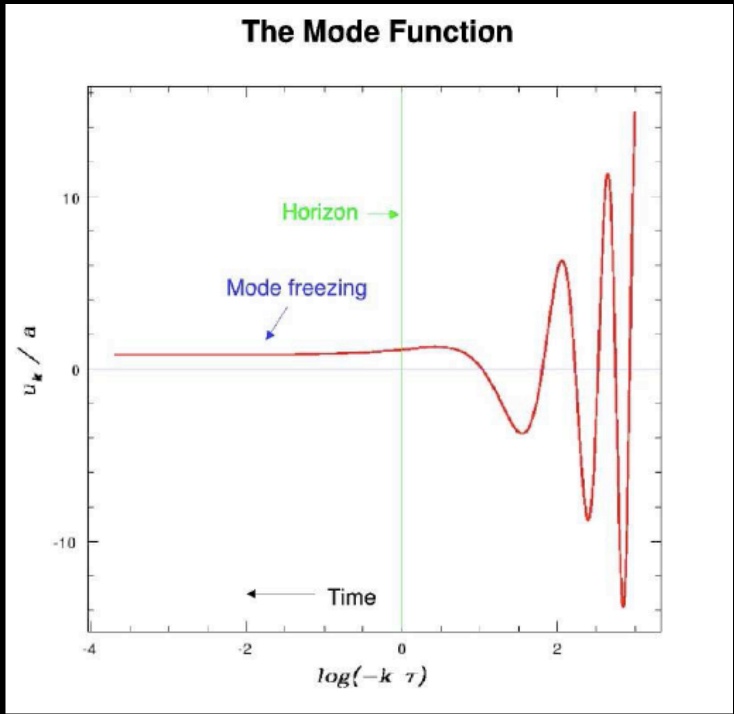
$$\langle 0 \left| (\delta\sigma(\mathbf{x},t))^2 \right| \rangle \quad = \quad \int \frac{d^3k}{(2\pi)^3} \left|\delta\sigma_k\right|^2$$

$$\equiv \quad \int \frac{dk}{k} \mathcal{P}_{\delta\sigma}(k)$$

$$\mathcal{P}_{\delta\sigma}(k) = \frac{k^3}{2\pi^2} \left|\delta\sigma_k\right|^2$$

$$\mathcal{P}_{\delta\sigma}(k) = \mathcal{A}^2 \left(\frac{k}{aH} \right)^{n-1}$$

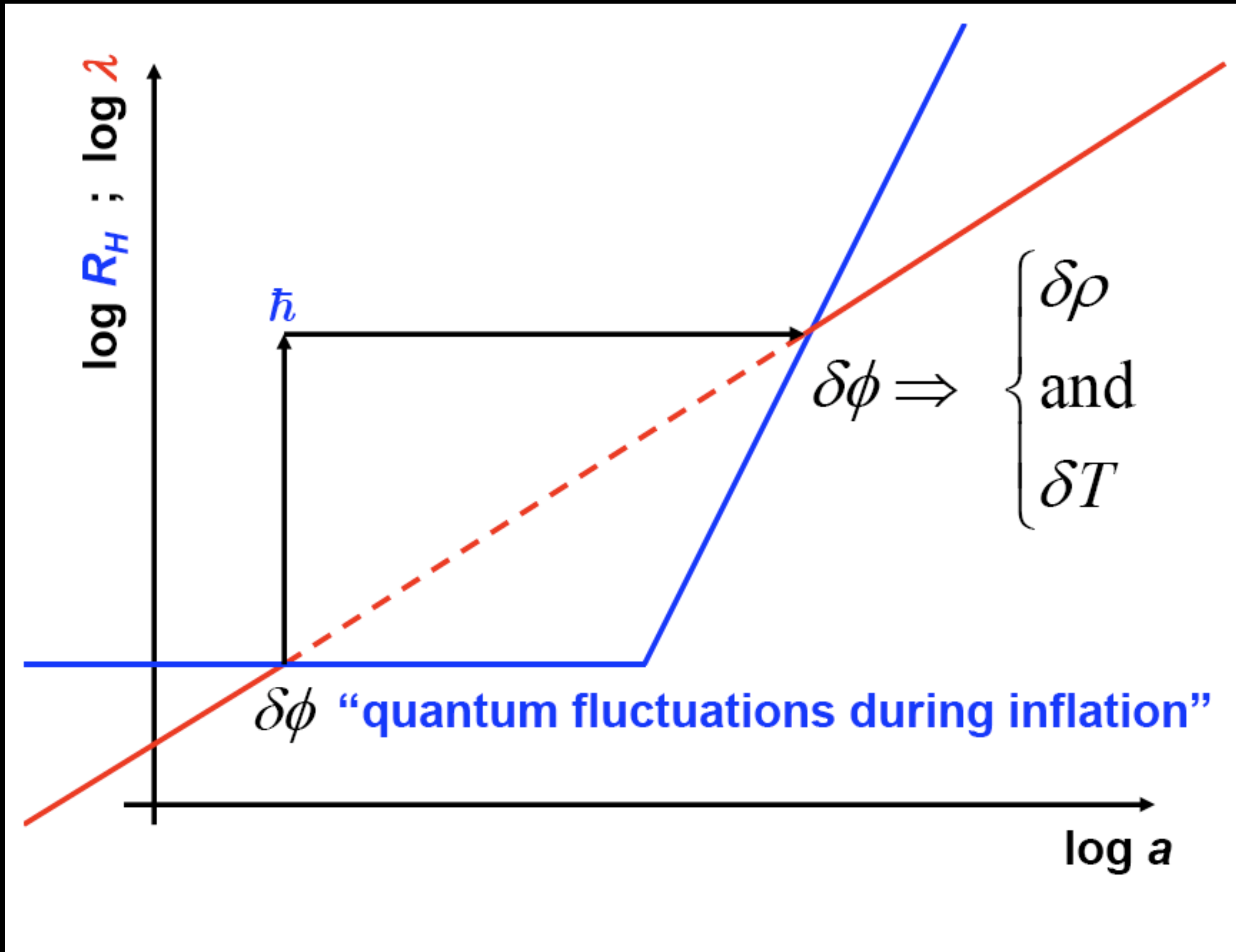
Perturbations of a (nearly) massless scalar field
are born as plane waves with wavelengths below the horizon.
As inflation proceeds, their wavelenghts are stretched
outside the horizon and get frozen



$$\mathcal{P}_{\delta\sigma} = \frac{k^3}{2\pi^2} |\delta\sigma_k|^2 = \left(\frac{H}{2\pi}\right)^2 \left(\frac{k}{aH}\right)^{n-1}$$

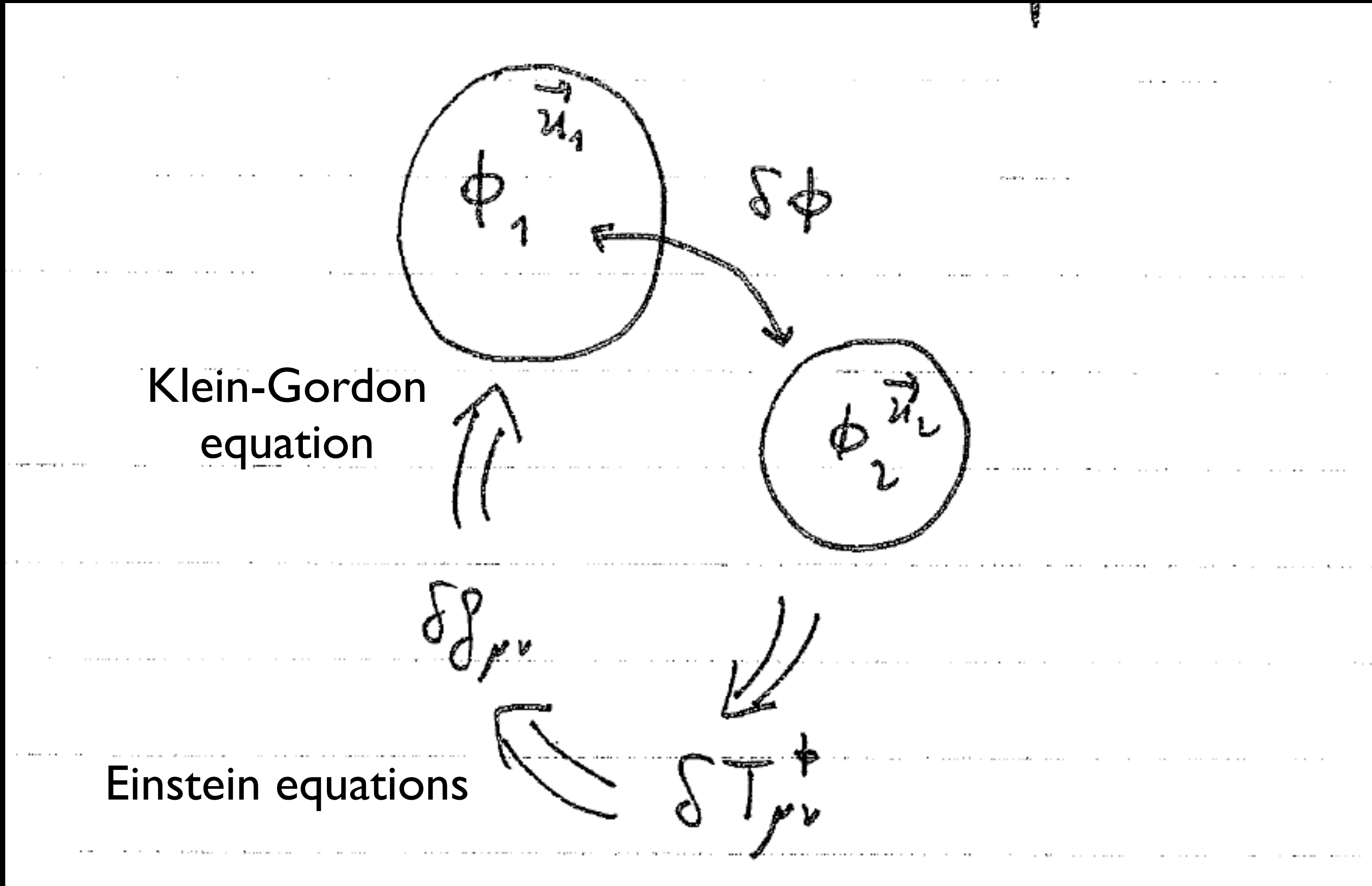
All massless scalar fields during a period of exponential inflation (pure de Sitter) are quantum mechanically excited with a power spectrum which is constant and flat on superhorizon scales (independent from the wavelength)

Perturbations are GAUSSIAN:
it is linear perturbation theory
and all oscillators evolve independently from each other



log R_H ; log λ
$\hbar$
$\delta\phi \Rightarrow \begin{cases} \delta\rho \\ \text{and} \\ \delta T \end{cases}$
$\delta\phi$ "quantum fluctuations during inflation"
log a

Have to include gravity



Counting degrees of freedom

1) $g_{\mu\nu}$ is a symmetric tensor, has 10 degrees of freedom, but we can perform a coordinate transformation $x^\mu \rightarrow x^\mu + \delta x^\mu$ and there remain 10-4=6 physical degrees of freedom

2) Helmholtz's theorem: $u_i = \partial_i v + v_i,\ \nabla \cdot \vec{v} = 0,\ v_{[i,j]} = 0$
there remain 2 vector degrees of freedom

3) Tensor perturbations have 6 degrees of freedom, but they are traceless and transverse, $h^i_j = 0,\ \partial^i h_{ij} = 0$, there remain 2 physical degrees of freedom

6-2-2=2 scalar degrees of freedom

We are only interested in slicings: $t \rightarrow t + \delta t \equiv \tilde{t}$

Take a scalar perturbation: $\tilde{f}(\tilde{t}) = f(t),\ \tilde{f}_0(\tilde{t}) = f_0(\tilde{t})$

$$
\begin{aligned}
\tilde{\delta f}(\tilde{t}) &= \tilde{f} - \tilde{f}_0(\tilde{t}) \\
&= f(t) - f_0(\tilde{t}) \\
&= f(t) - \dot{f}_0(t)\delta t - f_0(t) \\
&= \delta f - \dot{f}_0 \delta t
\end{aligned}
$$

$$\boxed{\delta f \rightarrow \delta f - \dot{f}_0 \delta t}$$

Take the gravitational potential in the metric:

$$ds^2 = \left[(1+2\Phi)dt^2 - a^2(1-2\psi)d\mathbf{x}^2\right]$$

$$\tilde{ds}^2 = ds^2 \Rightarrow \tilde{a}^2(\tilde{t})(1-2\tilde{\psi}) = a^2(t)(1-2\psi)$$

$$\tilde{a}^2(\tilde{t}) \simeq a^2(t) + 2\dot{a}a\delta t \Rightarrow \tilde{\psi} = \psi + H\delta t$$

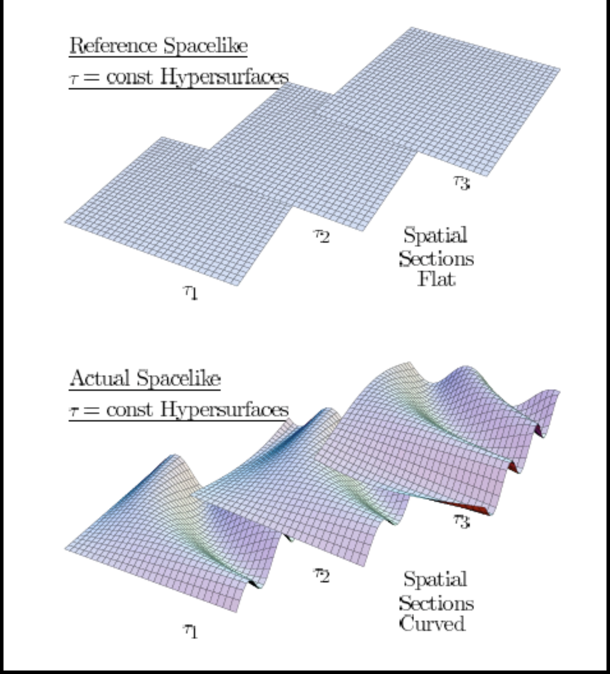
$$\psi \longrightarrow \psi + H\delta t$$

$$\Phi \longrightarrow \Phi - H\delta t - (\delta t)^{\cdot}$$

Including gravity

$$ds^2 = \left[(1+2\Phi)dt^2 - a^2(1-2\psi)d\mathbf{x}^2\right]$$

Need to define a gauge invariant quantity upon general coordinate transformations



$$\begin{aligned} t &\rightarrow t + \delta t, \\ \psi &\rightarrow \psi + H\,\delta t, \\ \delta\rho &\rightarrow \delta\rho - \dot{\rho}\,\delta t \end{aligned}$$

Comoving curvature perturbation

$$\zeta = -\psi - H\,\frac{\delta\rho}{\dot{\rho}}$$

Gravitational potential

Physical significance of the comoving curvature perturbation

$$\zeta = -\psi - H\,\frac{\delta\rho}{\dot\rho}$$

1) The curvature perturbation on slices of uniform energy density

$$\zeta = -\left.\psi\right|_{\delta\rho=0}\,, \quad {}^{(3)}R = \frac{4}{a^2}\nabla^2\psi$$

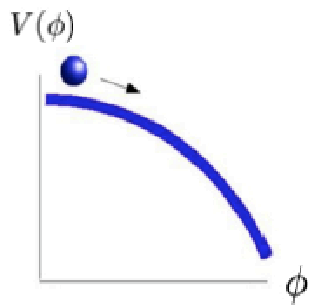
2) The energy density perturbation on flat slices

$$\zeta = -H\left.\frac{\delta\rho}{\dot\rho}\right|_{\psi=0} = H\left.\frac{\delta\rho}{3(\rho+P)}\right|_{\psi=0}$$

How to get Inflation

Inflation

For a review,see
D.H. Lyth and A.R.,
Phys. Rept. 314
(1999) 1



Potential → Vacuum Energy → Inflation

Friedmann equation:

slow roll

$$H^2 = \left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi}{3m_{\rm Pl}^2}\left[\frac{1}{2}\dot{\phi}^2 + V(\phi)\right] \simeq \text{const.}$$

Scalar field equation of motion:

$$\ddot{\phi} + 3\left(\frac{\dot{a}}{a}\right)\dot{\phi} + V'(\phi) = 0 \qquad a(t) \propto e^{\int H dt} \equiv e^N$$

How to get Inflation

Slow Roll Parameters

$\epsilon(\phi)$ Parameterizes equation of state:

$$\epsilon \equiv \frac{m_{\rm Pl}^2}{4\pi}\left[\frac{H'(\phi)}{H(\phi)}\right]^2 \simeq \frac{m_{\rm Pl}^2}{16\pi}\left[\frac{V'(\phi)}{V(\phi)}\right]$$

$$p = \rho\left(\frac{2}{3}\epsilon - 1\right)$$

Inflation ⟷ $\epsilon(\phi) < 1$

Second slow roll parameter:

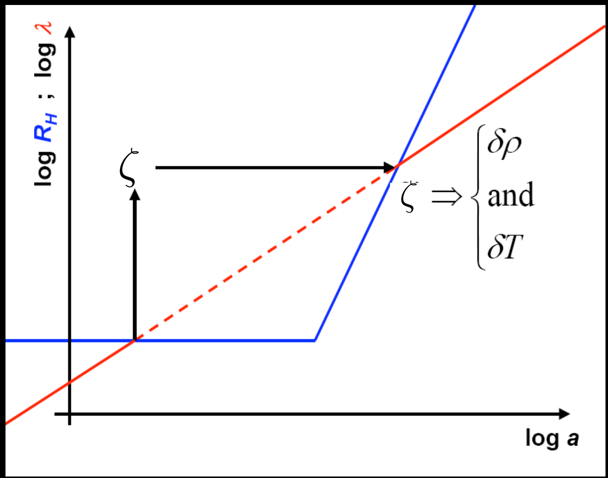
$$\eta \equiv \frac{m_{\rm Pl}^2}{4\pi}\left[\frac{H''(\phi)}{H(\phi)}\right] \simeq \frac{m_{\rm Pl}^2}{8\pi}\left[\frac{V''(\phi)}{V(\phi)}\right] - \frac{m_{\rm Pl}^2}{16\pi}\left[\frac{V'(\phi)}{V(\phi)}\right]$$

Slow-Roll parameters are small and vary slowly with time

$$\begin{aligned}\epsilon &= -\frac{\dot{H}}{H^2} = 4\pi G\frac{\dot{\phi}^2}{H^2} = \frac{1}{16\pi G}\left(\frac{V'}{V}\right)^2,\\ \eta &= \frac{1}{8\pi G}\left(\frac{V''}{V}\right) = \frac{1}{3}\frac{V''}{H^2},\\ \delta &= \eta - \epsilon = -\frac{\ddot{\phi}}{H\dot{\phi}}.\end{aligned}$$

$$\dot{\epsilon} \sim \left(\frac{\dot{\phi}\ddot{\phi}}{H^2} - \frac{\dot{\phi}^2}{H^3}\dot{H}\right)\frac{1}{M_p^2} \sim H(\epsilon\delta - \epsilon^2)$$

Comoving curvature perturbation generated by the one-single (slow-roll) field driving inflation



Quantum fluctuations on spatially flat hypersurfaces during inflation

$$\zeta = -\left(H\frac{\delta\rho}{\dot\rho}\right)_{k=aH} = -\left(H\frac{\delta\phi}{\dot\phi}\right)_{k=aH}$$

Curvature perturbation generated during inflation

$$\mathcal{P}_\zeta = \frac{1}{2}\left(\frac{H}{2\pi M_P \epsilon^{1/2}}\right)^2\left(\frac{k}{aH}\right)^{n_\zeta - 1},$$
$$n_\zeta = 1 + 2\eta - 6\epsilon$$

$$n_\zeta - 1 = \frac{d\ln\mathcal{P}_\zeta}{d\ln k} = \frac{d\ln H_k^4}{d\ln k} - \frac{d\ln\dot\phi_k^2}{d\ln k} = -4\epsilon + (2\eta - 2\epsilon) = 2\eta - 6\epsilon$$

Example: $$V(\phi) = \frac{1}{2} m^2 \phi^2$$

$$N = 8\pi G_N \int_{\phi_{\rm end}}^{\phi_N} d\phi \, \frac{V}{V'} \Rightarrow \phi_N \sim \sqrt{N} M_p$$

$$3H\dot{\phi} = -V' \Rightarrow \dot{\phi} \sim m M_p, \epsilon \sim 1/N$$

$$\zeta \sim \frac{H}{\sqrt{\epsilon} M_p} \sim \frac{m}{M_p} \sim 10^{-5} \Rightarrow m \sim 10^{12}\,\mathrm{GeV}$$

Tensor perturbations

$$ds^2 = dt^2 - a^2(\delta_{ij} + h_{ij})dx^i dx^j$$

$$\mathcal{L}_h = \frac{M_P^2}{2}\int d^4x\sqrt{-g}\frac{1}{2}\partial_\sigma h_{ij}\partial^\sigma h^{ij}$$

$$v_k = \frac{aM_P}{\sqrt{2}}h_k$$

Massless scalar field

$$v_k'' + \left(k^2 - \frac{a''}{a}\right)v_k = 0$$

$$\mathcal{P}_T(k) = \frac{8}{M_P^2}\left(\frac{H}{2\pi}\right)^2\left(\frac{k}{aH}\right)^{n_T}$$

$$n_T = -2\epsilon$$

Comments:

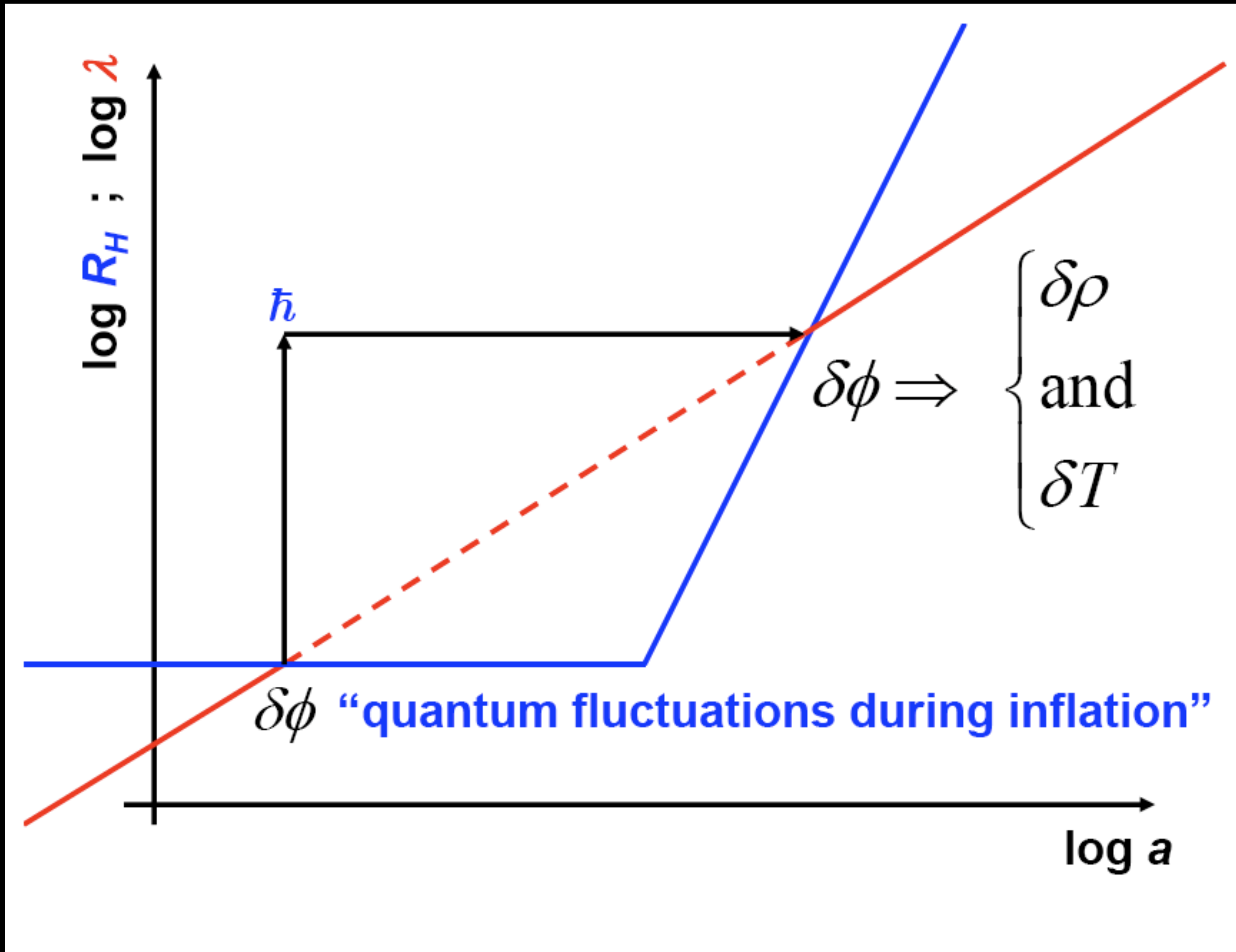
1) The amplitude of the tensor modes is proportional to the energy density of the inflaton field

2) For one-single field models of inflation there exists a CONSISTENCY RELATION

$$r \equiv \frac{\frac{1}{100}\mathcal{P}_T}{\frac{4}{25}\mathcal{P}_\zeta} = \epsilon = -\frac{n_T}{2}$$

The standard slow-roll scenario predicts:

- A (nearly) exact power law
- spectrum of (nearly) Gaussian
- super-Hubble radius
- scalar perturbations (seeds of structure) &
- tensor perturbations (gravitational waves)
- in their growing mode
- in a spatially flat universe



log R_H ; log λ
$\hbar$
$\delta\phi \Rightarrow \begin{cases} \delta\rho \\ \text{and} \\ \delta T \end{cases}$
$\delta\phi$ "quantum fluctuations during inflation"
log a

The comoving curvature perturbation is constant on superhorizon scales if the fluid is adiabatic

IT FOLLOWS FROM ENERGY CONSERVATION

$$\delta\left(\nabla_\mu T^{\mu\nu}\right)=0 \quad \Rightarrow \quad \delta\dot{\rho}+3H(\delta\rho+\delta P)-3\dot{\psi}(\rho+P)=0$$

$$\delta P \quad = \quad \delta P_{\rm nonad}+\frac{\dot{P}}{\dot{\rho}}\delta\rho$$

Go to a uniform energy density slice: $\delta\rho=0,\ \zeta=-\psi$

$$\dot{\zeta}=-\frac{H}{(\rho+P)}\delta P_{\rm nonad}$$

If the fluid is adiabatic, then $P=P(\rho)$ and $\delta P_{\rm nonad}=0$

Adiabatic vs isocurvature perturbations

Curvature (adiabatic) perturbations are there if:

$$\frac{\delta\rho_i}{\dot\rho_i} = \frac{\delta\rho_j}{\dot\rho_j} \text{ for every } i \text{ and } j$$

$$\frac{H\delta\rho_\gamma}{\dot\rho_\gamma} = \frac{H\delta\rho_m}{\dot\rho_m} = -\frac{\delta\rho_\gamma}{4\rho_\gamma} = -\frac{\delta\rho_m}{3\rho_m}$$

$$\frac{\delta\rho}{\dot\rho} = \frac{\delta P}{\dot P} \Rightarrow P = P(\rho)$$

Isocurvature perturbations are present if some of the following combination is nonvanishing:

$$S_{ij} = -3H\left(\frac{\delta\rho_i}{\dot\rho_i} - \frac{\delta\rho_j}{\dot\rho_j}\right) = 3(\zeta_i - \zeta_j)$$

Example: take two fluids

$$\zeta = \sum_i \frac{\dot{\rho}_i}{\dot{\rho}} \zeta_i$$

$$\dot{\zeta} = \left(\frac{\ddot{\rho}_2}{\dot{\rho}} - \frac{\dot{\rho}_2 \ddot{\rho}}{\dot{\rho}^2} \right) (\zeta_2 - \zeta_1)$$

The comoving curvature perturbation is not conserved on superhorizon scale if an isocurvature component is present

The curvature perturbation may come from fields different from the inflaton

- ***coupled fields during slow-roll during inflation***

  Starobinski & Yokoyama; Sasaki & Stewart; Mukhanov & Steinhardt; Linde, Garcia-Bellido & Wands.... (1995)

- ***curvaton decay after inflation***

  weakly-coupled, late-decaying scalar field

  Enqvist & Sloth; Lyth & Wands; Moroi & Takahashi (2001)

- ***inhomogeneous / modulated reheating or preheating***

  inflaton decay-rate modulated by another light field

  Dvali, Gruzinov & Zaldariaga; Kofman (2003); Kolb, A.R. & Vallinotto (2004)

- ***inhomogeneous end of inflation***

  Lyth, A.R. (2006)

Curvature perturbation from isocurvature fields during inflation (curvaton)

- Take a scalar field $\sigma(\mathbf{x}, t)$ other than the inflaton field; it does not dominate the energy density during inflation
- Its potential is $V(\sigma) = \frac{1}{2}m^2\sigma^2$
- During inflation it is quantum mechanically excited: $\delta\rho_\sigma \sim m^2\bar{\sigma}\delta\sigma$ and $\frac{\delta\rho_\sigma}{\rho_\sigma} \sim \frac{\delta\sigma}{\bar{\sigma}}$
- When it decays into radiation, its fluctuations are transferred to radiation

$$\zeta \sim \frac{\delta\sigma}{\bar{\sigma}} \sim \frac{H}{\bar{\sigma}}$$

Inflation provides
the initial seeds
for the cosmological perturbations
we see in the Universe

Inflation provides the initial conditions for the gravitational potential

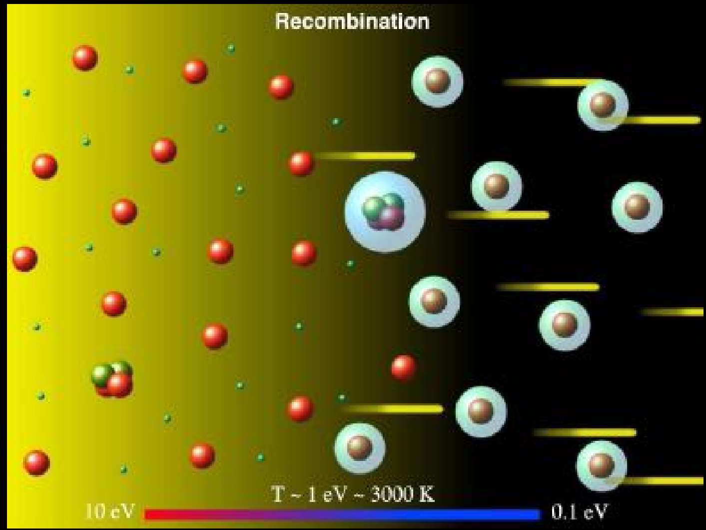
Einstein equations indicate that, on superhorizon scales,

$$\psi = \Phi,\ \ \delta\rho = -2\Phi$$

$$\begin{aligned} \zeta &= -\psi + \frac{\delta\rho}{3(\rho+P)} = -\psi + \frac{\delta\rho}{3(1+w)\rho} = -\frac{5+3w}{3(1+w)}\Phi \\ &= \begin{cases} -\frac{3}{2}\Phi\ (\mathrm{RD}) \\ -\frac{5}{3}\Phi\ (\mathrm{MD}) \end{cases} \end{aligned}$$

The gravitational potential inherits the flat spectrum generated during inflation

Hydrogen Recombination & Last Scattering Surface

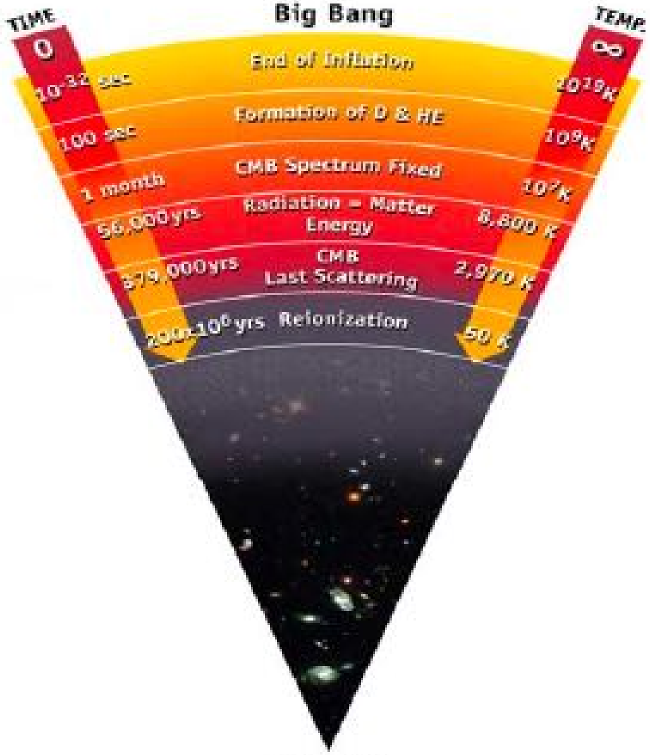


Matter is ionized at temperatures higher than the hydrogen ionization energy of 13.6 eV

$$\frac{n_e n_p}{n_H} = \left(\frac{m_e T}{2\pi}\right)^{3/2} e^{-E_{\rm ion}/T}$$

The Universe becomes transparent to photons when

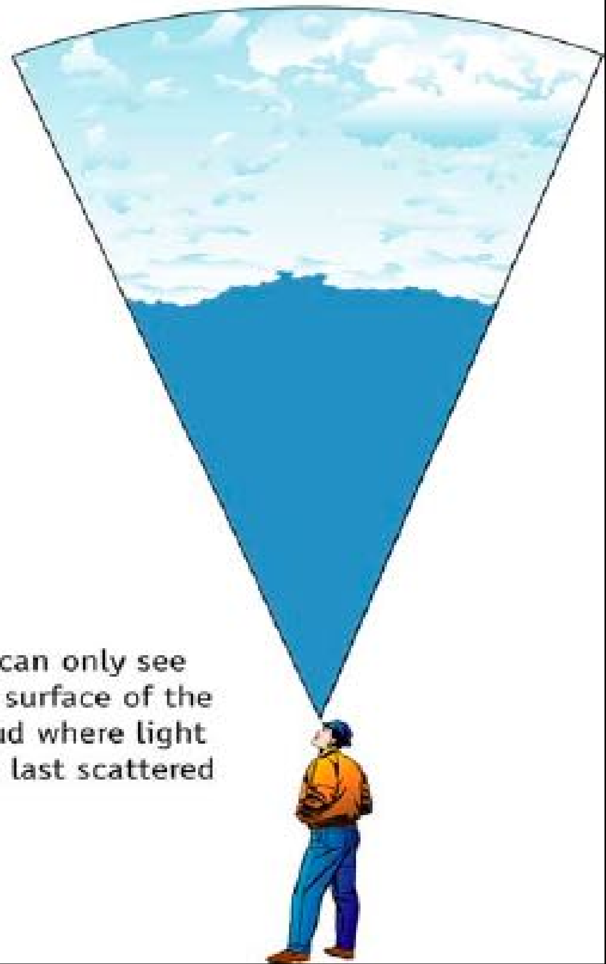
$$(\sigma_{e\gamma} n_e)^{-1} \sim t,\ \sigma_{e\gamma} = 8\pi\alpha^2/3m_e^2,\ T_{\rm LS} \simeq 0.26\ \mathrm{eV}$$



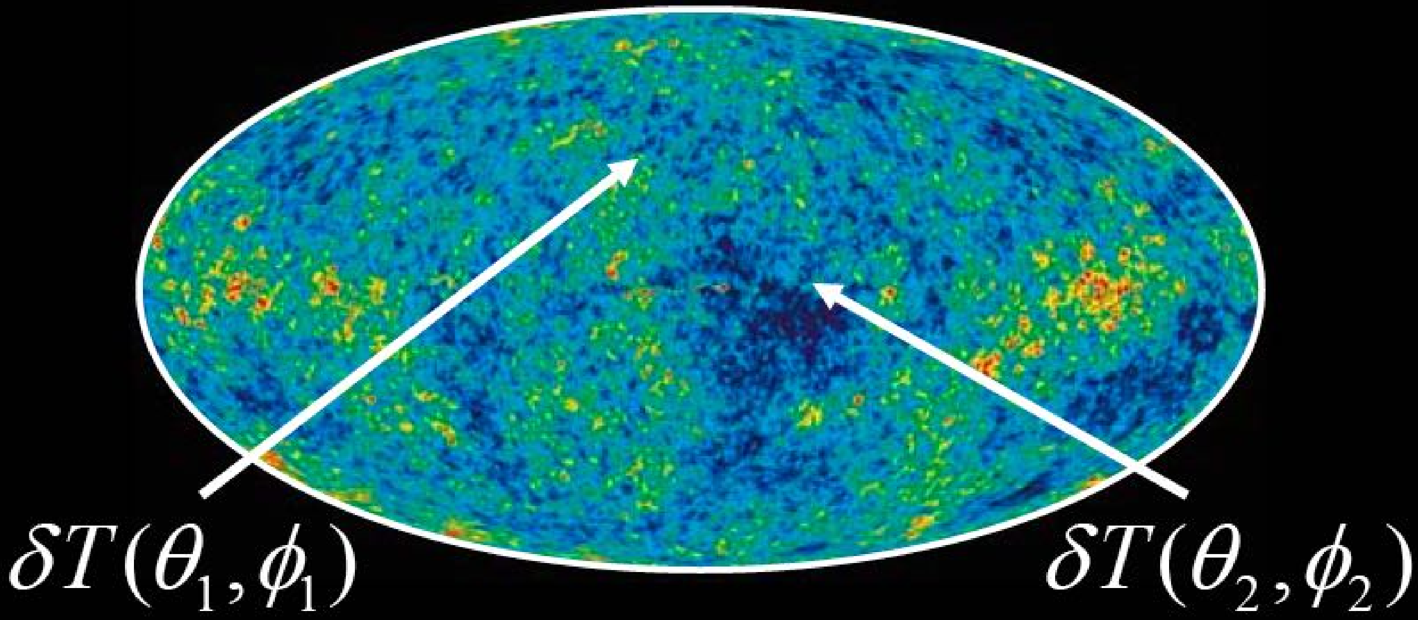
PRESENT
13.7 Billion Years after the Big Bang

The cosmic microwave background Radiation's "surface of last scatter" is analogous to the light coming through the clouds to our eye on a cloudy day.

We can only see the surface of the cloud where light was last scattered



CMB anisotropy

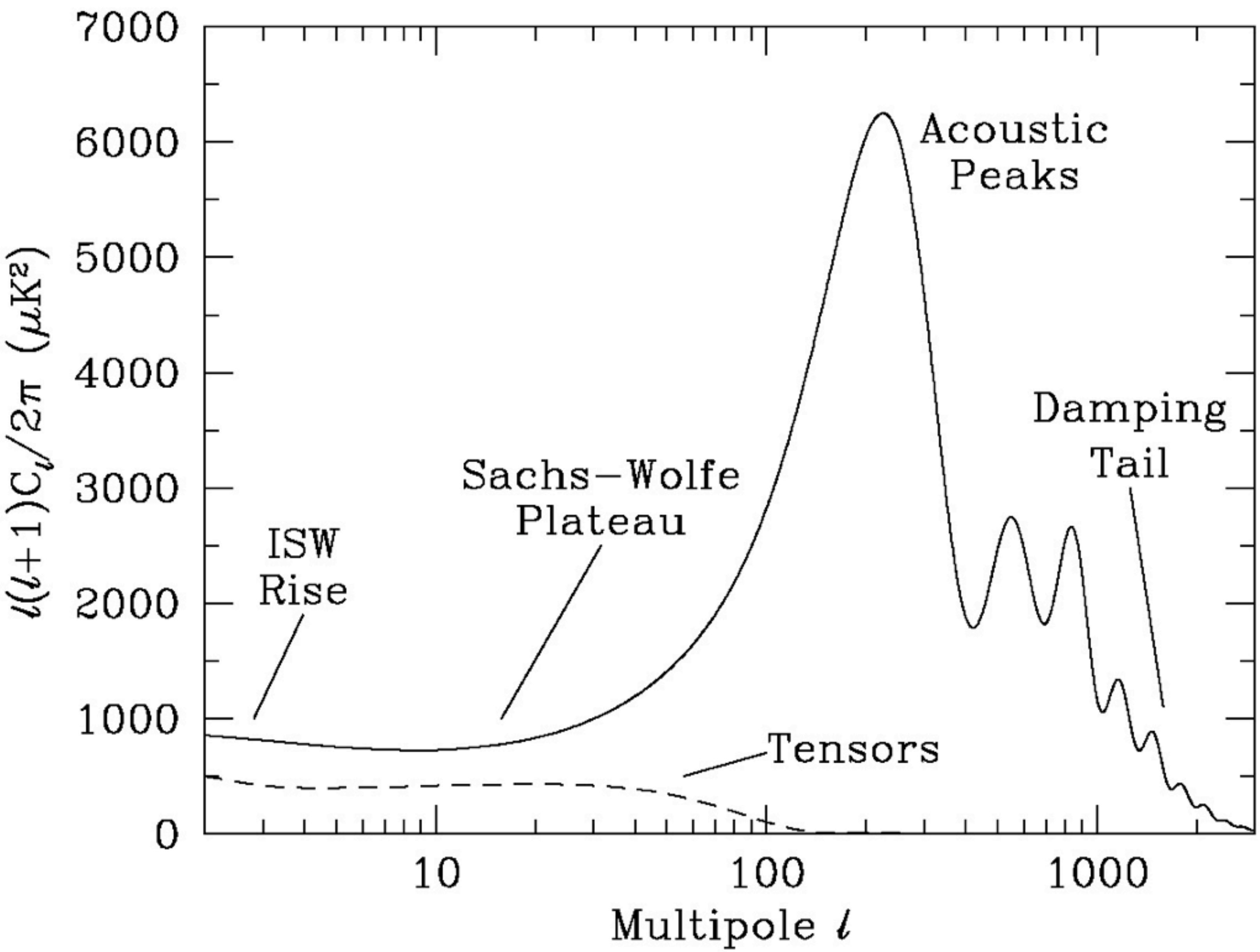


$$\frac{\Delta T}{T}(x_0,\tau_0,\mathbf{n}) = \sum_{\ell m} a_{\ell m}(x_0) Y_{\ell m}(\mathbf{n})$$

$$\langle a_{\ell m} a_{\ell' m'}\rangle = \delta_{\ell\ell'}\delta_{mm'}C_\ell$$

$$\langle \frac{\Delta T}{T}(\mathbf{n})\frac{\Delta T}{T}(\mathbf{n}')\rangle = \sum_{\ell} \frac{(2\ell+1)}{4\pi} C_\ell P_\ell(\mathbf{n}\cdot\mathbf{n}')$$

(ensemble averages)



7000
6000
5000
4000
3000
2000
1000
0
$l(l+1)C_l/2\pi$ (μK^2)
10
100
1000
Multipole l
Acoustic Peaks
Damping Tail
Sachs–Wolfe Plateau
ISW Rise
Tensors

The total CMB anisotropy

$$\Delta(\mathbf{k}, \mathbf{n}, \eta) = (\Delta_0 + 4\Phi + 4\mathbf{v} \cdot \mathbf{n}) + 4 \int_0^{\eta_0} (\Phi + \psi)'$$

Sachs-Wolfe effect

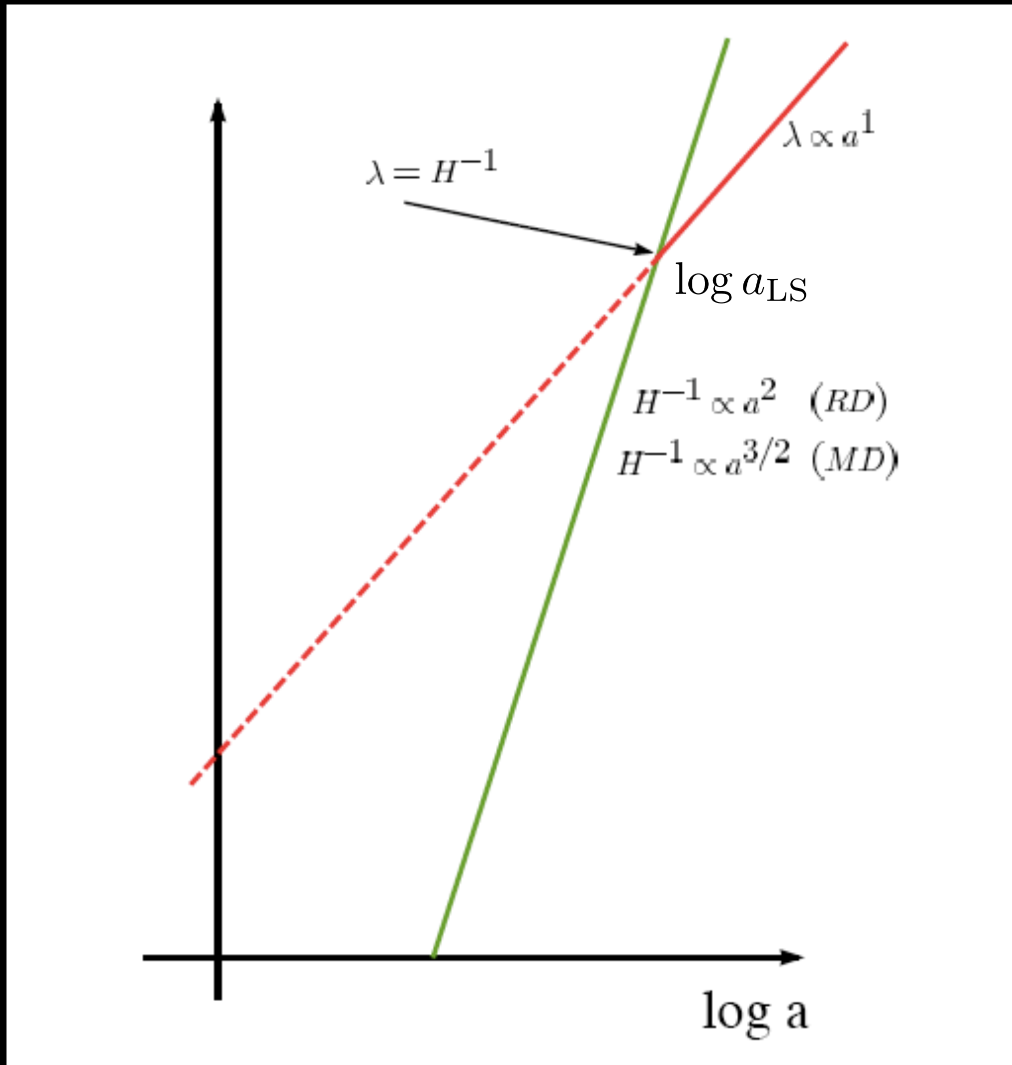
Doppler effect

Integrated Sachs-Wolfe effect

$$\Delta = \frac{1}{4} \frac{\delta\rho_\gamma}{\rho_\gamma}$$

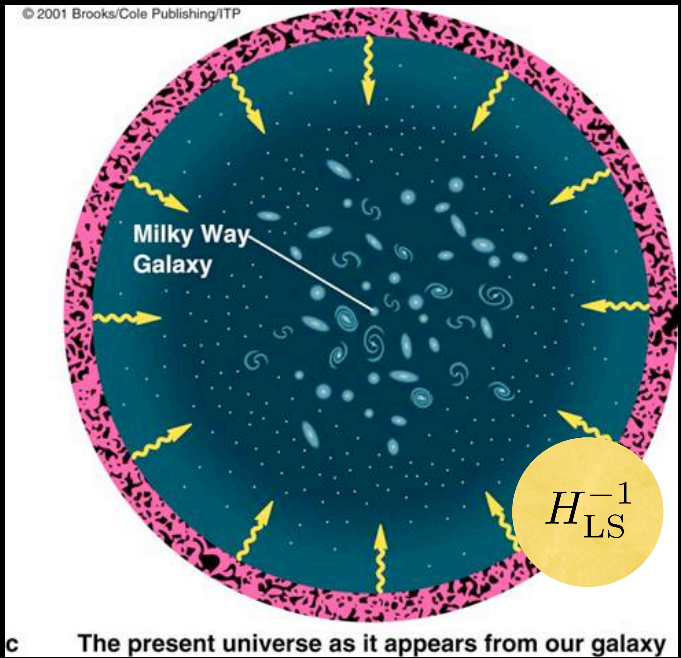
Φ and ψ are gravitational potentials

CMB anisotropy
at
scales larger than
the horizon
at last scattering



$\lambda \propto a^1$
$\lambda = H^{-1}$
$\log a_{\rm LS}$
$H^{-1} \propto a^2 \quad (RD)$
$H^{-1} \propto a^{3/2} \quad (MD)$
log a

Horizon at Last Scattering



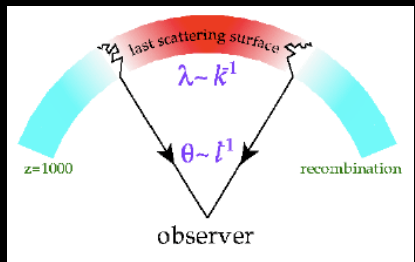
Comoving distance between us and the last scattering surface

$$d\tau = dt/a$$

$$\int_{t_{\rm LS}}^{t_0} \frac{dt}{a} = \int_{\tau_{\rm LS}}^{\tau_0} d\tau = (\tau_0 - \tau_{\rm LS})$$

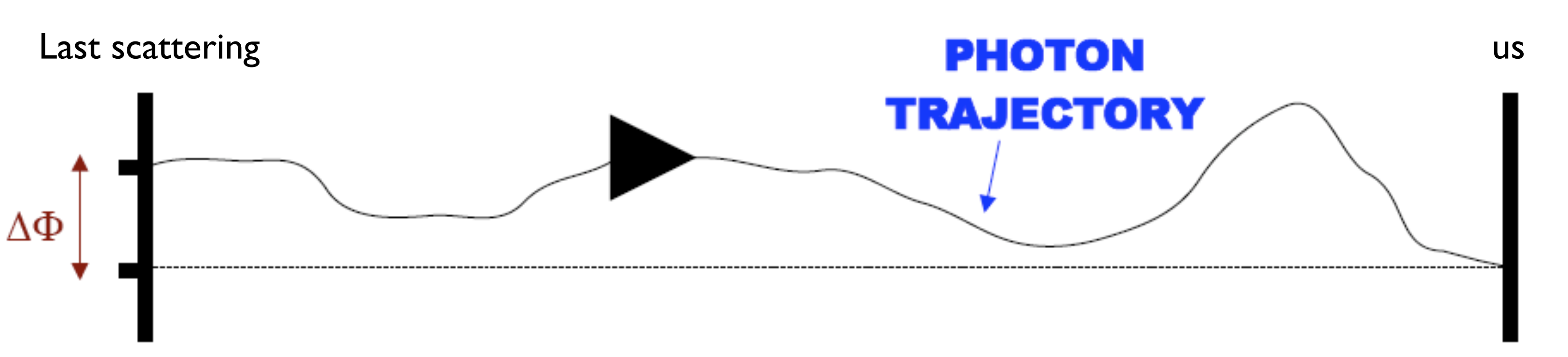
Angle subtended by a given comoving length scale

$$\theta \simeq \frac{\lambda}{(\tau_0 - \tau_{\rm LS})}$$



Sound Horizon

$$\theta_{\rm HOR} \simeq c_s \frac{\tau_{\rm LS}}{(\tau_0 - \tau_{\rm LS})} \simeq c_s \frac{\tau_{\rm LS}}{\tau_0} \simeq c_s \left(\frac{T_0}{T_{\rm LS}}\right)^{1/2} \simeq 1^{\rm o}$$



Last scattering
PHOTON
TRAJECTORY
us
$\Delta\Phi$

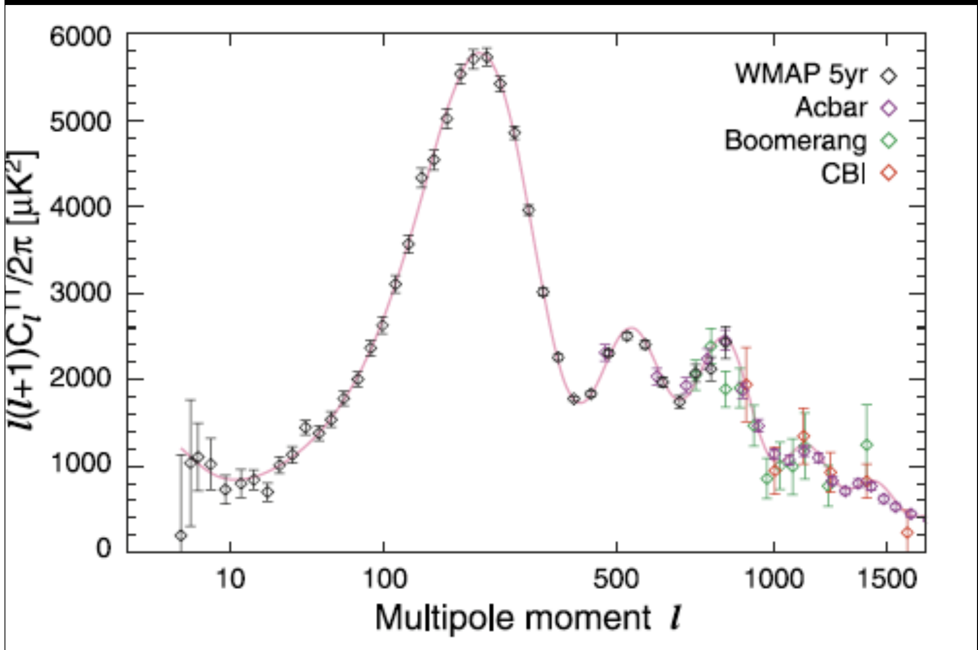
Sachs-Wolfe Plateau

For modes beyond the horizon at last scattering and adiabatic conditions:

$$
\begin{aligned}
\frac{\delta T(\mathbf{n})}{T} &= \frac{\Delta(\mathbf{n})}{4} = \left(\frac{\Delta}{4} + \Phi\right)(\eta_{\rm LS}) \\
&= \left(\frac{1}{4}\frac{\delta\rho_\gamma}{\rho_\gamma} + \Phi\right)(\eta_{\rm LS}) = \left(\frac{1}{3}\frac{\delta\rho_m}{\rho_m} + \Phi\right)(\eta_{\rm LS}) \\
&= \left(-\frac{2}{3}\Phi + \Phi\right)(\eta_{\rm LS}) = \frac{1}{3}\Phi(\eta_{\rm LS}) \\
&= -\frac{1}{5}\zeta_{\rm inf}
\end{aligned}
$$

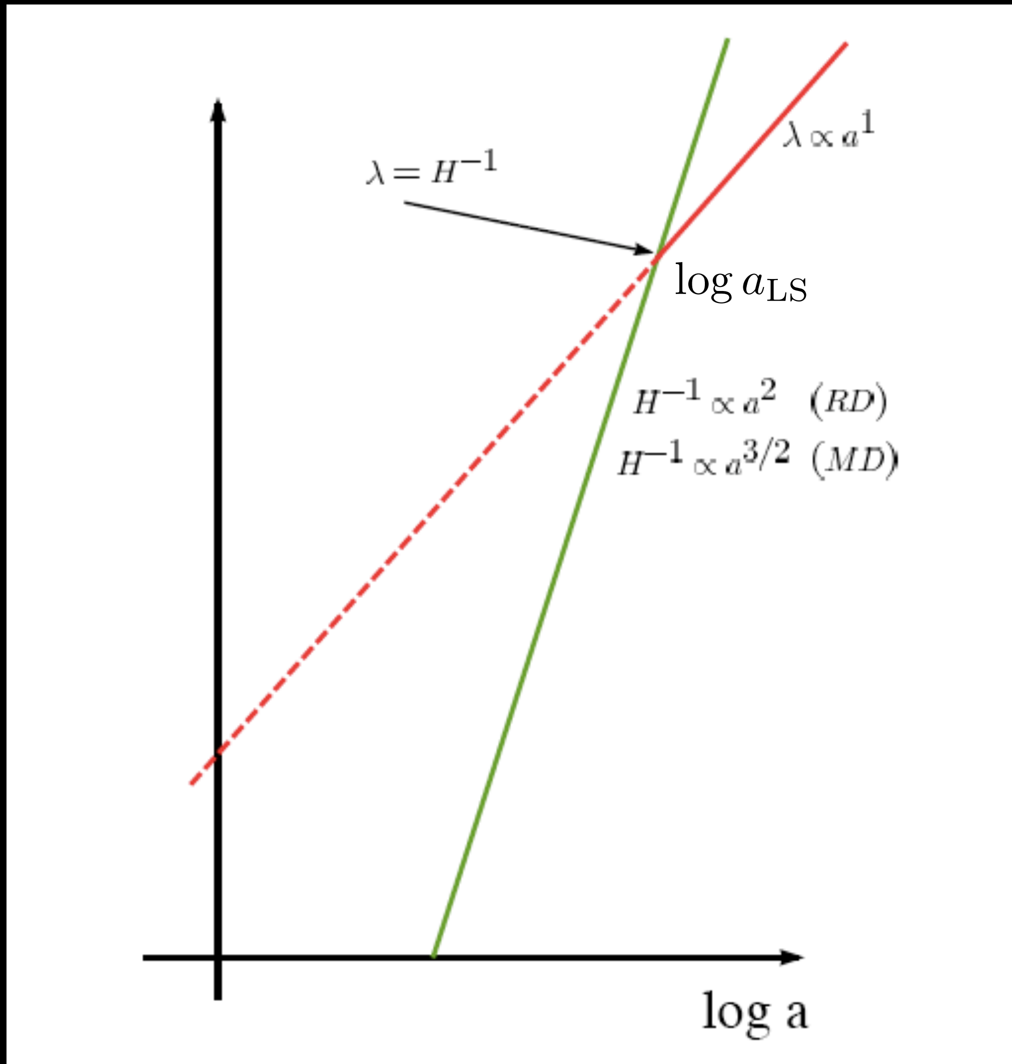
$$C_\ell = \frac{2}{\pi}\int \frac{dk}{k}\langle\frac{1}{25}\,|\zeta_k|^2\rangle k^3 \mathrm{j}_\ell^2(k(\eta_0-\eta_{\rm LS}))$$

$$\pi\ell(\ell+1)C_\ell = \frac{1}{50}\frac{1}{M_P^2}\left(\frac{H}{2\pi\epsilon^{1/2}}\right)^2$$



$$\left(\frac{V}{\epsilon}\right)^{1/4} \simeq 6.7\times10^{16}\,\mathrm{GeV}$$

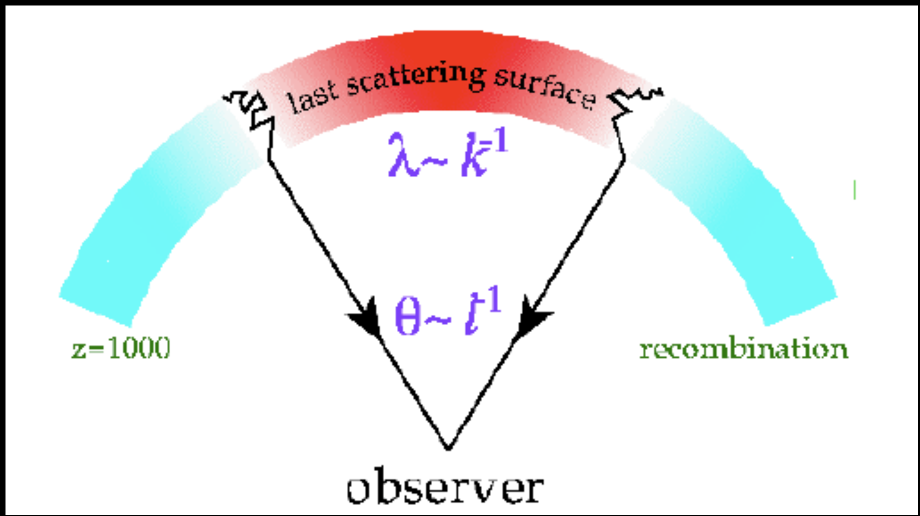
CMB anisotropy
at
scales smaller than
the horizon
at last scattering



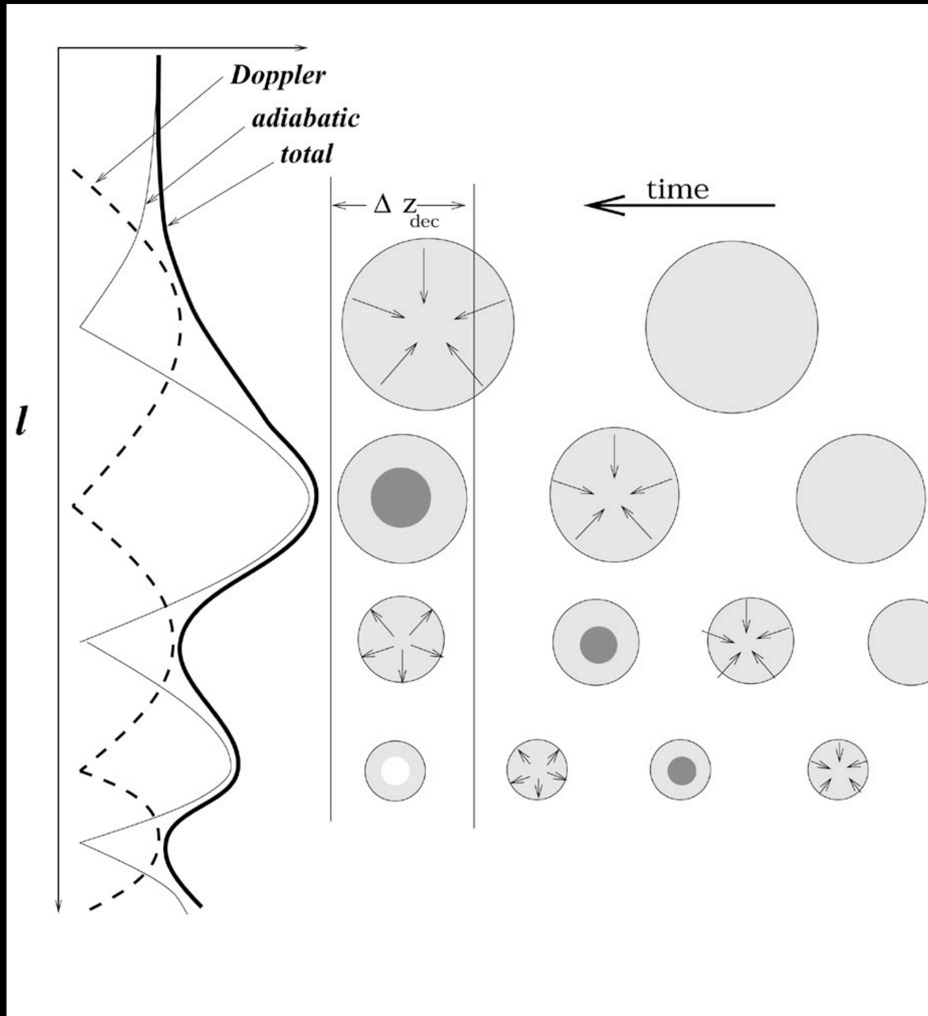
$\lambda = H^{-1}$
$\lambda \propto a^1$
$\log a_{\rm LS}$
$H^{-1} \propto a^2$ (RD)
$H^{-1} \propto a^{3/2}$ (MD)
log a

Acoustic peaks

- At recombination, baryon-photon fluid undergoes "acoustic oscillations" $A \cos c_s k\eta + B \sin c_s k\eta$
- Compressions and rarefactions change the temperature T_γ
- Peaks in ΔT_γ corresponds to extrema of compressions and rarefactions



Acoustic peaks



Dynamics of the photon-baryon fluid (electrons are kept in equilibrium through the Coulomb scatterings with protons)

The photon distribution satisfies the Boltzmann equation

$$\frac{df}{d\eta} = C[f](\text{Thomson scatterings})$$

$$f(x^i, p, n^i, \eta) = 2\left[\exp\left\{\frac{p}{T(\eta)(1+\Theta(x^i, n^i\eta)}\right\} - 1\right]^{-1}$$

$$\frac{\partial \Delta}{\partial \eta} + n^i \frac{\partial \Delta}{\partial x^i} + 4n^i \frac{\partial \Phi}{\partial x^i} - 4\frac{\partial \psi}{\partial \eta} = -\tau' \left[\Delta_0 + \frac{1}{2}\Delta_2 P_2(\hat{\mathbf{v}}\cdot\mathbf{n}) - \Delta + 4\mathbf{v}\cdot\mathbf{n}\right]$$

$$\Delta = 4\Theta \qquad \Delta_\ell = \frac{1}{(-i)^\ell}\int_{-1}^{1}\frac{d\mu}{2}P_\ell(\mu)\Delta(\mu), \;\; \mu = \hat{\mathbf{v}}\cdot\mathbf{n}$$

By integrating over the solid angle, we get:

Energy continuity equation

$$\Delta_0' + \frac{4}{3}\partial_i v_\gamma^i - 4\psi' = 0, \;\; \frac{4}{3}v_\gamma^i = \int \frac{d\Omega}{4\pi}\Delta n^i$$

Velocity continuity equation

$$\begin{aligned} v_\gamma'^{\,i} + \frac{3}{4}\partial_j \Pi_\gamma^{ij} + \frac{1}{4}\Delta_0 + \partial^i\Phi &= -\tau'\left(v^i - v_\gamma^i\right) \\ \Pi_\gamma^{ij} &= \int \frac{d\Omega}{4\pi}\left(n^i n^j - \frac{1}{3}\delta^{ij}\right)\Delta \end{aligned}$$

Momentum continuity equation for baryons

$$v^i = v_\gamma^i + \frac{R}{\tau'}\left(v'^{\,i} + \mathcal{H}v^i + \partial^i\Phi\right), \;\; R = \frac{3}{4}\frac{\rho_b}{\rho_\gamma}$$

Acustic Oscillations of the photon-baryon fluid (beneath the horizon)

$$v_\gamma^{i'} + \mathcal{H}\frac{R}{1+R}v_\gamma^i + \frac{1}{4}\frac{\partial^i \Delta_0}{1+R} + \partial^i \Phi = 0$$

$$\left(\Delta_0^{''} - 4\psi^{''}\right) + \frac{\mathcal{H}R}{1+R}\left(\Delta_0' - 4\psi'\right) - c_s^2 \nabla^2 \left(\Delta_0 - 4\psi\right) = \frac{4}{3}\nabla^2\left(\Phi + \frac{\psi}{1+R}\right)$$

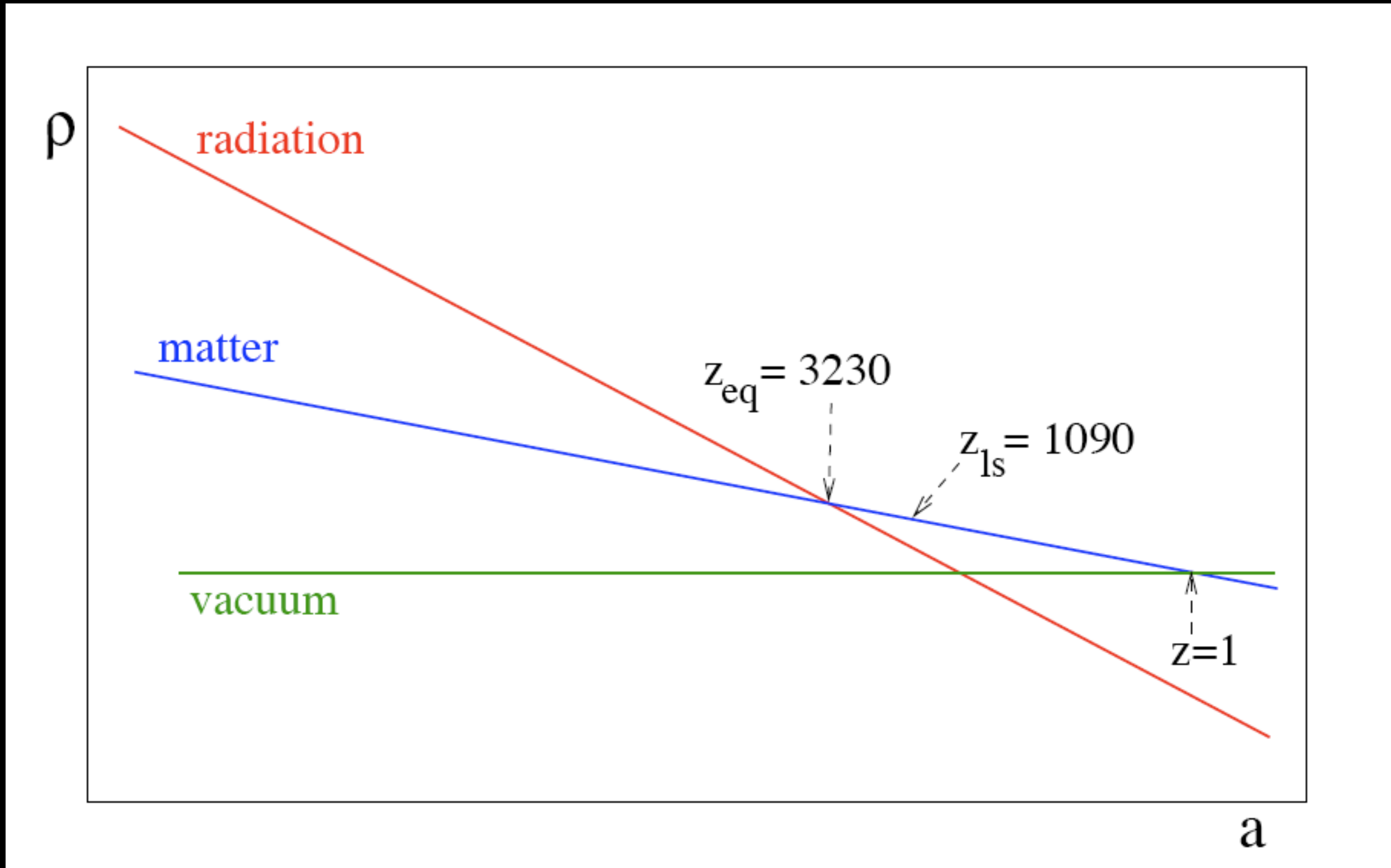
redshfit Hubble drag pressure infall

$$c_s = 1/\sqrt{3(1+R)}$$

Pattern of oscillations:

$$\begin{aligned} [1+R(\eta)]^{1/4}\left(\Delta_0 - 4\psi\right) &= A\cos[kr_s(\eta)] + B\sin[kr_s(\eta)] \\ &- \frac{4k}{\sqrt{3}}\int_0^\eta d\eta' \left[1+R(\eta')\right]^{3/4}\left[\Phi(\eta') + \frac{\psi(\eta')}{1+R}\right]\sin\left[k(r_s(\eta) - r_s(\eta')\right] \\ r_s(\eta) &= \int_0^\eta d\eta' c_s(\eta') \end{aligned}$$

To study the solutions we have to see if the modes enter the horizon before or after matter-radiation equality



First, fix the initial (adiabatic) conditions

$$\begin{aligned}\Phi &= -\frac{1}{2}\Delta_0 \; [(00) - \text{Einstein equation}] \\ \Delta_0 &- 4\psi = \text{constant} \; [\text{continuity equation}]\end{aligned}$$

$$\begin{aligned}(\Delta - 4\psi) &= -6\Phi\cos(\omega_0\eta) - 8\frac{k}{\sqrt{3}}\int_0^\eta d\eta'\,\Phi(\eta')\sin[\omega_0(\eta - \eta')], \\ \omega_0 &= kc_s,\; c_s = 1/\sqrt{3(1+R_*)},\, \psi = \Phi\end{aligned}$$

Time behaviour of the gravitational perturbation

$$3\mathcal{H}\left(\mathcal{H}\Phi + \dot{\Phi}\right) + \nabla^2\Phi = -4\pi G_N a^2 \delta\rho$$

$$\ddot{\Phi} + 3\mathcal{H}\dot{\Phi} + \left(2\dot{\mathcal{H}} + \mathcal{H}^2\right)\Phi = 4\pi G_N \delta P$$

Using $\delta P = c_s^2 \delta\rho$

$$\ddot{\Phi} + 3\mathcal{H}(1 + c_s^2)\dot{\Phi} + (2\dot{\mathcal{H}} + \mathcal{H}^2 + 3c_s^2\mathcal{H}^2)\Phi + c_s^2\partial^i\partial_i\Phi = 0$$

$$\Phi_m = \text{constant for all scales} = -\frac{5}{3}\zeta = \frac{9}{10}\Phi_\gamma(0)$$

$$\Phi_\gamma = 3\Phi_\gamma(0)\frac{\sin(k\eta/\sqrt{3}) - (k\eta/\sqrt{3})\cos(k\eta/\sqrt{3})}{(k\eta/\sqrt{3})^3}$$

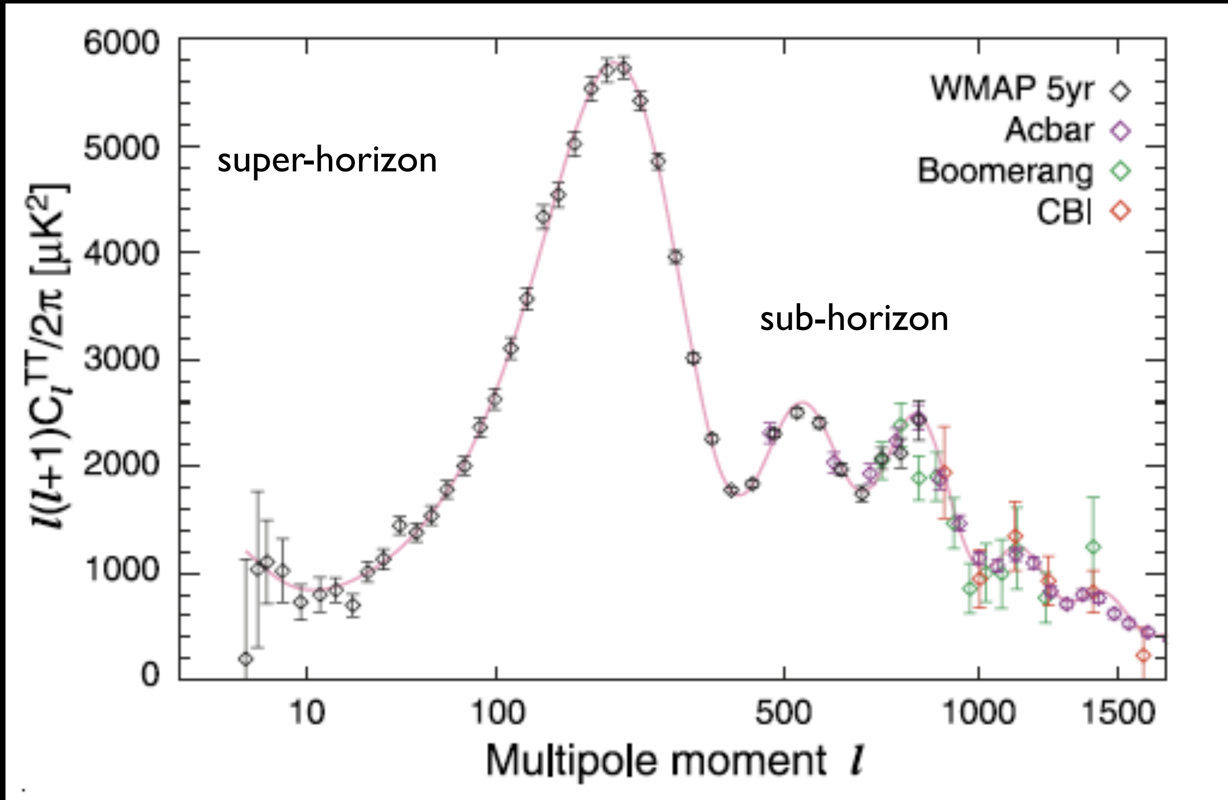
For modes which enter the horizon when the Universe is MD

$$\Delta_0 - 4\psi = \frac{6}{5}\Phi_\gamma(0)\cos(\omega_0\eta) - \frac{36}{5}\Phi_\gamma(0)$$

For modes which enter the horizon when the Universe is RD

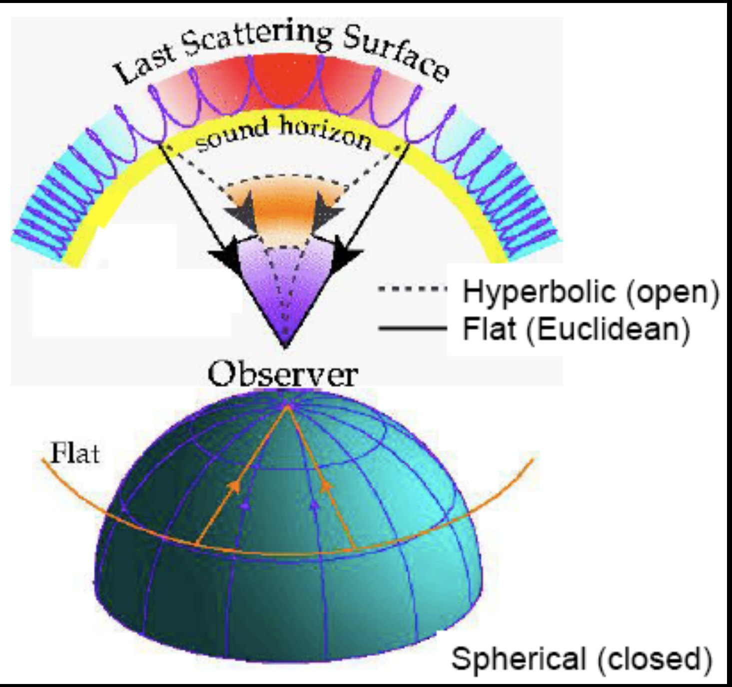
$$\Delta_0 - 4\psi = 6\Phi_\gamma(0)\cos(\omega_0\eta)$$

$$\omega_0 = c_s\,k$$

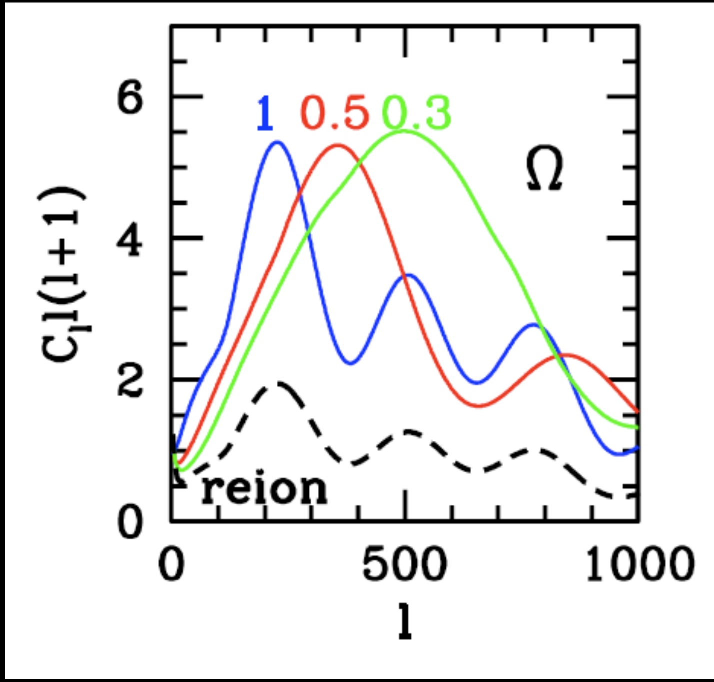

6000
5000
4000
3000
2000
1000
0
10
100
500
1000
1500
Multipole moment l
l(l+1)C_l^TT/2π [μK²]
WMAP 5yr
Acbar
Boomerang
CBI
super-horizon
sub-horizon

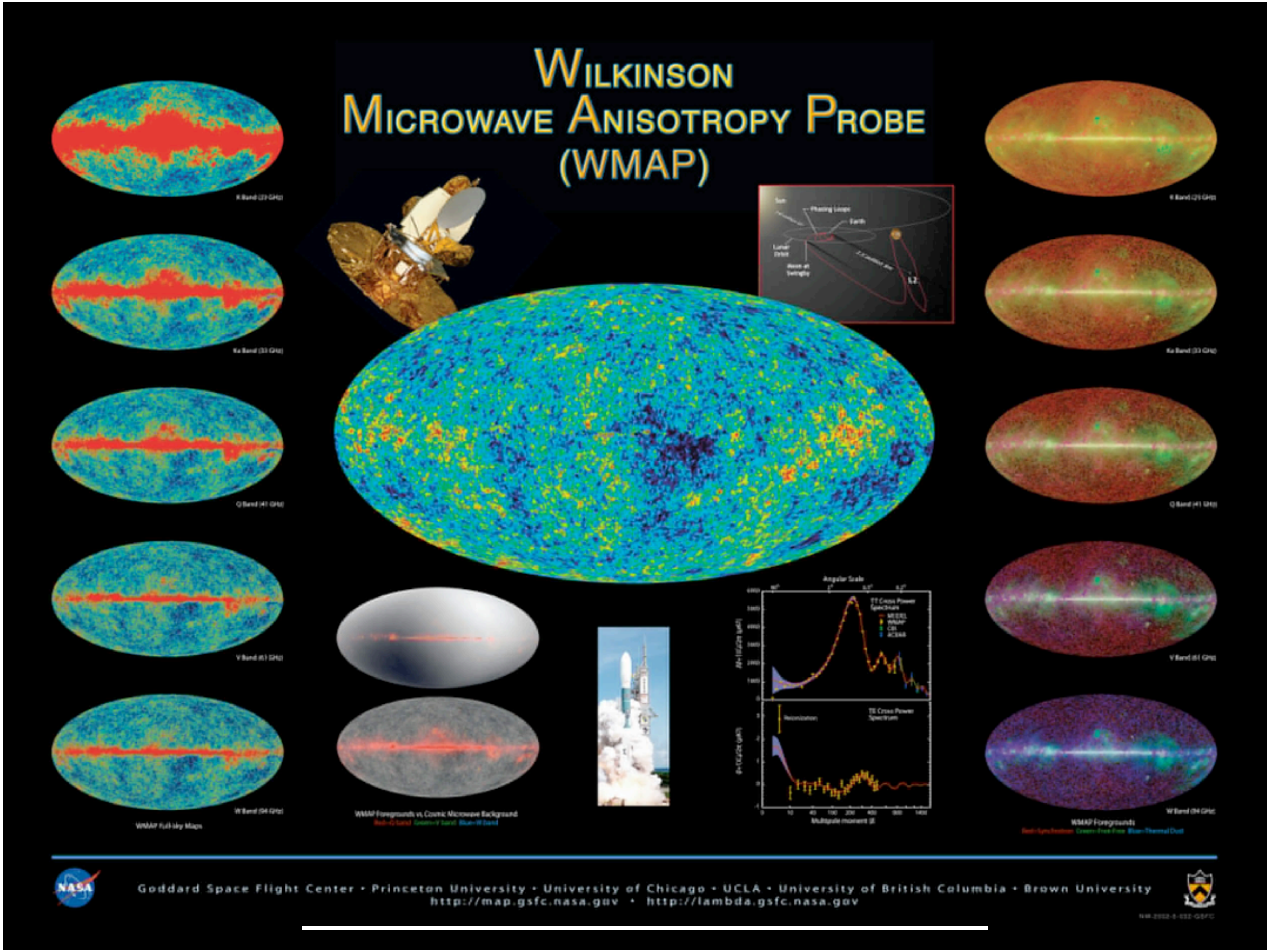
Position of the first peak

Modes caught in the extrema of their oscillation at recombination will have enhanced fluctuations, yielding a fundemental scale or frequency related to the Universe sound horizon



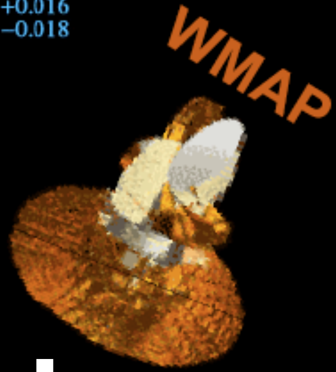
$$\ell_{\text{first peak}} \simeq \frac{220}{\sqrt{\Omega_{\text{tot}}}}$$





Wilkinson
Microwave Anisotropy Probe
(WMAP)
W Band (94 GHz)
WMAP Full-sky Maps
WMAP Foregrounds vs Cosmic Microwave Background
Angular Scale
TT Cross Power Spectrum
MODEL
WMAP
CBI
ACBAR
Reionization
TE Cross Power Spectrum
Multipole moment (l)
WMAP Foregrounds
Goddard Space Flight Center • Princeton University • University of Chicago • UCLA • University of British Columbia • Brown University
http://map.gsfc.nasa.gov • http://lambda.gsfc.nasa.gov

Precision Cosmology



$\Omega_{tot} = 1.02^{+0.02}_{-0.02}$

$w < -0.78$ (95% CL)

$\Omega_\Lambda = 0.73^{+0.04}_{-0.04}$

$\Omega_b h^2 = 0.0224^{+0.0009}_{-0.0009}$

$\Omega_b = 0.044^{+0.004}_{-0.004}$

$n_b = 2.5 \times 10^{-7}{}^{+0.1\times10^{-7}}_{-0.1\times10^{-7}}$ cm^{-3}

$\Omega_m h^2 = 0.135^{+0.008}_{-0.009}$

$\Omega_m = 0.27^{+0.04}_{-0.04}$

$\Omega_\nu h^2 < 0.0076$ (95% CL)

$m_\nu < 0.23$ eV (95% CL)

$T_{cmb} = 2.725^{+0.002}_{-0.002}$ K

$n_\gamma = 410.4^{+0.9}_{-0.9}$ cm^{-3}

$\eta = 6.1 \times 10^{-10}{}^{+0.3\times10^{-10}}_{-0.2\times10^{-10}}$

$\Omega_b \Omega_m^{-1} = 0.17^{+0.01}_{-0.01}$

$\sigma_8 = 0.84^{+0.04}_{-0.04}$ Mpc

$\sigma_8 \Omega_m^{0.5} = 0.44^{+0.04}_{-0.05}$

$A = 0.833^{+0.086}_{-0.083}$

$n_s = 0.93^{+0.03}_{-0.03}$

$dn_s/d\ln k = -0.031^{+0.016}_{-0.018}$

$r < 0.71$ (95% CL)

$z_{dec} = 1089^{+1}_{-1}$

$\Delta z_{dec} = 195^{+2}_{-2}$

$h = 0.71^{+0.04}_{-0.03}$

$t_0 = 13.7^{+0.2}_{-0.2}$ Gyr

$t_{dec} = 379^{+8}_{-7}$ kyr

$t_r = 180^{+220}_{-80}$ Myr (95% CL)

$\Delta t_{dec} = 118^{+3}_{-2}$ kyr

$z_{eq} = 3233^{+194}_{-210}$

$\tau = 0.17^{+0.04}_{-0.04}$

$z_r = 20^{+10}_{-9}$ (95% CL)

$\theta_A = 0.598^{\circ}{}^{+0.002}_{-0.002}$

$d_A = 14.0^{+0.2}_{-0.3}$ Gpc

$l_A = 301^{+1}_{-1}$

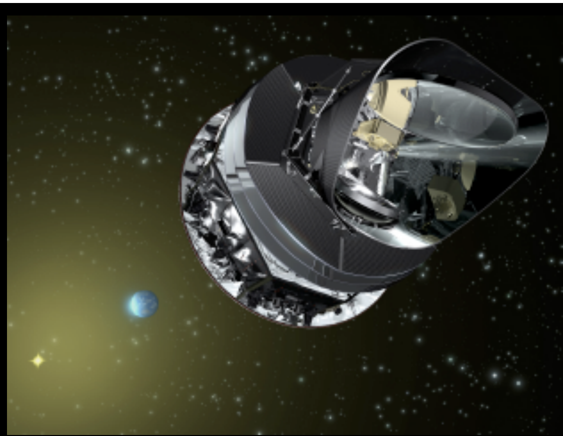
$r_s = 147^{+2}_{-2}$ Mpc

INFLATION

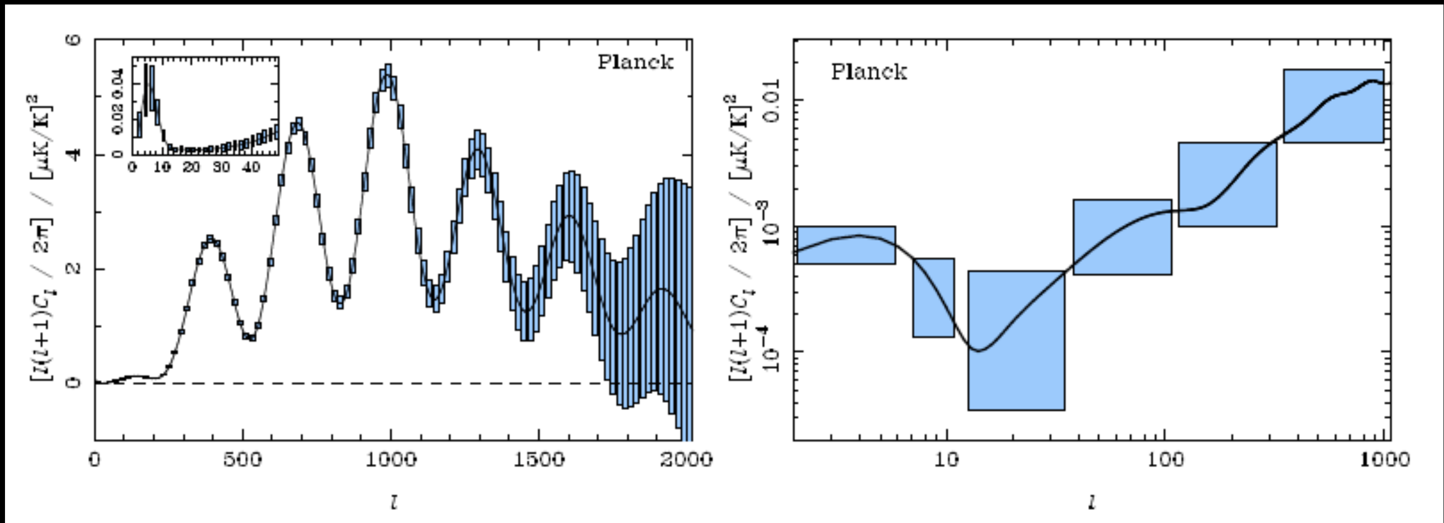
The Future

- CMB polarization
- Non-Gaussianity

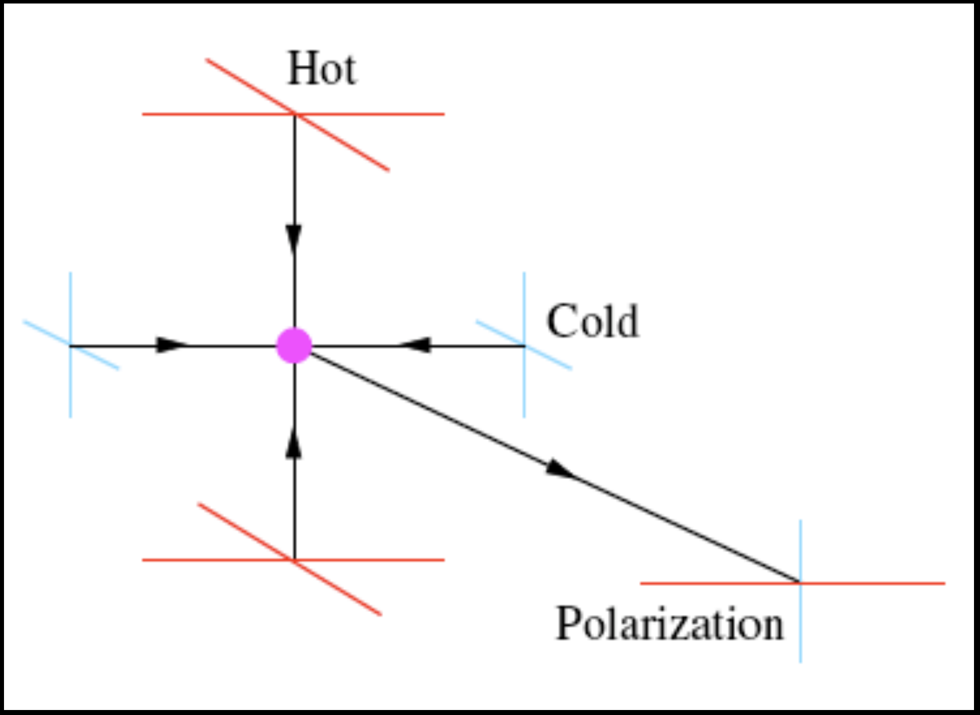
Planck



- Lunch in April 29, 2009
- Fully sky imaging from L2 in nine frequency bands (30-587 GHz)
- Polarization may be sensitive to $r \sim 0.1$



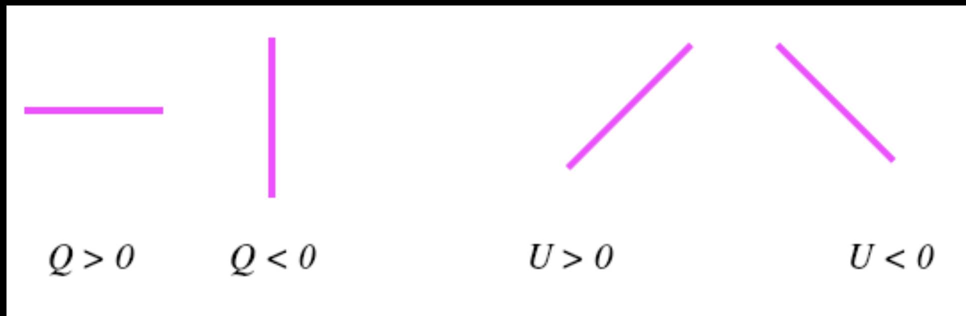
CMB anisotropy is polarized



CMB Polarization

For a plane wave along the z-direction, symmetric trace-free (STF) correlation tensor of electric field defines (transverse) linear polarization tensor:

$$\mathcal{P}_a \equiv \begin{pmatrix} \frac{1}{2}\langle E_x^2 - E_y^2 \rangle & \langle E_x E_y \rangle \\ \langle E_x E_y \rangle & -\frac{1}{2}\langle E_x^2 - E_y^2 \rangle \end{pmatrix} = \frac{1}{2}\begin{pmatrix} Q & U \\ U & -Q \end{pmatrix}$$



Under a rotation in the (x-y)-plane

$$Q \pm iU \rightarrow (Q \pm iU)e^{-\mp\alpha} \Rightarrow (Q + iU) \text{ is spin 2}$$

E- and B-modes

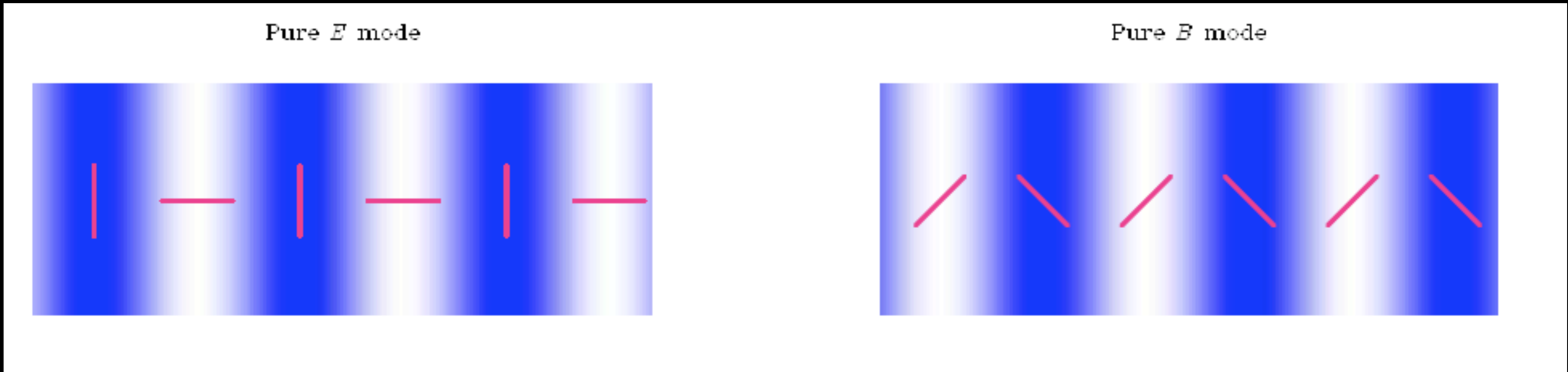
$$\mathcal{P}_{ab}(\mathbf{n}) = \nabla_{\langle a}\nabla_{b\rangle}P_E + \epsilon^{c}_{\ (a}\nabla_{b)}\nabla_c P_B$$

$$Q + iU = \bar{\eth}\bar{\eth}(P_E - P_B)$$

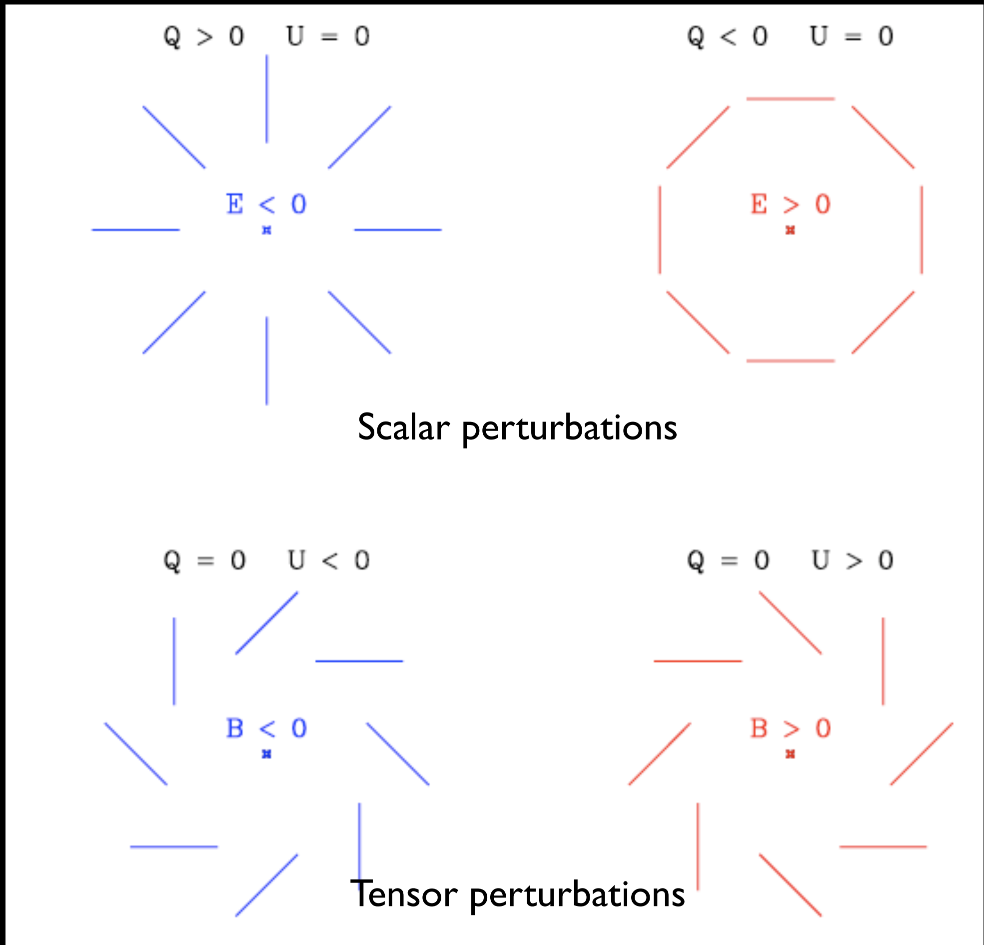
$$\bar{\eth}_s\eta = -\sin^{-s}\theta(\partial_\theta - i\,\mathrm{cosec}\,\partial_\phi)(\sin^s\theta\,\eta)$$

Expand in spin-weight harmonics

$$P_{E(B)} = \sum_{\ell m}\sqrt{\frac{(\ell-2)!}{(\ell+2)!}}E_{\ell m}(B_{\ell m})Y_{\ell m}(\mathbf{n}) \Rightarrow (Q \pm iU) = \sum_{\ell m}(E_{\ell m} \mp B_{\ell m})_{\mp 2}Y_{\ell m}(\mathbf{n})$$



If parity is respected, only three correlations: $C_\ell^E,\ C_\ell^B,\ C_\ell^{TE}$

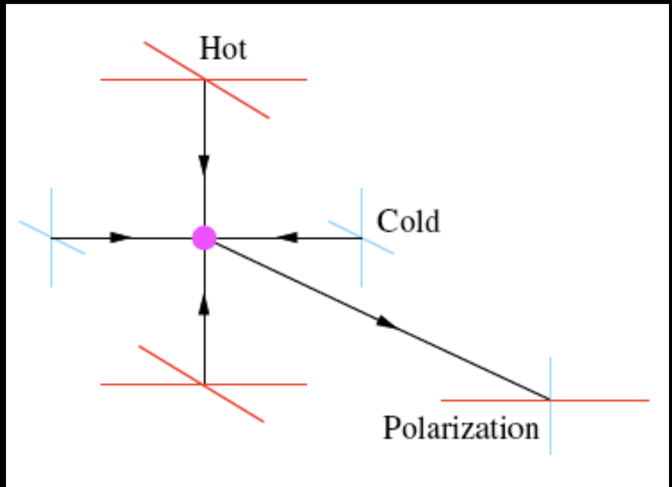

Q > 0 U = 0
E < 0
Q < 0 U = 0
E > 0
Scalar perturbations
Q = 0 U < 0
B < 0
Q = 0 U > 0
B > 0
Tensor perturbations

CMB Polarization from scalar perturbations

Thomson scattering of radiation quadrupole produces linear polarization, which is conserved by free-streeming, but suppressed during reionization

Due to Doppler effect, eletron scatterers see the photon-baryon fluid temperature anisotropy carrying a nonvanishing quadrupole

$$\delta T(x_0, \mathbf{n}) = \mathbf{n} \cdot [\mathbf{v}(x) - \mathbf{v}(x_0)] \simeq \lambda_T \mathbf{n}_i \mathbf{n}_j \partial_i v_j(x_0)$$

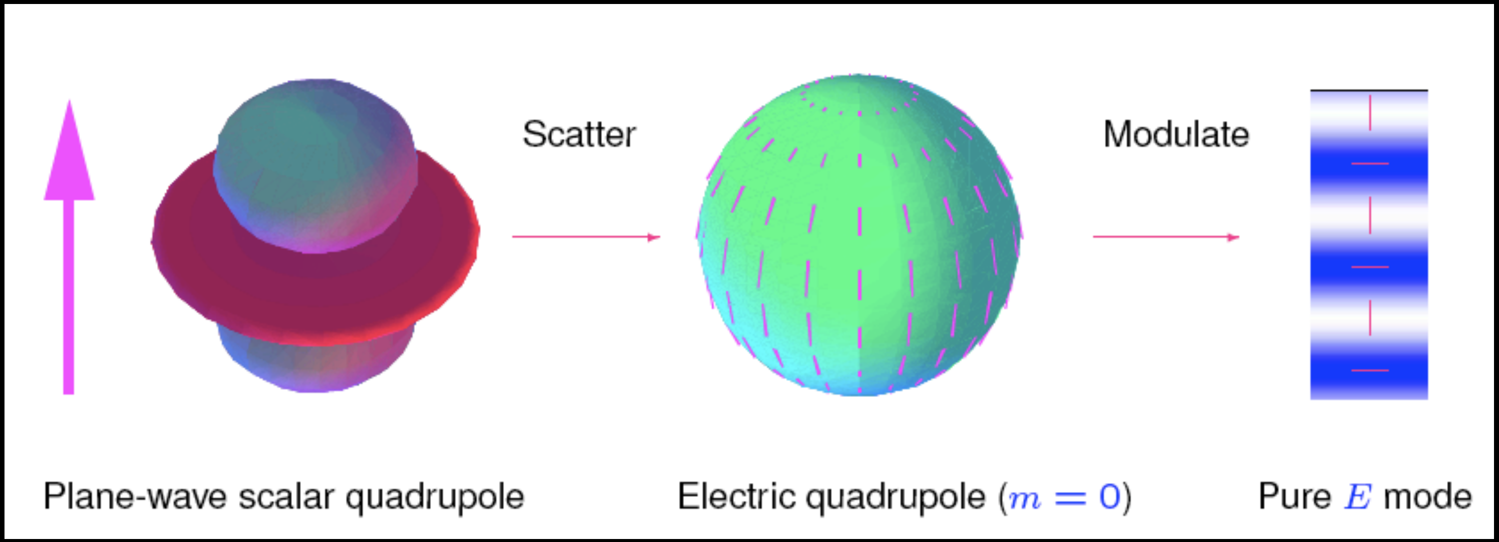


$$\begin{aligned}(Q + iU) &\propto \sigma_T \int d\Omega' (\mathbf{m} \cdot \mathbf{n}')^2 T(\mathbf{n}') \propto \delta\tau_{\rm LS} \mathbf{m}^i \mathbf{m}^j \partial_i v_j(\mathrm{LS}) \\ \text{scattering matrix } P &= -3/4 \sigma_T (\mathbf{m} \cdot \mathbf{n}')^2, \ \mathbf{m} = \mathbf{e_1} + i\mathbf{e_2}\end{aligned}$$

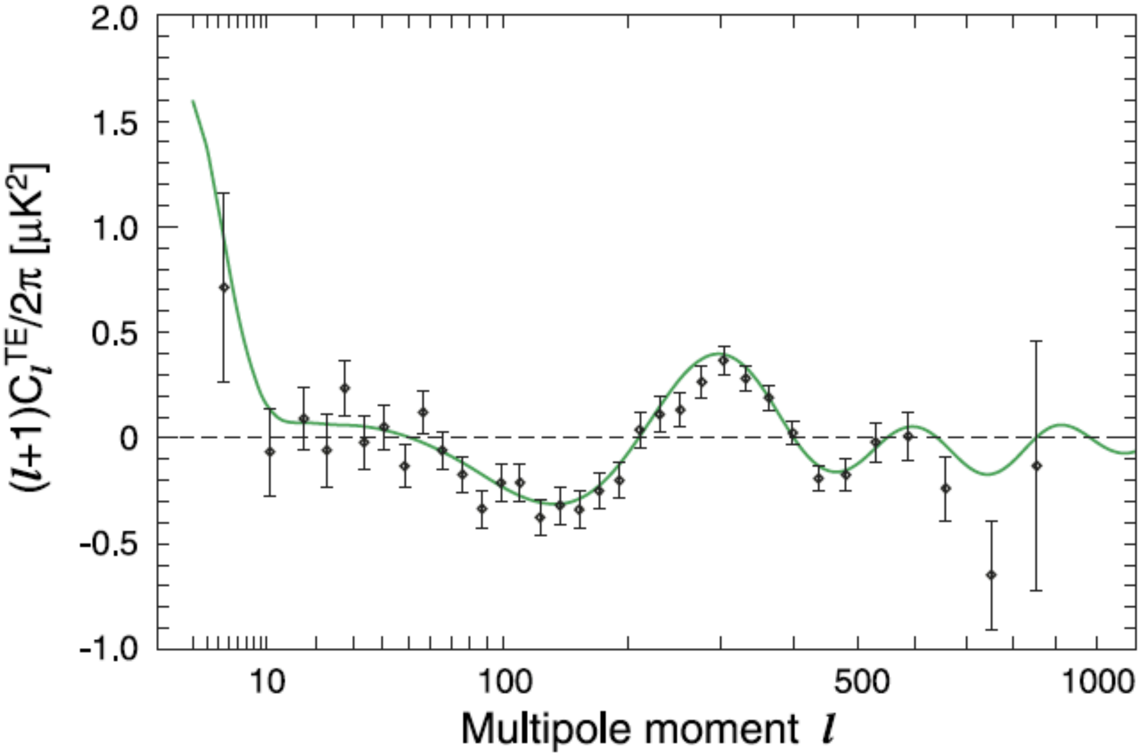
Physics of CMB Polarization: scalar perturbations

A single plane wave of scalar perturbation has:

$$\Theta_{2m} \propto Y^*_{2m}(\mathbf{k}) \Rightarrow dQ \propto \sin^2\theta \text{ and } dU = 0 \text{ as } \mathbf{k} \text{ along } z$$



Only E-mode which traces baryon velocity perturbation



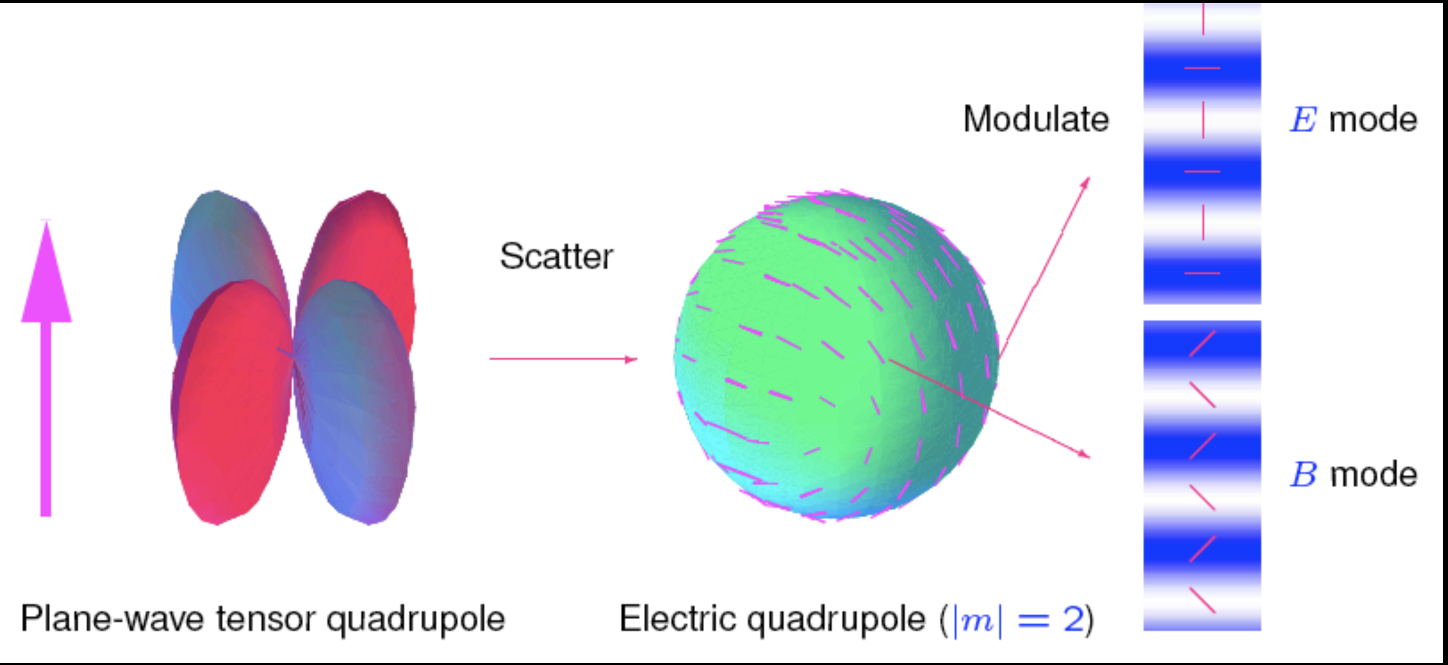
2.0
1.5
1.0
0.5
0
-0.5
-1.0
10
100
500
1000
Multipole moment l
$(l+1)C_l^{TE}/2\pi$ [μK^2]

CMB Polarization from tensor perturbations

Take a gravity wave propagating along the z-axis. The frequency shift in the temperature is given by

$$\frac{1}{\nu}\frac{d\nu}{d\eta} = \frac{1}{2}\mathbf{n}^i\mathbf{n}^j h_{ik}^{(\pm)} = \frac{1}{2}\sin^2\theta e^{\pm 2i\phi}\dot{h}e^{i\mathbf{k}\cdot\mathbf{x}}$$

$$\Rightarrow \quad dQ \propto (1+\cos^2\theta)\cos 2\phi, \text{ and } dU = -\cos\theta\sin 2\phi$$

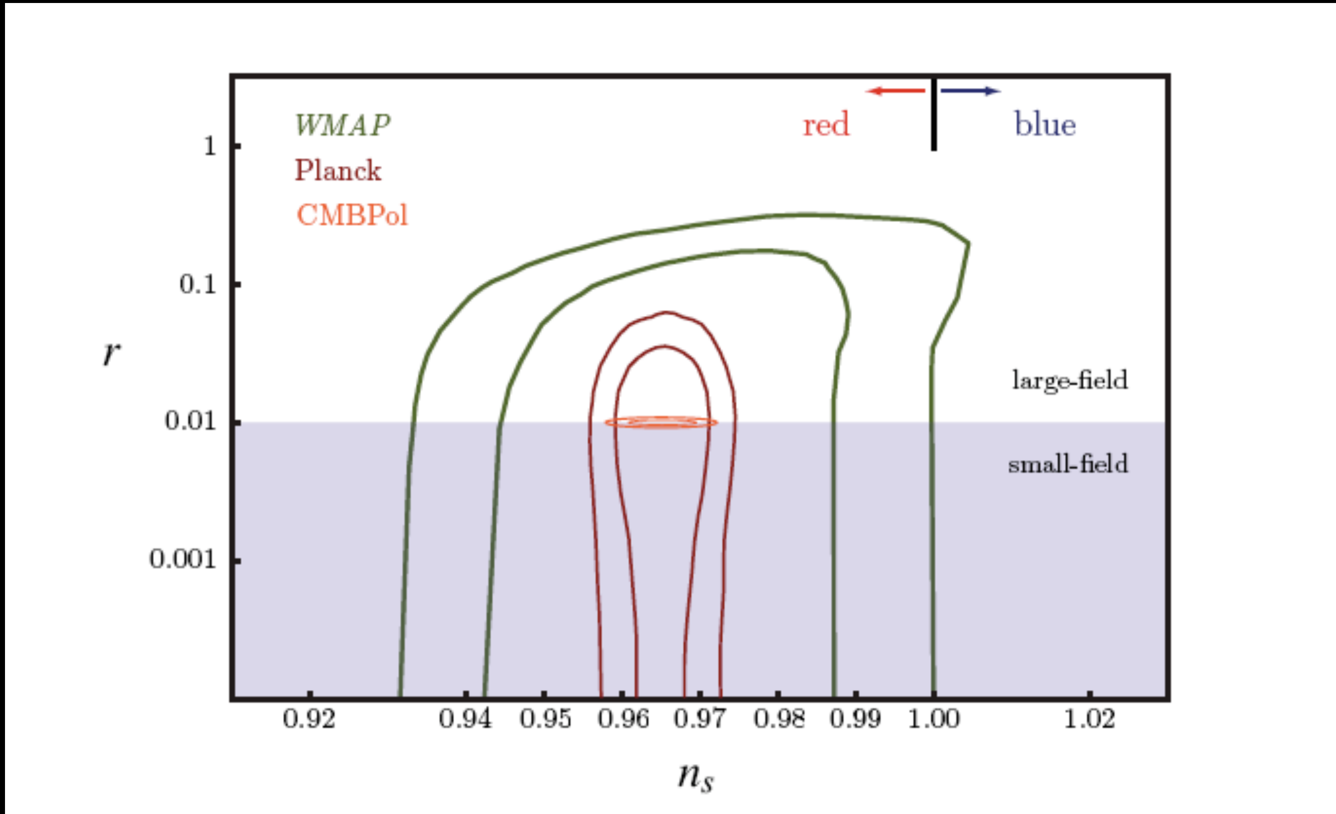


Both E- and B-modes with roughly same amplitude

Testing the energy scale of Inflation

CMBpol: approved by NASA on Feb. 18, 2008,

http://astro.fnal.gov/cmb/, Weiss document



$$[\ell(\ell+1)C_{B\ell}/2\pi]^{1/2} \simeq 0.024(E_{\rm inf}/10^{16}\,{\rm GeV})\,\mu{\rm K}$$

Observation of the B-mode polarization
from inflationary gravity waves requires

$$r \simeq 10^{-2} \left(\frac{\Delta\phi}{m_{\rm Pl}} \right) > 10^{-2}$$

Non-Gaussianity

Characterizing the cosmological perturbations

- The WMAP data are telling us that primordial fluctuations are very close to being Gaussian.

- It may not be so easy to explain that CMB is Gaussian unless we have a compelling early universe model that predicts Gaussian primordial fluctuations: Inflation

What if we discover
in the future
that perturbations
are non-Gaussian?

Gaussian ⟺ free (i.e. non-interacting) field, linear theory

- Collection of independent harmonic oscillators (no mode-mode coupling)
- NG requires more than linear theory

"*... the linear perturbations are so surprisingly simple that a perturbation analysis accurate to second order may be feasible ...*" (Sachs & Wolfe 1967)

Why do we expect some NG, i.e. some Non-Linearity ?

- The observed sky is NG: astrophysical sources (point sources and galactic emission, low level contaminaton of galactic foreground leads to detectable NG, but negligible effects in the angular power spectrum)
- Secondary anisotropies (lensing, SZ, .etc: known to exist)
- Variance of the noise is spatially variable, increasing the variance of the NG estimator
- Gravity itself is nonlinear
- Primordial contribution

How large is the predicted value of NG ?

It depends on the primordial contribution: it is the contribution generated either during or after inflation, when the comoving curvature perturbation becomes finally constant (in time) on super-horizon scales

It is the real science driver

Phenomenlogical approach:

$$\zeta(x) = \zeta_g(x) - \frac{3}{5} f_{\rm NL} \left(\zeta_g^2(x) - \langle \zeta_g^2 \rangle \right)$$

The expanding parameter is roughly $f_{\rm NL}\zeta_g$

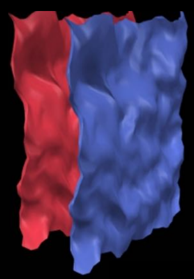
The non-linear parameter is usually momentum dependent

It is not directly connected to the measurable quantity, the CMB anisotropy

Second scenario: Inflation is non-standard (DBI, ghost inflation,....)

Third scenario: inflation does not take place, instead ekpyrotic,....

	Canonical	Non canonical
One field	• Single field inflation with canonical kinetic term $f_{\mathrm{NL}}^{\mathrm{local,equil}} = O(\varepsilon, \eta) \sim 0.01$	• K-inflation, DBI-inflation,… $f_{\mathrm{NL}}^{\mathrm{equil}} \sim 1/c_s^2 \sim 100$ • Break in slow-roll…
More fields	• Multi-field inflation $f_{\mathrm{NL}}^{\mathrm{local}} \sim \frac{1}{16} r + \mathrm{nl\ evol}$ ~ 0.01	• DBI-multi-field inflation $f_{\mathrm{NL}}^{\mathrm{equil/local}} \sim 1/c_s^2$ • Curvaton-like models $f_{\mathrm{NL}}^{\mathrm{local}} \sim \frac{5}{4} \left(\rho/\rho_{\mathrm{curvaton}}\right)_{\mathrm{dec}} > 1$ • New ekpyrotic $f_{\mathrm{NL}}^{\mathrm{local}} > (n_s - 1)^{-1} \quad (n_s > 1)$

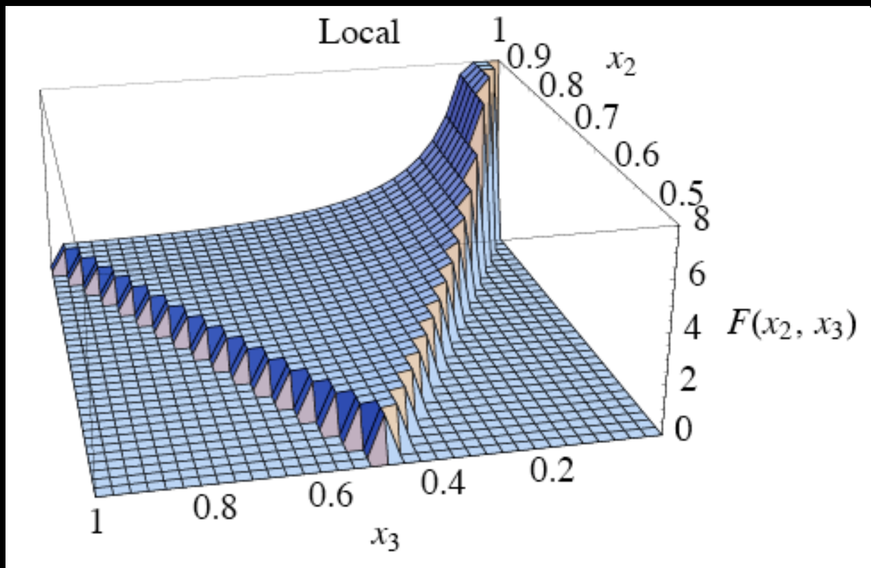


N. Bartolo, E. Komatsu, S. Matarrese and A.R., Phys. Rept. 402, 103 (2004)

The Bispectrum

$$\langle \zeta_{\vec{k}_1} \zeta_{\vec{k}_2} \zeta_{\vec{k}_3} \rangle = \delta\left(\vec{k}_1 + \vec{k}_2 + \vec{k}_3\right) B_\zeta(k_1, k_2, k_3)$$

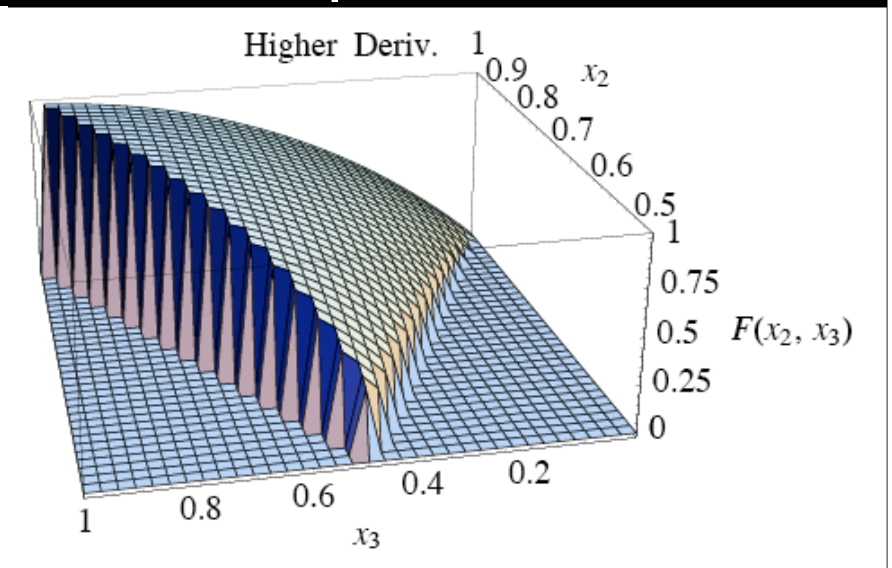
Local



$$B_\zeta(k_1, k_2, k_3) \propto f_{\rm NL}\left[P(k_1)P(k_2) + \text{perm.}\right]$$

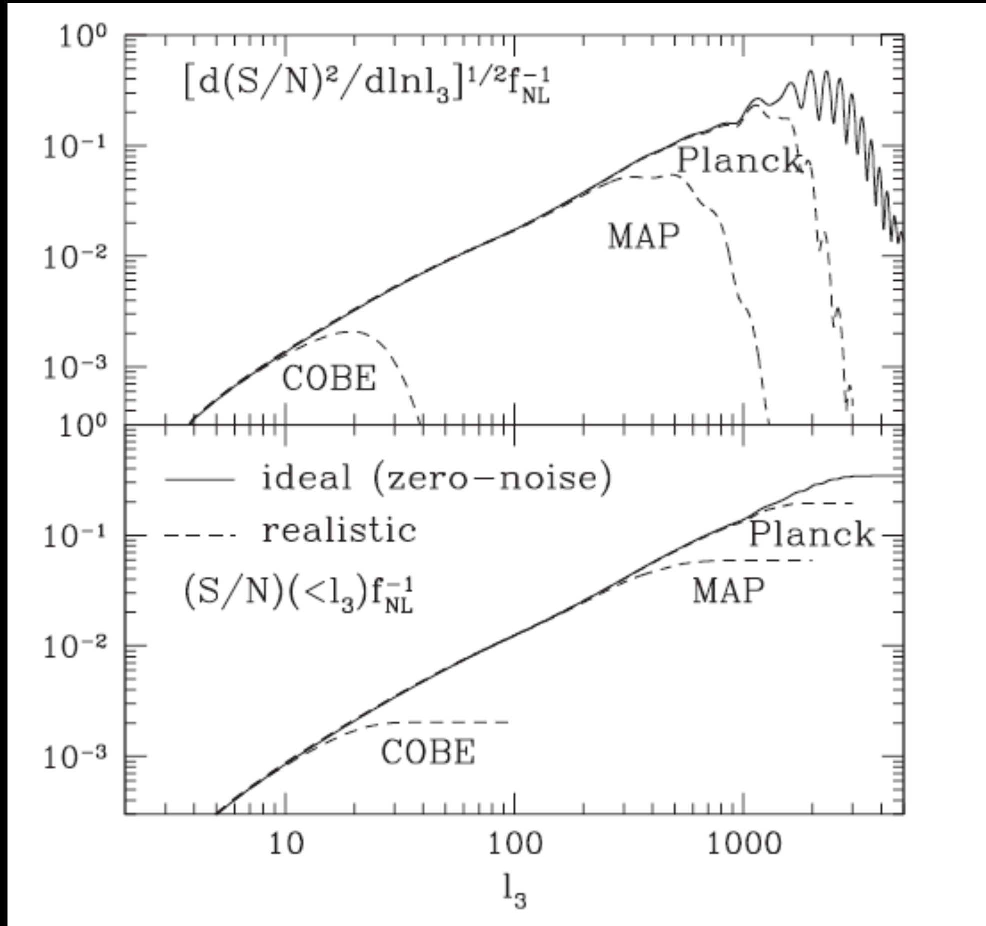
$$k_1 \ll k_2, k_3$$

Equilateral



D. Babich et al., (2005)

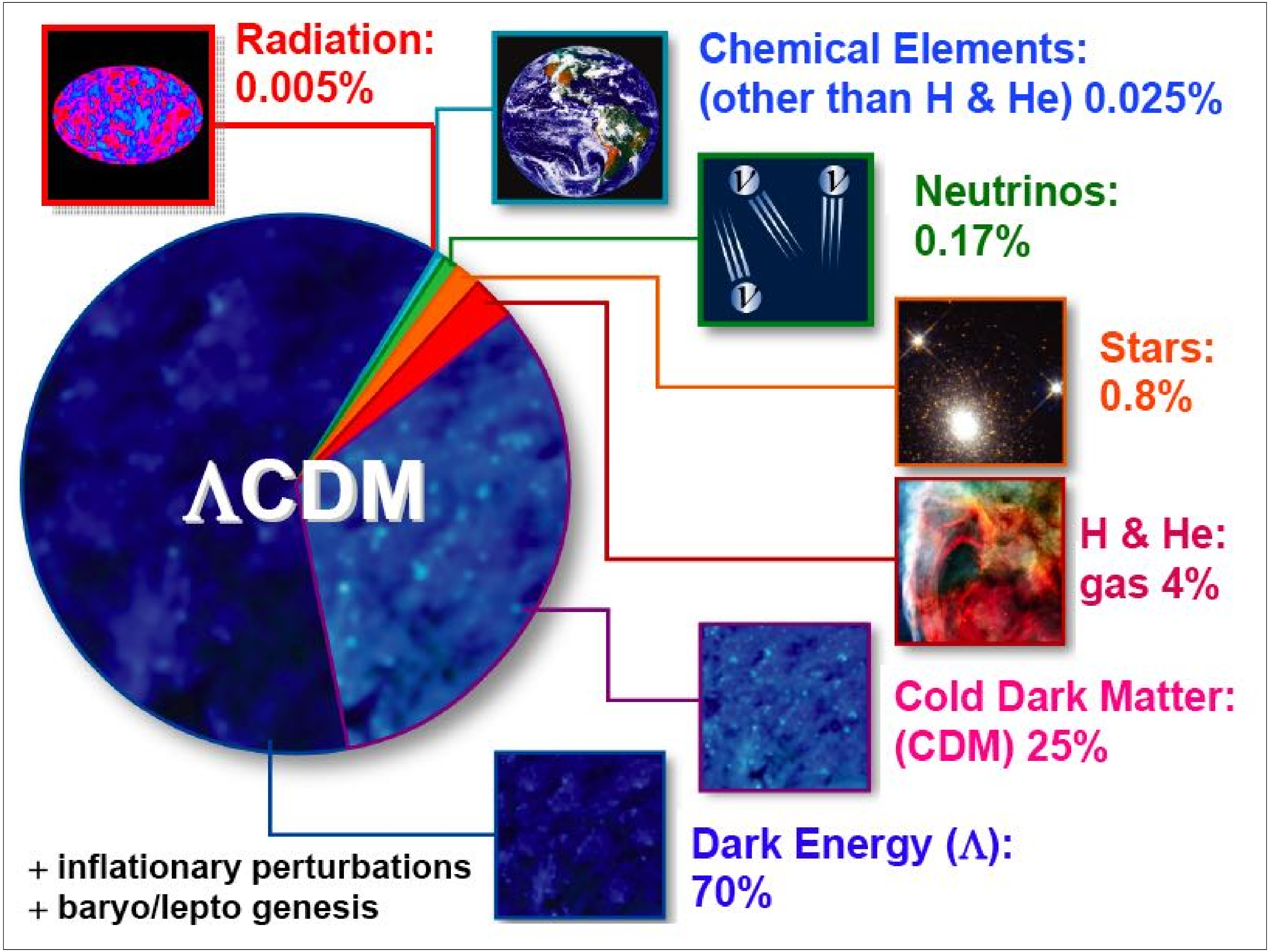
$$\left(\frac{S}{N}\right)_{\rm prim} \sim 10^{-4} f_{\rm NL}\,\ell$$



$$\Delta f_{\rm NL} \sim 20,\ \ell_{\rm max} \sim 500\ \text{(WMAP)}$$
$$\Delta f_{\rm NL} \sim 3,\ \ell_{\rm max} \sim 3000\ \text{(Planck)}$$
$$\Delta f_{\rm NL} \sim 2,\ \text{(ideal experiment)}$$

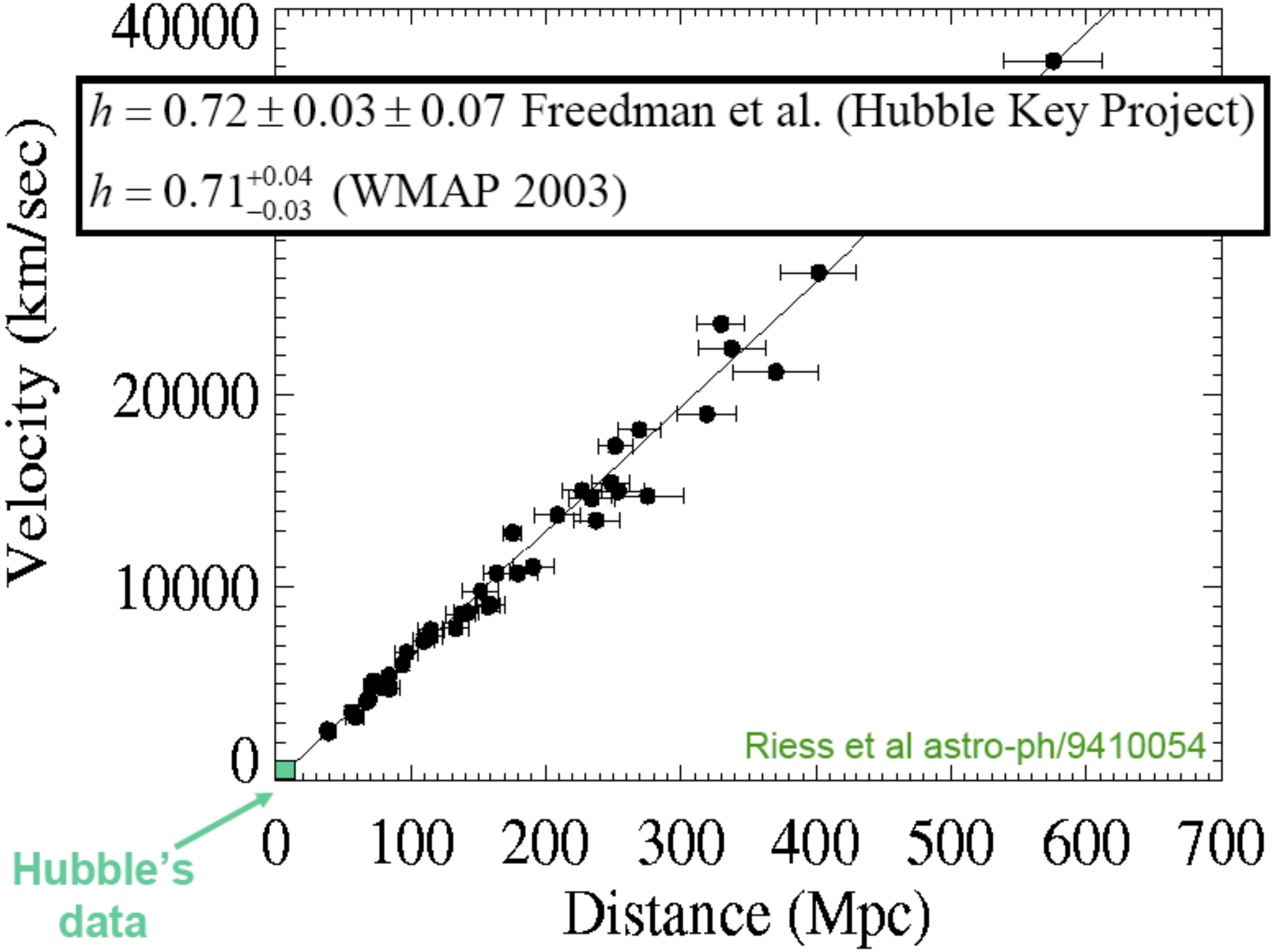
N. Bartolo, E. Komatsu, S. Matarrese and A.R.,
Phys. Rept. 402, 103 (2004)

Lecture three:
the Dark Puzzles



Radiation:
0.005%
Chemical Elements:
(other than H & He) 0.025%
Neutrinos:
0.17%
Stars:
0.8%
ΛCDM
H & He:
gas 4%
Cold Dark Matter:
(CDM) 25%
Dark Energy (Λ):
70%
+ inflationary perturbations
+ baryo/lepto genesis

Dark Energy



40000
20000
10000
0
Velocity (km/sec)
$h = 0.72 \pm 0.03 \pm 0.07$ Freedman et al. (Hubble Key Project)
$h = 0.71^{+0.04}_{-0.03}$ (WMAP 2003)
Riess et al astro-ph/9410054
Hubble's data
0
100
200
300
400
500
600
700
Distance (Mpc)

Distance-Redshift Relation

$F = \frac{L}{4\pi d_L^2}$ defines luminosity distance, know L, measure F

$4\pi d_L^2$ area of ^2S centered on source at time of detection

$$ds^2 = dt^2 - a^2(t)\left[\frac{dr^2}{1-kr^2} + r^2 d\Omega^2\right] \Rightarrow \text{area} = 4\pi a_0^2 r^2$$

Energy redshifted: $(1+z)$
Time interval redshifted: $(1+z)$
Flux redshifted: $(1+z)^2$

$$d_L^2 = a_0^2 r^2 (1+z)^2$$

Distance-Redshift Relation

Light travels on geodesics

$$ds^2 = 0 \Rightarrow \int \frac{dr}{\sqrt{1-kr^2}} = \int \frac{dt}{a(t)} = \int \frac{da}{H(a)a^2}$$

$$\int_0^r \frac{dr'}{\sqrt{1-kr'^2}} = \int_0^z \frac{a^{-1}(t_0)H_0^{-1} \quad dz'}{\sqrt{(1-\Omega_0)(1+z')^2 + \Omega_M(1+z')^3 + \Omega_w(1+z')^{3(1+w)} + \ldots}}$$

Program:

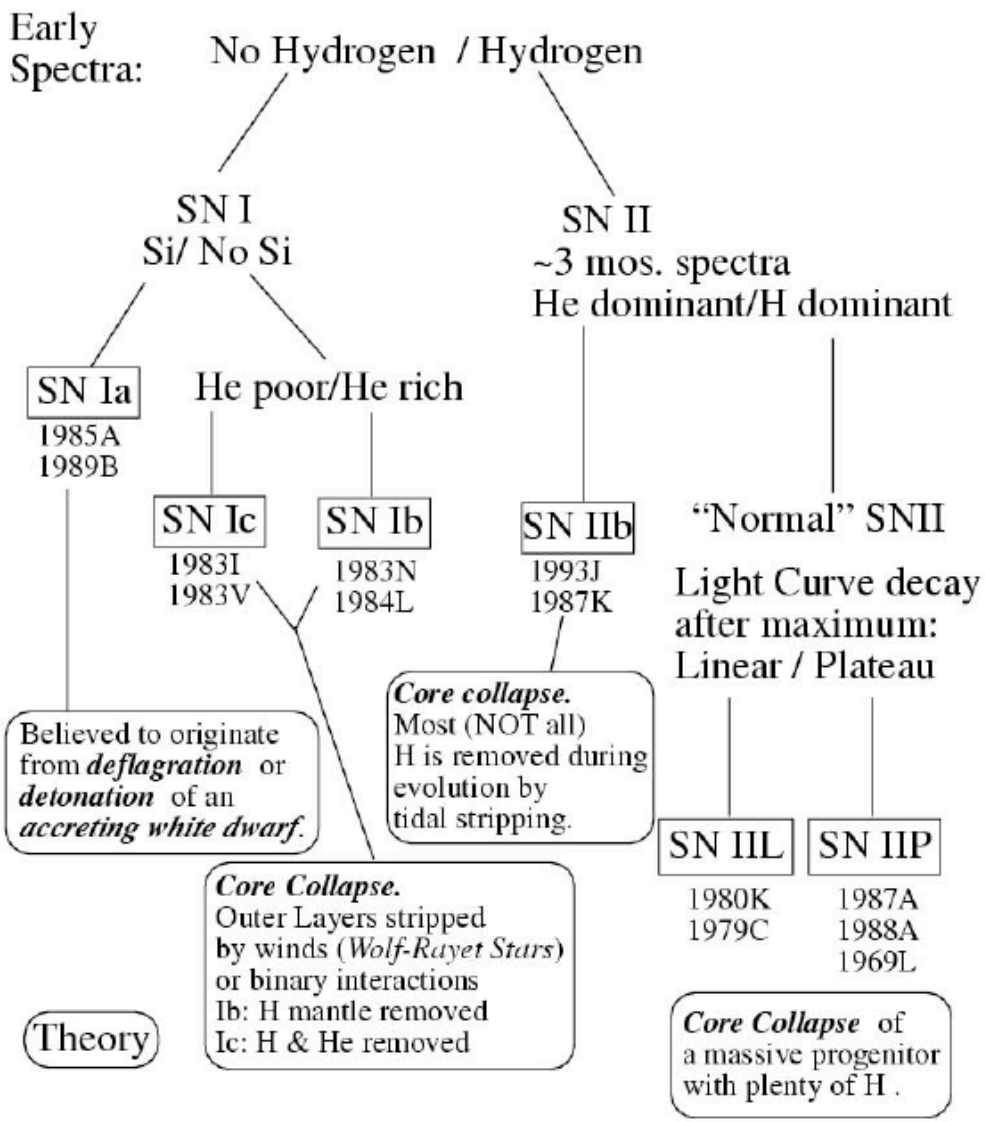
- measure d_L (via $d_L^2 = L/4\pi F$) and z
- input a model cosmology (Ω_i) and calculate $a_0\, r$
- compare to data
- need bright "standard candle"

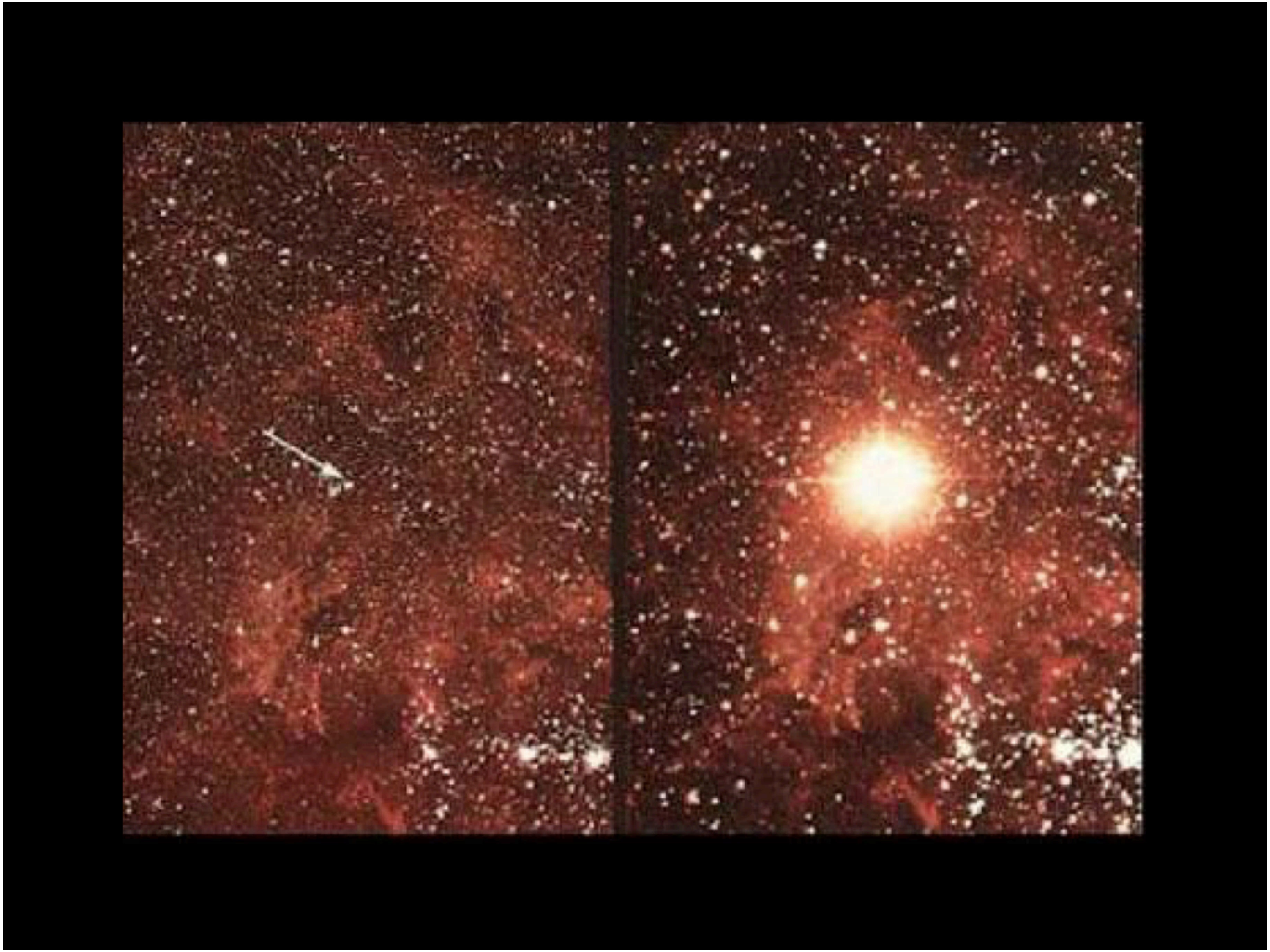
Distance-Redshift Relation

$$d_L(z) \;=\; \frac{1}{H_0}\left[z + (1-q_0)\frac{z^2}{2} + \left(-j_0 + 3q_0^2 - 1 - \frac{k}{a_0^2 H_0^2}\right)\frac{z^3}{6} + \mathcal{O}(z^4)\right.$$

$$q \;\equiv\; -(\ddot{a}/a)/H^2, \text{ jerk } j \equiv (\dddot{a}/a)/H^3$$

Supernova Taxonomy





Monastic Chronicles re: Supernova 1006:

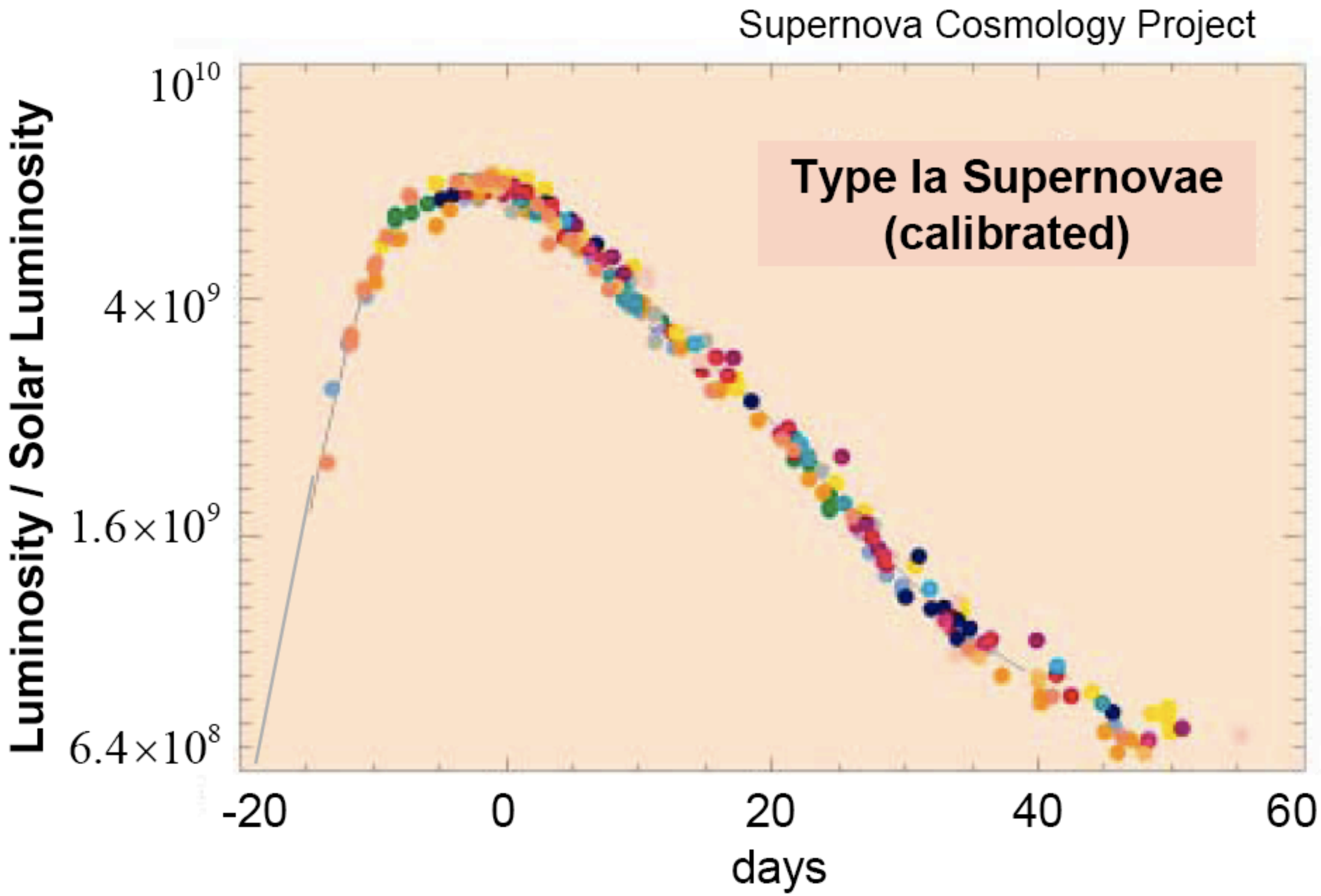
"in 1006 there was a very great famine and a comet appeared for a long time …."

"at the same time a comet, which always announces human shame, appeared in the southern regions, which was followed by a great pestilence…"

"three years after the king was raised to the throne, a comet with a horrible appearance was seen in the southern part of the sky, emitting flames this way and that…"

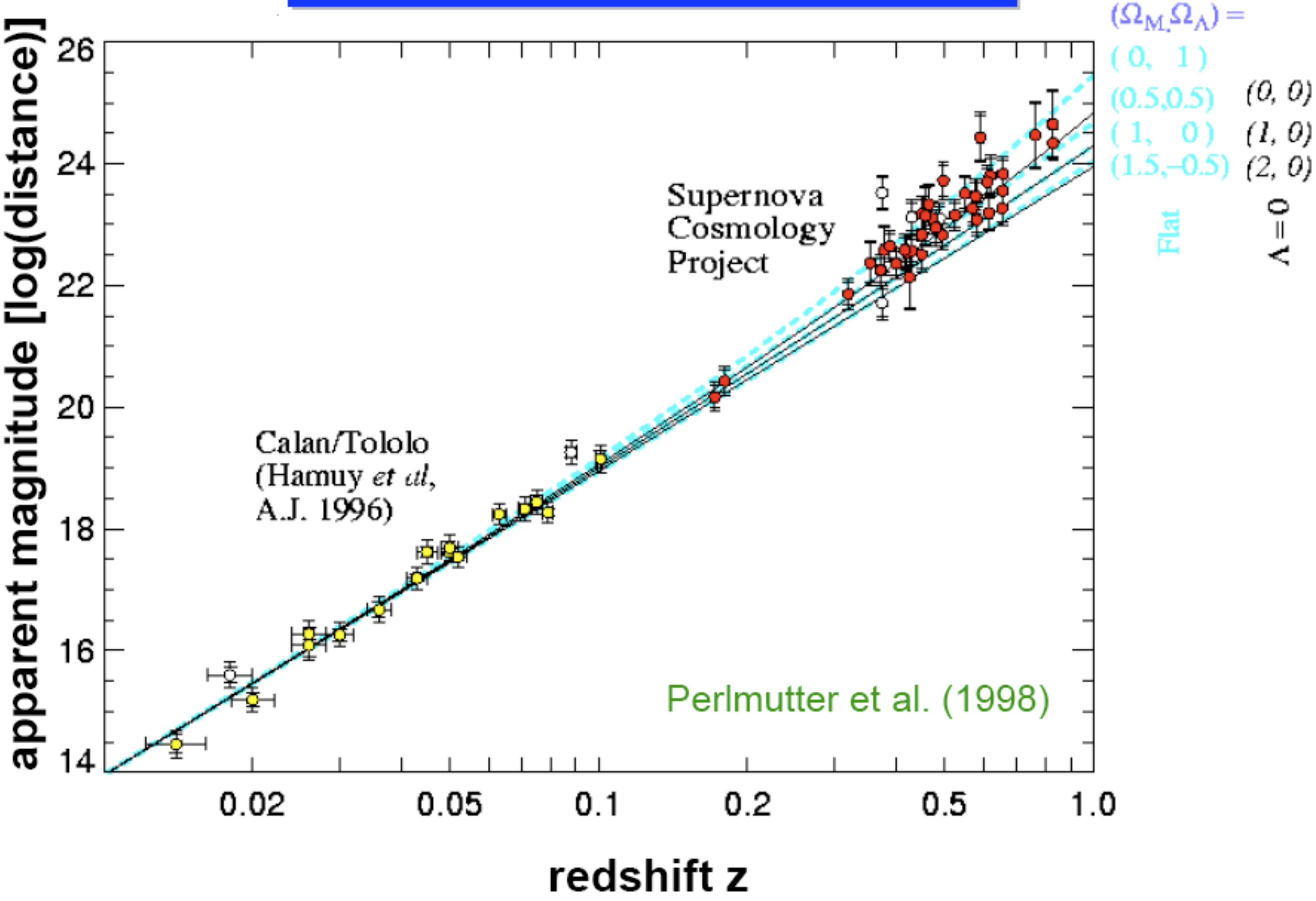
Georg Busch (German painter) in 1572:

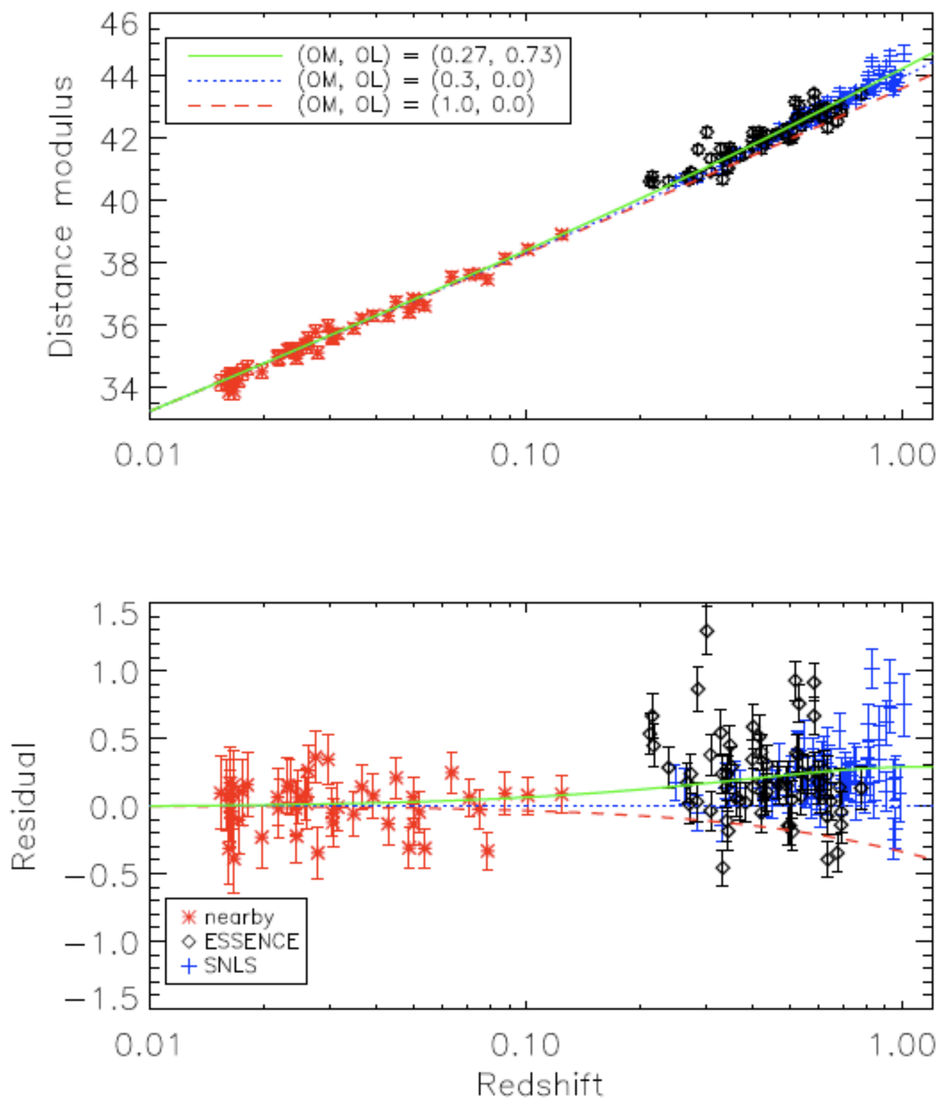
"It is a sign that we will be visited by all sorts of calamities such as inclement weather, pestilence, and Frenchmen."



Supernova Cosmology Project
Type Ia Supernovae
(calibrated)
Luminosity / Solar Luminosity
10^10
4×10^9
1.6×10^9
6.4×10^8
-20
0
20
40
60
days

Type Ia supernova



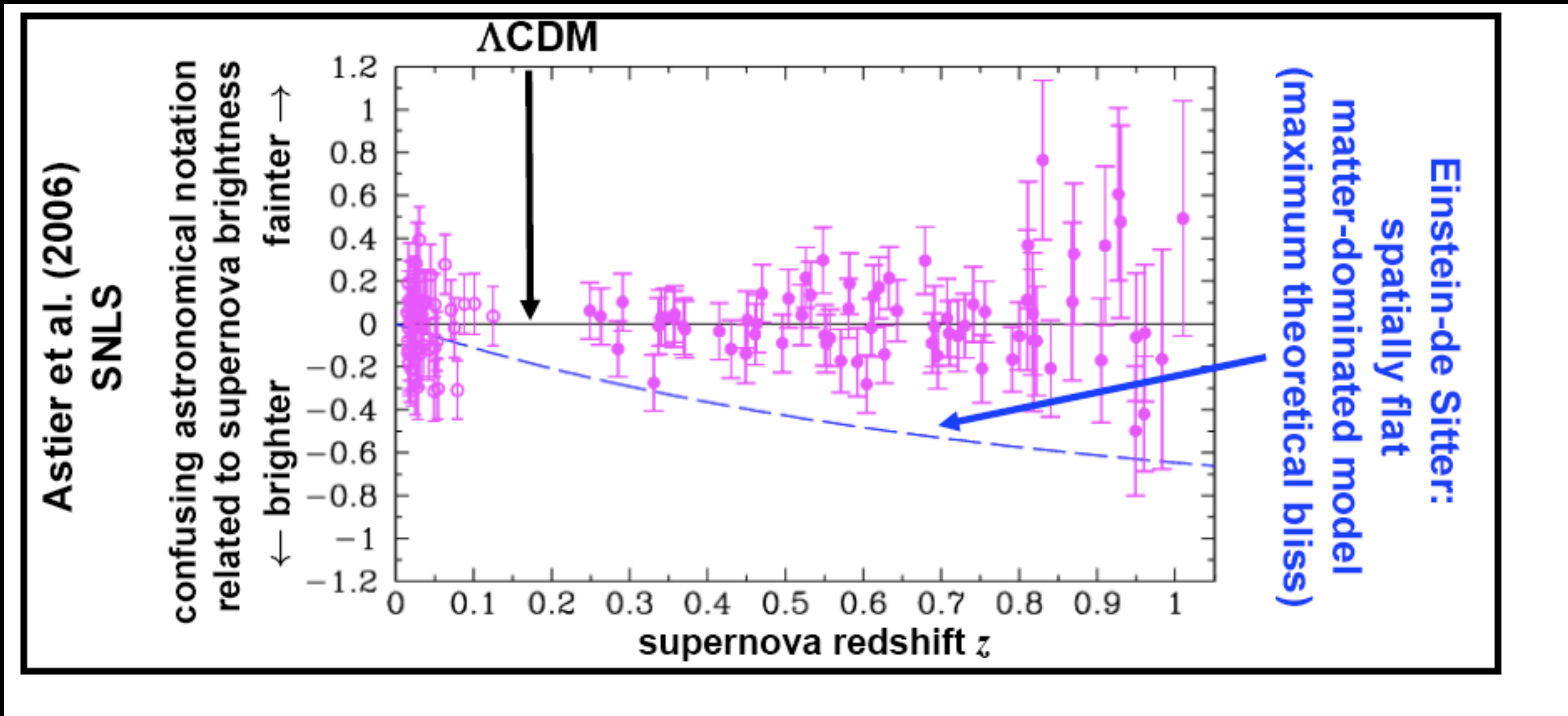


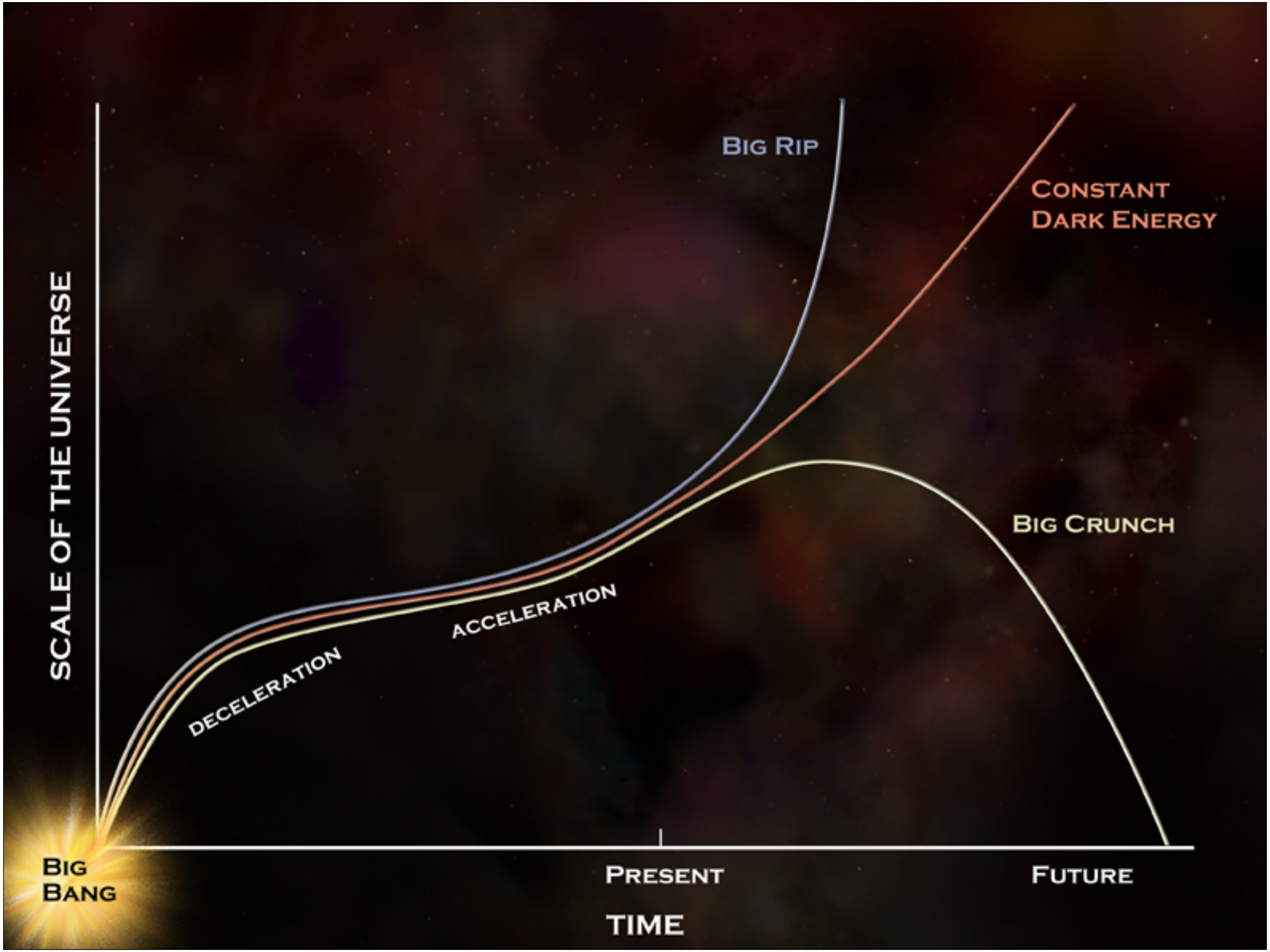
(OM, OL) = (0.27, 0.73)
(OM, OL) = (0.3, 0.0)
(OM, OL) = (1.0, 0.0)
Distance modulus
46
44
42
40
38
36
34
0.01
0.10
1.00
Residual
1.5
1.0
0.5
0.0
−0.5
−1.0
−1.5
nearby
ESSENCE
SNLS
Redshift

Evidence for acceleration

$$d_L(z) = \frac{1}{H_0}\left[z + (1-q_0)\frac{z^2}{2} + \left(-j_0 + 3q_0^2 - 1 - \frac{k}{a_0^2 H_0^2}\right)\frac{z^3}{6} + \mathcal{O}(z^4)\right)$$

$$q \equiv -(\ddot{a}/a)/H^2, \text{ jerk } j \equiv (\dddot{a}/a)/H^3$$




Big Rip
Constant
Dark Energy
Big Crunch
Scale of the Universe
Deceleration
Acceleration
Big
Bang
Present
Future
Time

Taking sides:

$$G_{00}(\mathrm{FRW}) = 8\pi\, G\, T_{00}$$

1) Modify the RHS of Einstein equations
   a) Cosmological constant
   b) Not constant (scalar field)

2) Modify the LHS of Einstein equations
   a) Beyond Einstein (mod. of gravity)
   b) Just Einstein (BR of inhomog.)

The dark side of the Universe

70% of the energy density of the Universe is in the form of dark energy

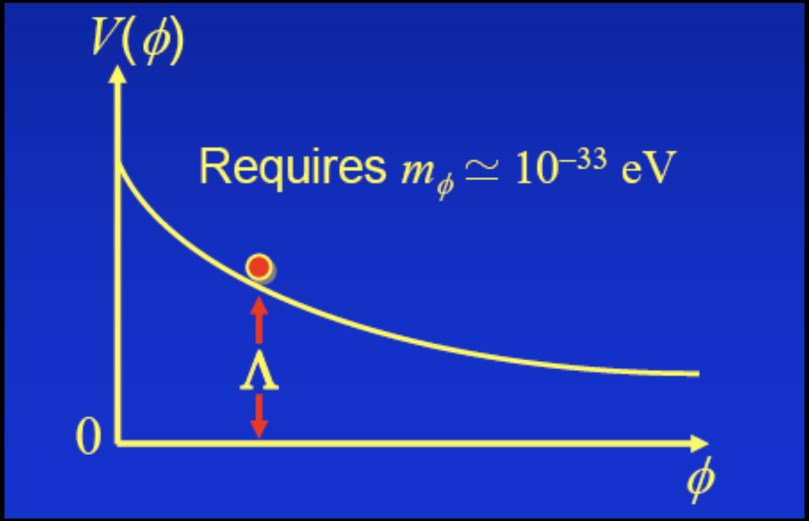
$$\ddot{a} > 0 \Leftrightarrow w \equiv P/\rho < -1/3$$

How do we know DE exists?

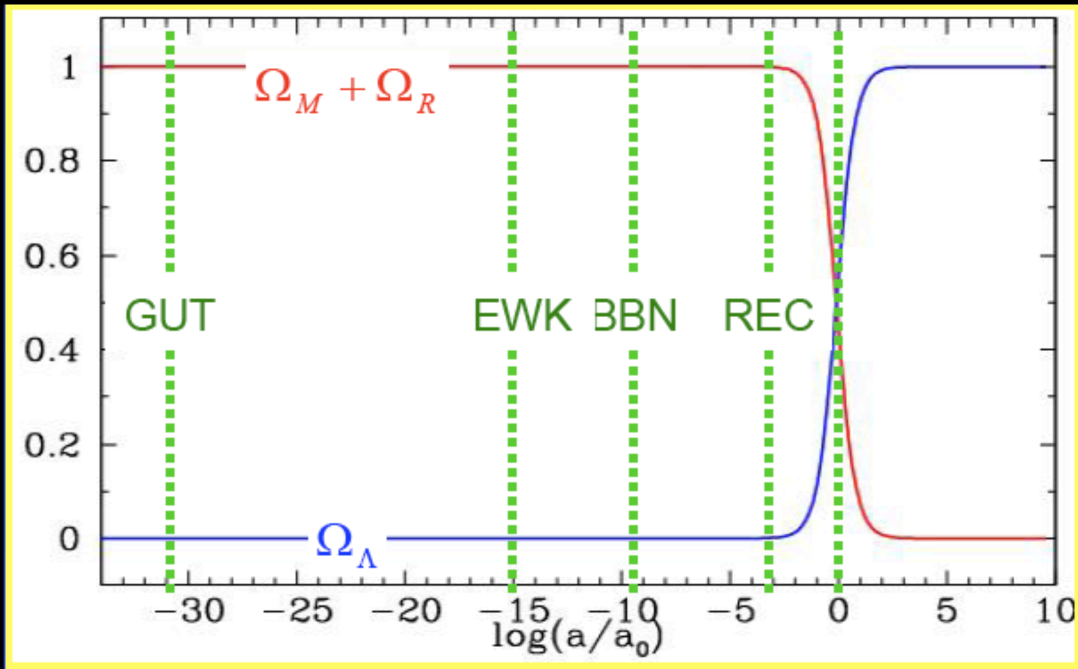
- Assume FRW model of cosmology: $H^2 = 8\pi G\rho/3 - k/a^2$
- Assume energy and pressure content: $\rho = \rho_M + \rho_\gamma + \rho_\Lambda + \cdots$
- Input cosmological parameters
- Compute observables: $d_L(z),\ d_A(z),\ H(z)$
- Model cosmology fits with ρ_Λ , but not without ρ_Λ
- All evidence for DE is INDIRECT : the observed Hubble rate is not the one predicted through all the previous steps

Modify the RHS: CC/Quintessence

- Many possible contributions?
- Why then is the total so small?
- Perhaps some unknown dynamics sets the total CC to zero, but we are not there yet



Why now?


$\Omega_M + \Omega_R$
Ω_Λ
GUT
EWK
BBN
REC
1
0.8
0.6
0.4
0.2
0
−30
−25
−20
−15
−10
−5
0
5
10
$\log(a/a_0)$

Modify the LHS: non-standard gravity

$$F_g = G_N \frac{m_1 m_2}{r^2} \text{ per } r < r_c$$

$$F_g = G_N \frac{m_1 m_2}{r^3} \text{ per } r > r_c$$

Degravitation

$$G_{\mathrm{N}}^{-1}\left(L^2\Box\right) G_{\mu\nu} = 8\,\pi\, T_{\mu\nu}$$

All these class of theories predict the presence of extra longitudinal degrees of freedom of the graviton which becomes strongly coupled at some distance

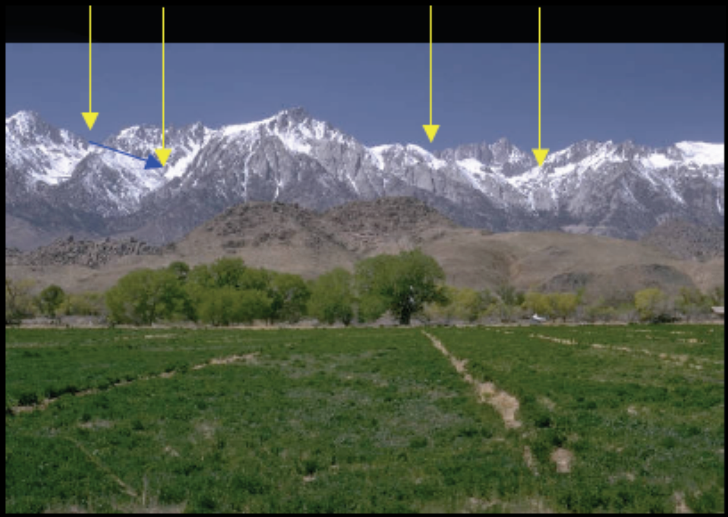
Anthropic/Landscape

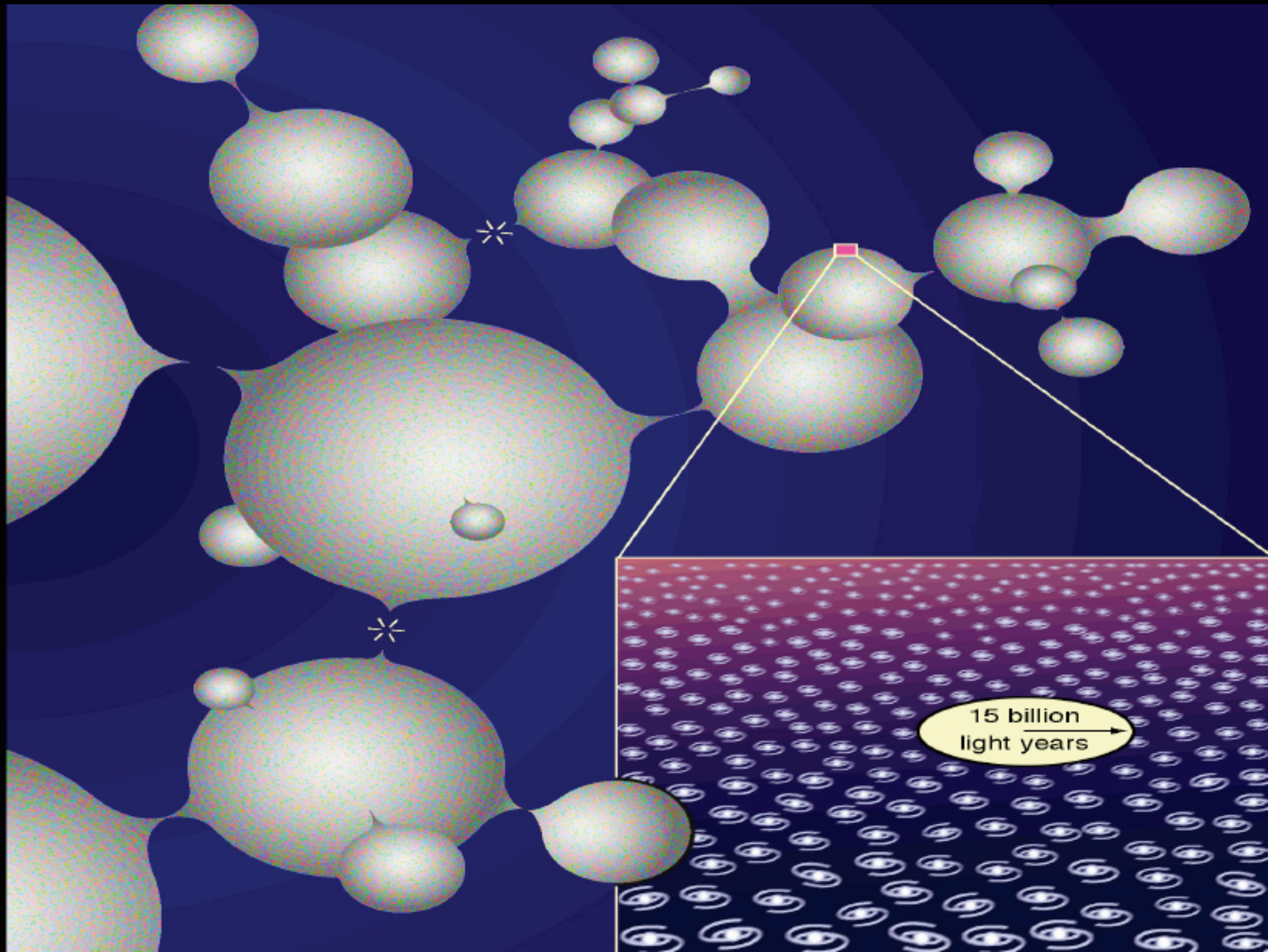
- Many sources of vacuum energy
- String Theory has many vacua $> 10^{500}$
- Some of them correspond to a cancellation leading to the observed small cosmological constant

Galaxies require (Weinberg)

$$\Lambda < 10^{-118}\, M_{\rm Pl}^4$$

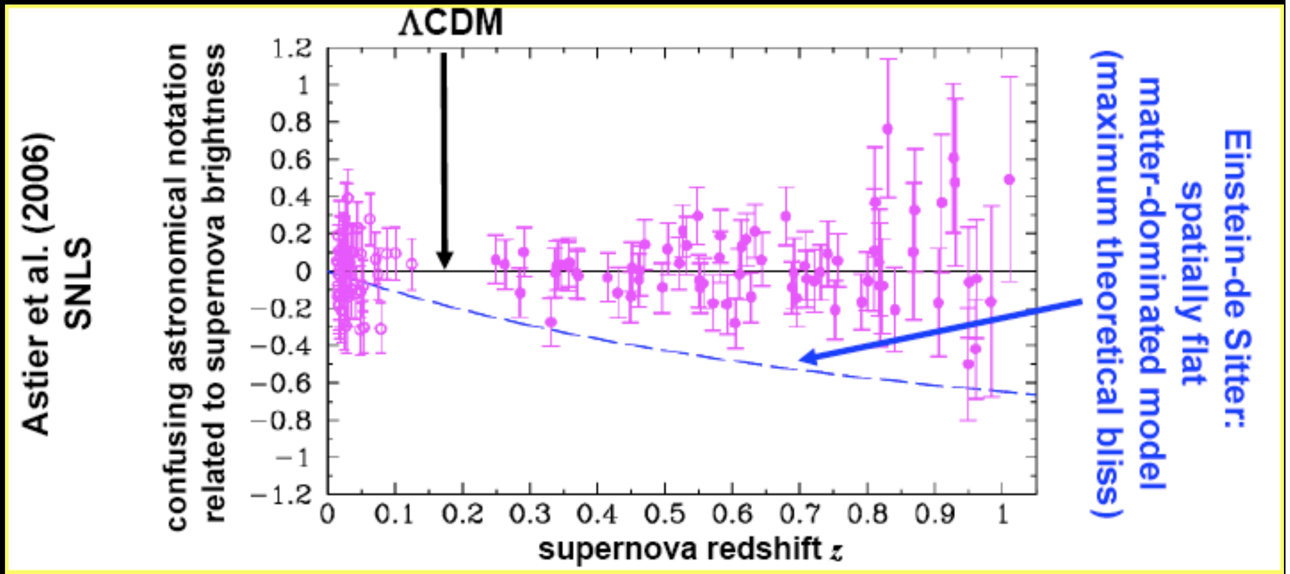
- Although the are exponentially uncommon, they are preferred because...
- More common values of the CC results in an inhospitable Universe



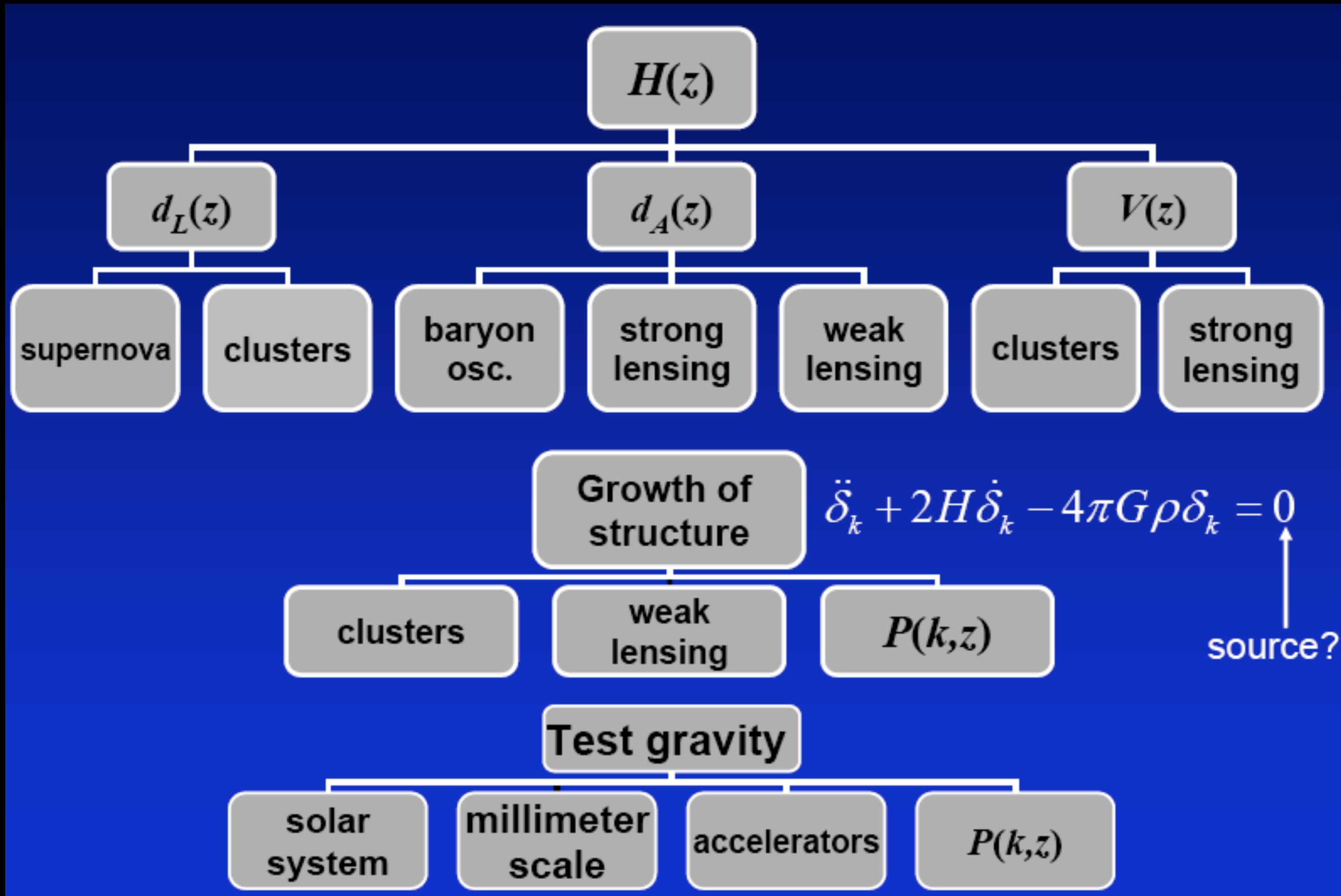

15 billion
light years

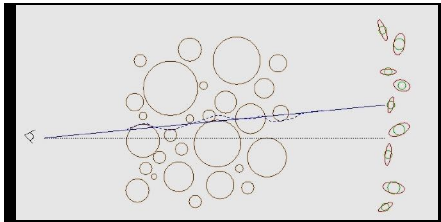
Evidence for Dark Energy

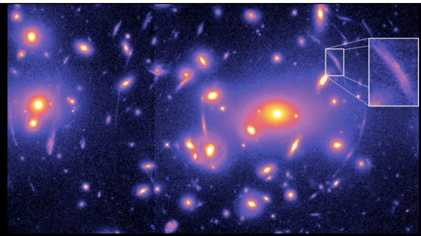
- Hubble Diagram (SNe)
- Baryon acoustic oscillations
- Weak lensing
- Galaxy clusters
- Age of the Universe
- Structure formation



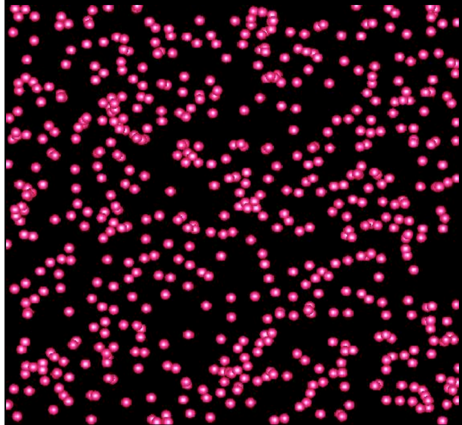
Observational strategy

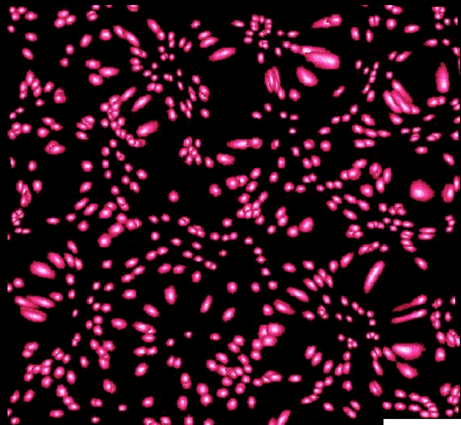






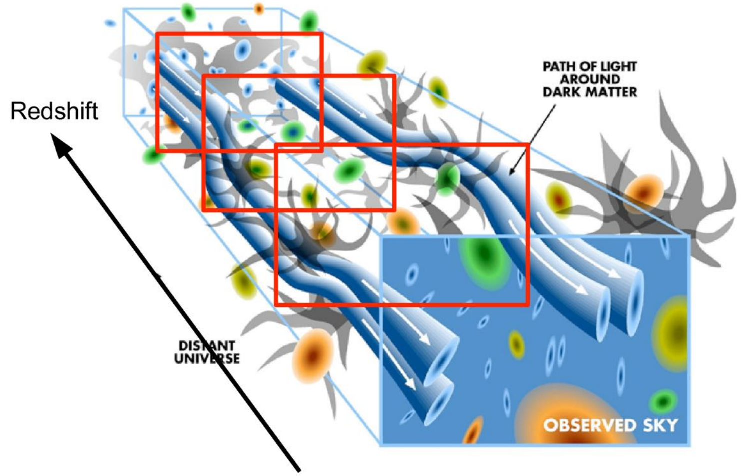
Distortion of background images by foreground matter





Unlensed   Lensed

- **Tomography** = bin galaxies by redshift



Cosmological Perturbations are sensitive to energy content and to modified gravity

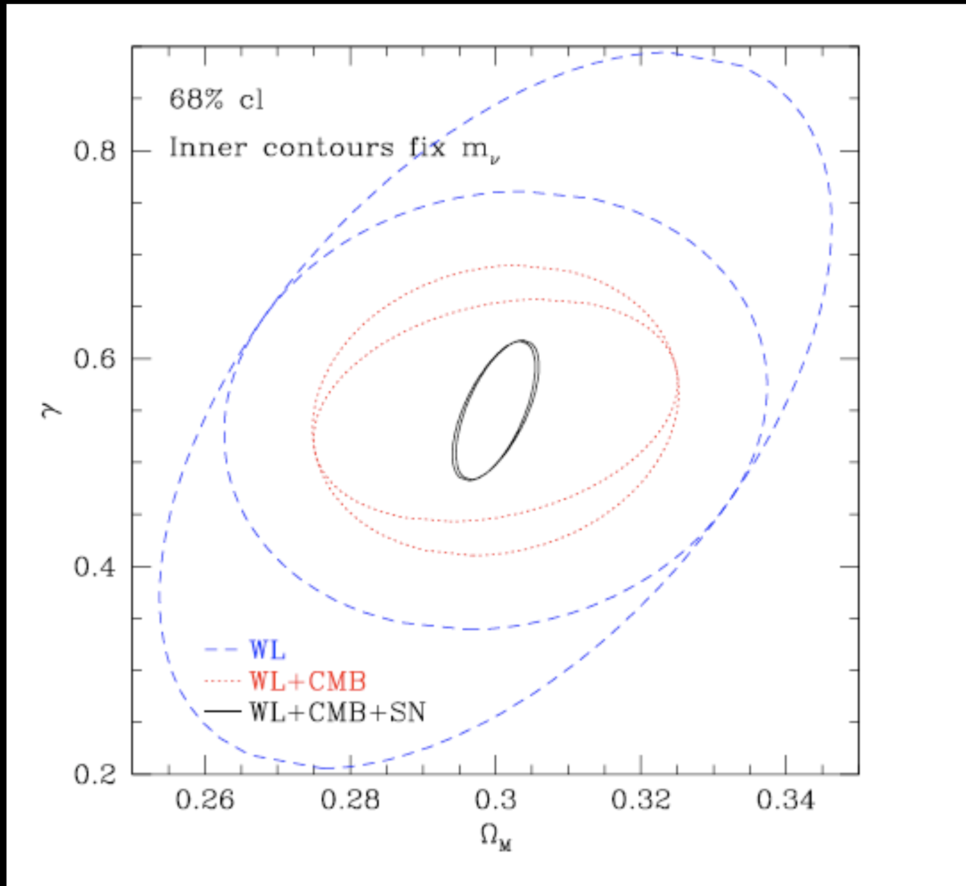
$$\ddot{\delta}_m + 2H\dot{\delta}_m = \frac{3}{2}H^2\delta_m, \;\; \delta_m = \delta\rho_m/\rho_m$$

$$\delta_m(a) = D(a) = \text{growth function}, \; D(a) = a \text{ in MD}$$

Perturbations can be probed at different epochs:

1) CMB, z ~ 1100
2) 21 cm, z ~ 10-20
3) Ly-alpha forest, z ~ 2-4
4) Weak lensing, z ~ 0.3-2
5) Galaxy clustering, z ~ 0-2

$$w(a) = w_0 + (1 - a)w_a$$

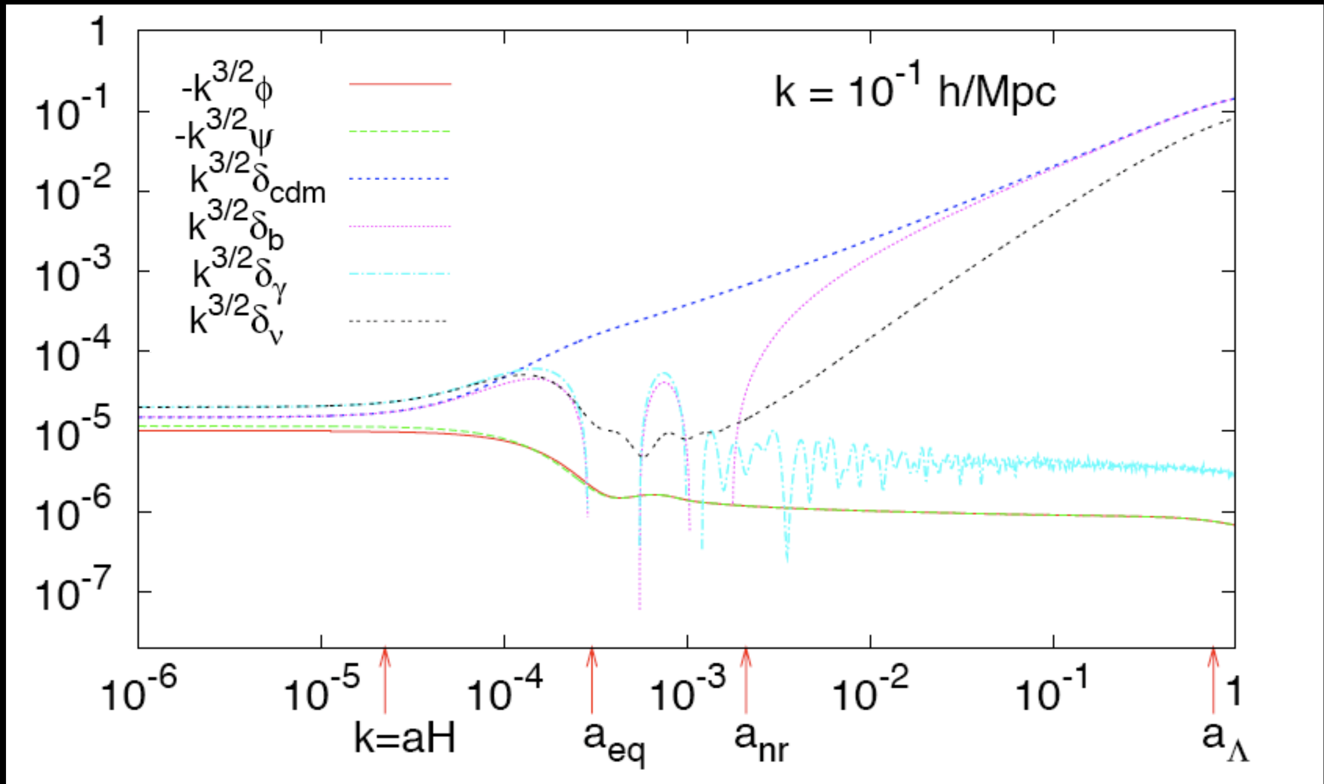


$$g(a) \equiv \delta_m/a = e^{\int_0^a d\ln a' \left[\Omega_M^\gamma(a') - 1\right]}$$

D. Huterer, E. Linder, 2005

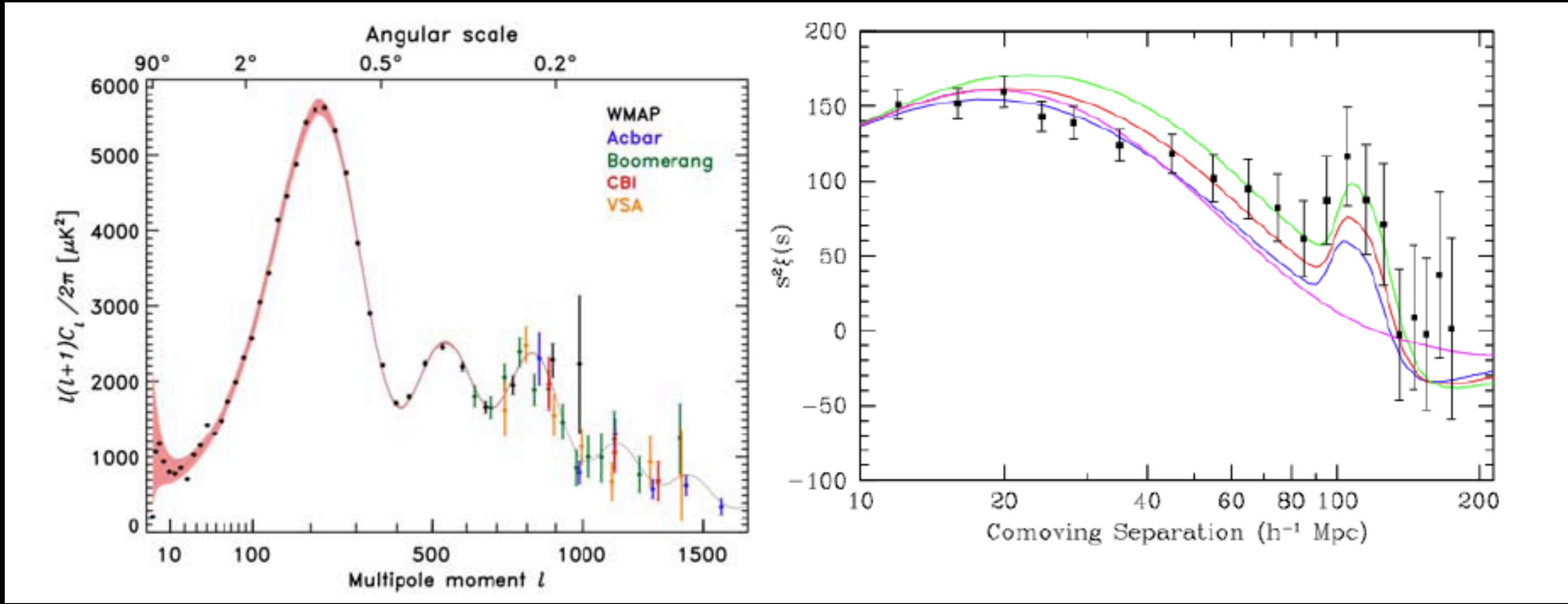
Acoustic Baryonic Oscillations

$$\frac{\delta\rho_{\rm M}}{\rho_{\rm M}} = \frac{\delta\rho_{\rm B} + \delta\rho_{\rm DM}}{\rho_{\rm B} + \rho_{\rm DM}} = \frac{\Omega_{\rm B}}{\Omega_{\rm M}}\frac{\delta\rho_{\rm B}}{\rho_{\rm B}} + \frac{\Omega_{\rm DM}}{\Omega_{\rm M}}\frac{\delta\rho_{\rm DM}}{\rho_{\rm DM}}$$



$$\langle \left(\frac{\delta\rho_{\rm M}}{\rho_{\rm M}}\right)^2 \rangle = \frac{\Omega_{\rm DM}}{\Omega_{\rm M}} B(k) + \frac{\Omega_{\rm B}}{\Omega_{\rm M}} C(k)\cos(kr_s)$$

Acoustic Baryonic Oscillations



Each overdense region is an overpressure that launches a spherical sound wave. Wave travels outward at sound speed. Photons decouple, travel to us and are observable as CMB acoustic peaks. For matter, sound speed plummets, wave stalls, total distance travelled 150 Mpc imprinted on power spectrum.

DE enters in the determination of the angular distance

Main current/future BAO surveys

Name	Telescope	$N(z) / 10^6$	Dates	Status
SDSS/2dFGRS	SDSS/AAT	0.8	Now	Done
WiggleZ	AAT(AAOmega)	0.4	2007-2011	Running
FastSound	Subaru(FMOS)	0.6	2009-2012	Proposal
BOSS	SDSS	1.5	2009-2013	Proposal
HETDEX	HET(VIRUS)	1	2010-2013	Part funded
WFMOS	Subaru	>2	2013-2016	Part funded
ADEPT	Space	>100	2012+	JDEM
SKA	SKA	>100	2020+	Long term

Most data will come at z ~ 1 (U-band bottleneck for LBGs)

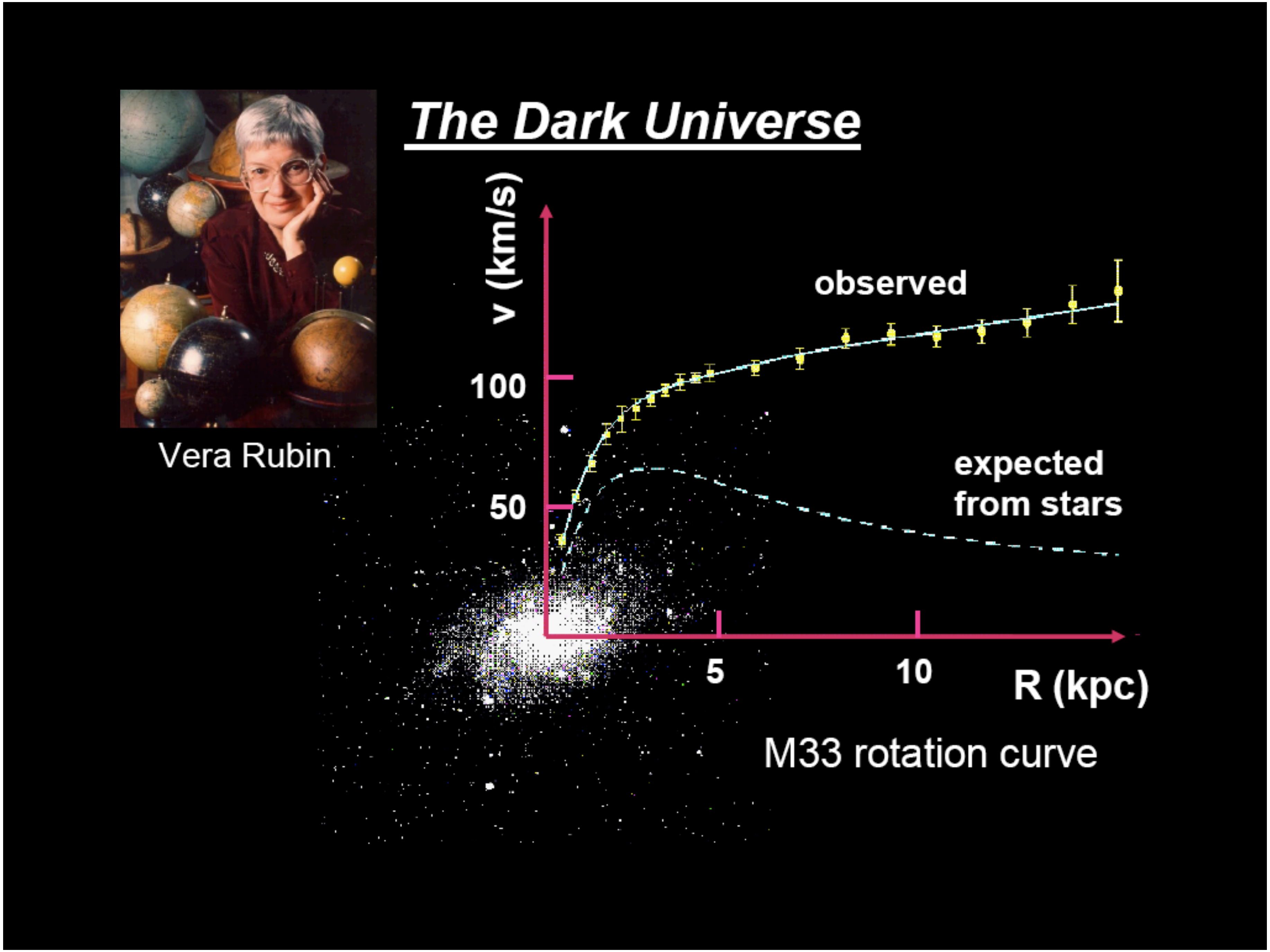
Σ WiggleZ/FastSound/BOSS = 2m by ~2012 (~7% on w)

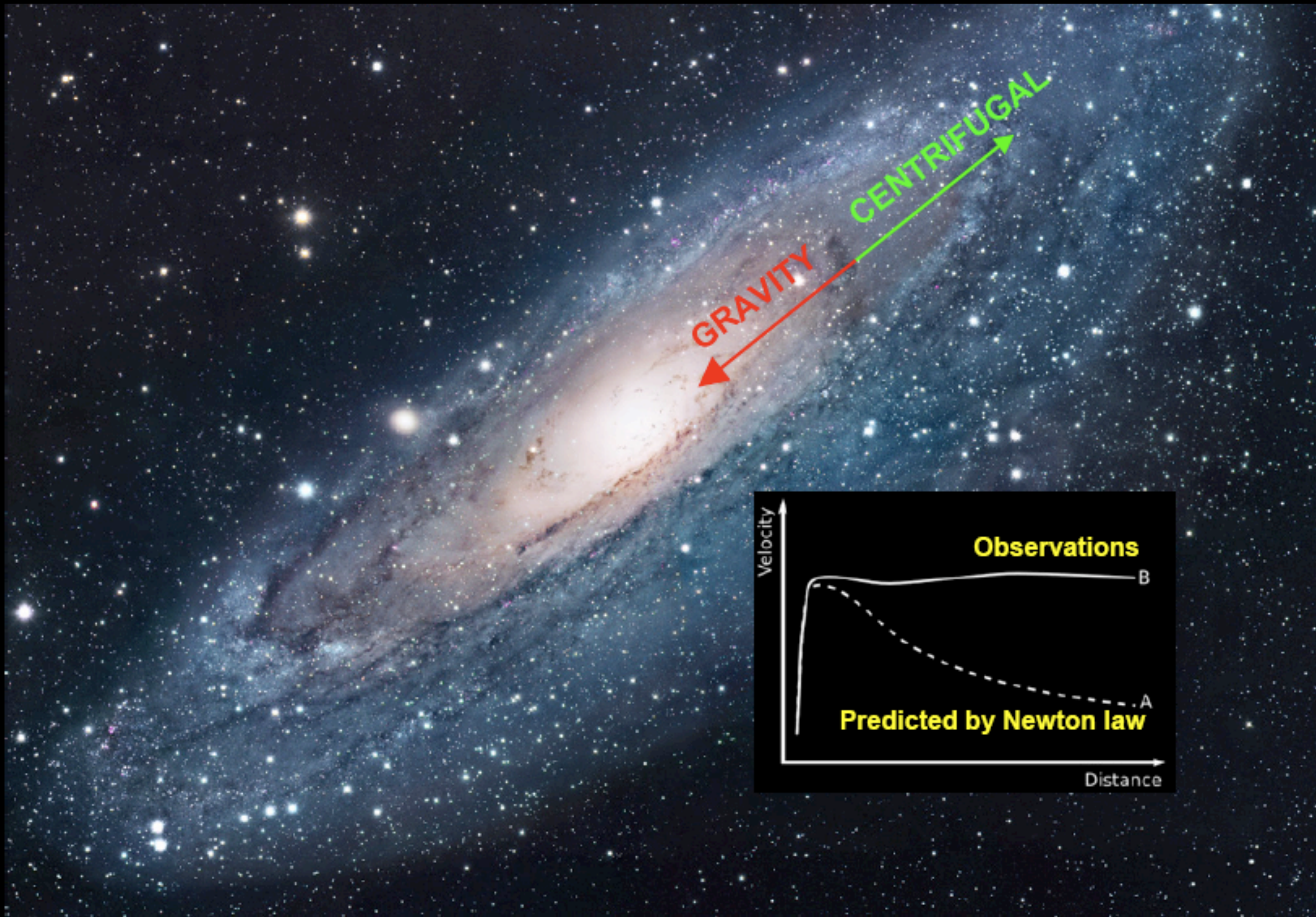
What's Ahead

	2008	2010	2015	2020
Lensing	CFHTLS SUBARU	DES, VISTA	DUNE LSST	SKA
	DLS SDSS ATLAS KIDS	Hyper suprime Pan-STARRS	JDEM	
BAO	FMOS LAMOST	DES, VISTA,VIRUS	WFMOS LSST	SKA
	SDSS ATLAS	Hyper suprime Pan-STARRS	JDEM	
SNe	CSP ESSENCE	DES	LSST	
	SDSS CFHTLS	Pan-STARRS	JDEM	
Clusters	AMI APEX SPT	DES		
	XCS SZA AMIBA ACT			
CMB	WMAP 2/3 WMAP 5 yr			
	Planck	Planck 4yr		

Roger Davies

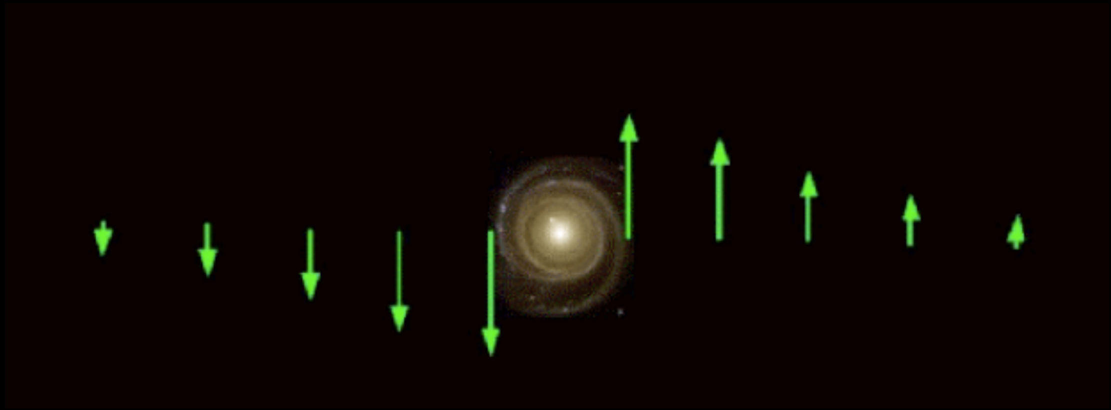
Dark Matter


The Dark Universe
Vera Rubin
v (km/s)
100
50
observed
expected
from stars
5
10
R (kpc)
M33 rotation curve

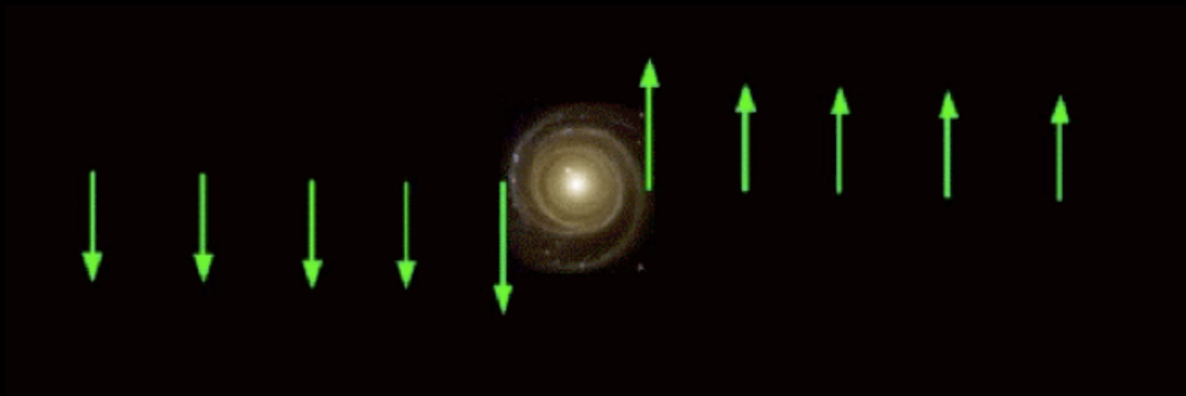


The Andromeda Galaxy (M31)

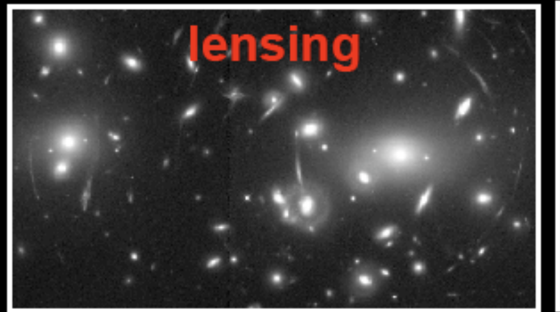
What we should see

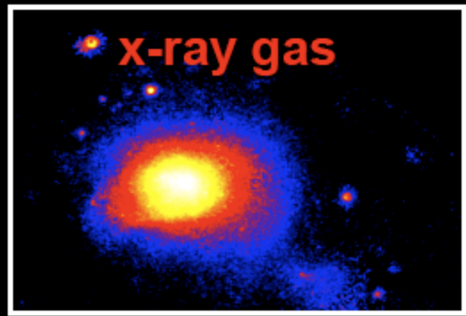


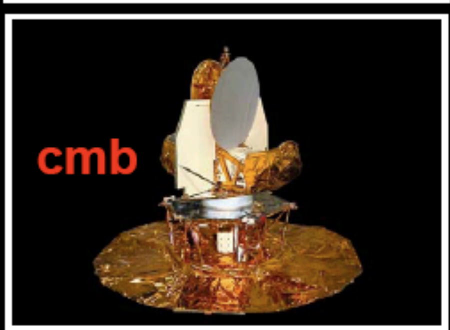
What we do see

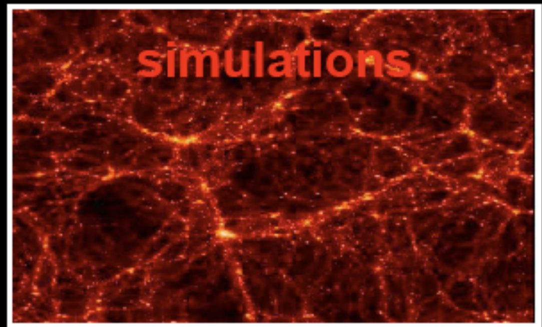


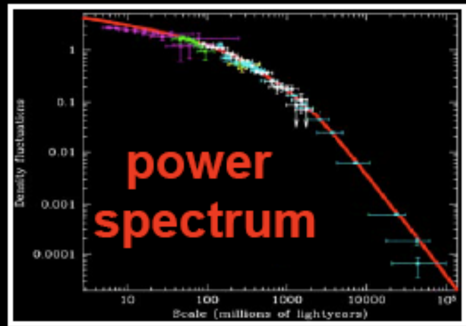
$\Omega_M \sim 0.3$





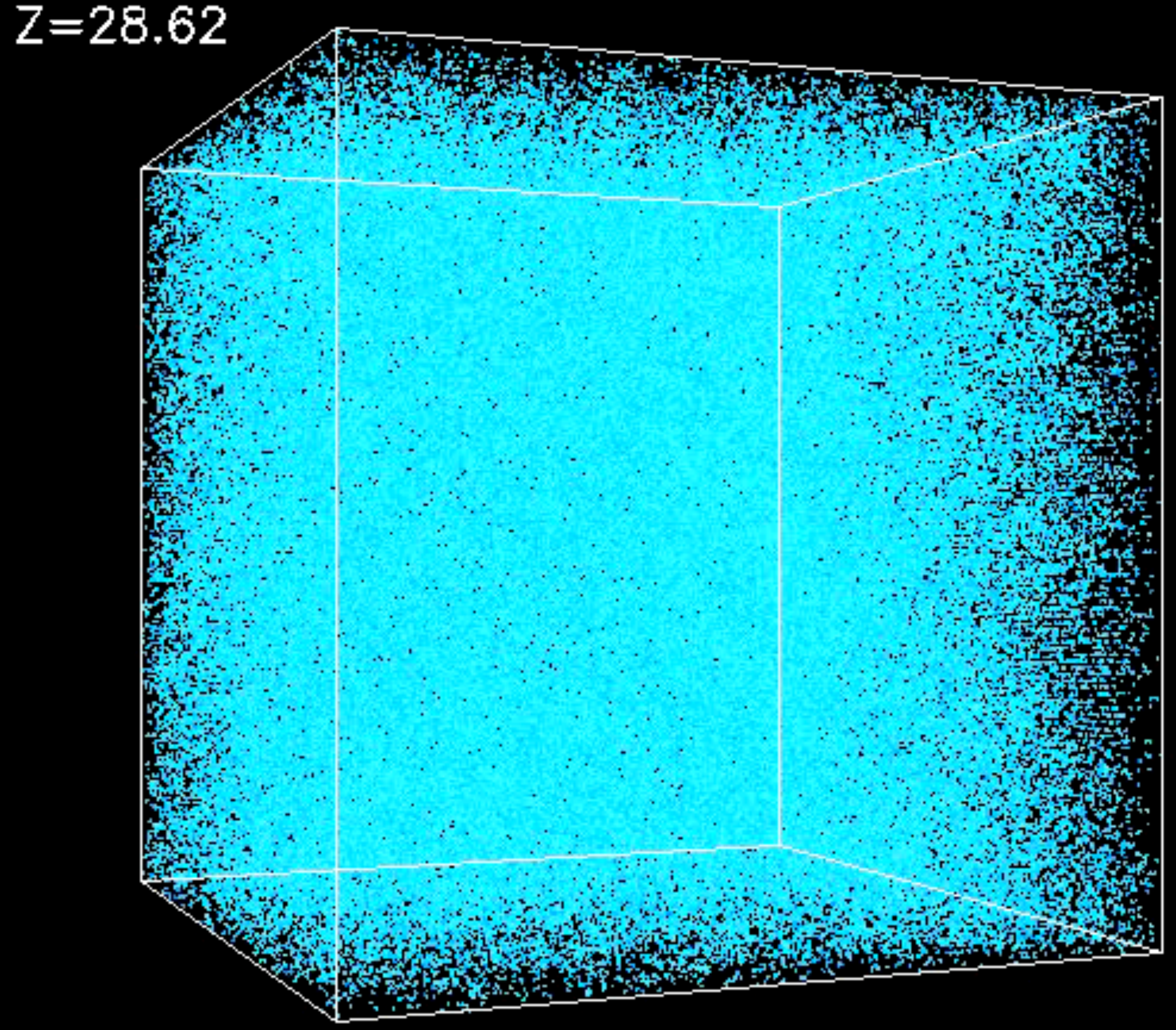






The cornerstones of structure formation

- Initial seeds provided by primordial inflation: spectrum of perturbations nearly flat and nearly gaussian
- Density perturbations grow because of the gravitational instability. They grow like the scale factor at the linear level
- In the CDM scenario, the first objects to collapse and form dark matter haloes are of low mass
- Merger trees: a halo that exists at a given time will have been constructed by the merging of smaller fragments over time
- When haloes merge, their cores survive as distinct subhaloes for some time. In group/cluster scale haloes, these will mark the locations of the galaxies



The Millenium Simulation Project:

http://www.mpa-garching.mpg.de/galform/virgo/millennium/

The structure in the Universe

Perturbing around the average energy density we may define the density contrast

$$\delta(\mathbf{x},t) \equiv \frac{\rho(\mathbf{x},t)-\overline{\rho}}{\overline{\rho}} = \int \frac{d^3k}{(2\pi)^3}\,\delta_{\mathbf{k}}(t)\,e^{-i\,\mathbf{k}\cdot\mathbf{x}}$$

The power spectrum is defined by

$$\langle \delta_{\mathbf{k}}\,\delta_{\mathbf{k}'}\rangle = (2\pi)^3\,P_\delta(k)\,\delta(\mathbf{k}-\mathbf{k}')$$

$$\boxed{\Delta_\delta(k) = \frac{k^3\,P_\delta(k),}{2\pi^2}, \;\; P_\delta = A\,k^n\,T(k)}$$

$$n \simeq 1,\; T(k) = \text{transfer function}$$

Matter perturbations

They can be found from the (00) Einstein equation
(Poisson equation)

For modes well inside the horizon
during the MD period, matter perturbations grow

$$\nabla^2\Phi = -4\pi G_N a^2 \delta\rho_m = -\frac{3}{2}H^2 a^2 \frac{\delta\rho_m}{\rho_m}$$

$$\frac{\delta\rho_m}{\rho_m} \propto (Ha)^{-2}\Phi \propto a \times a^0 = a$$

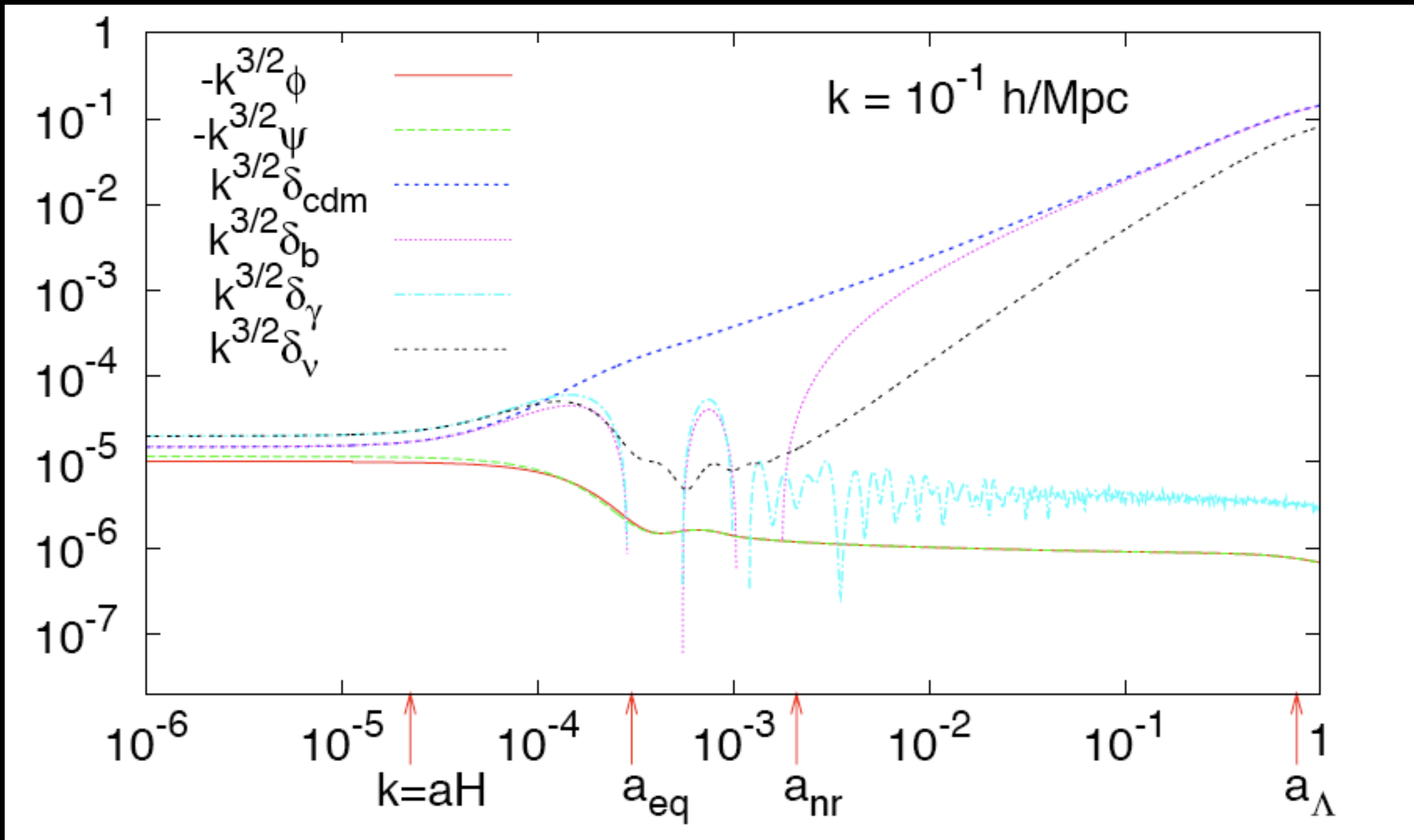
For modes well inside the horizon
during the RD period, matter perturbations are frozen

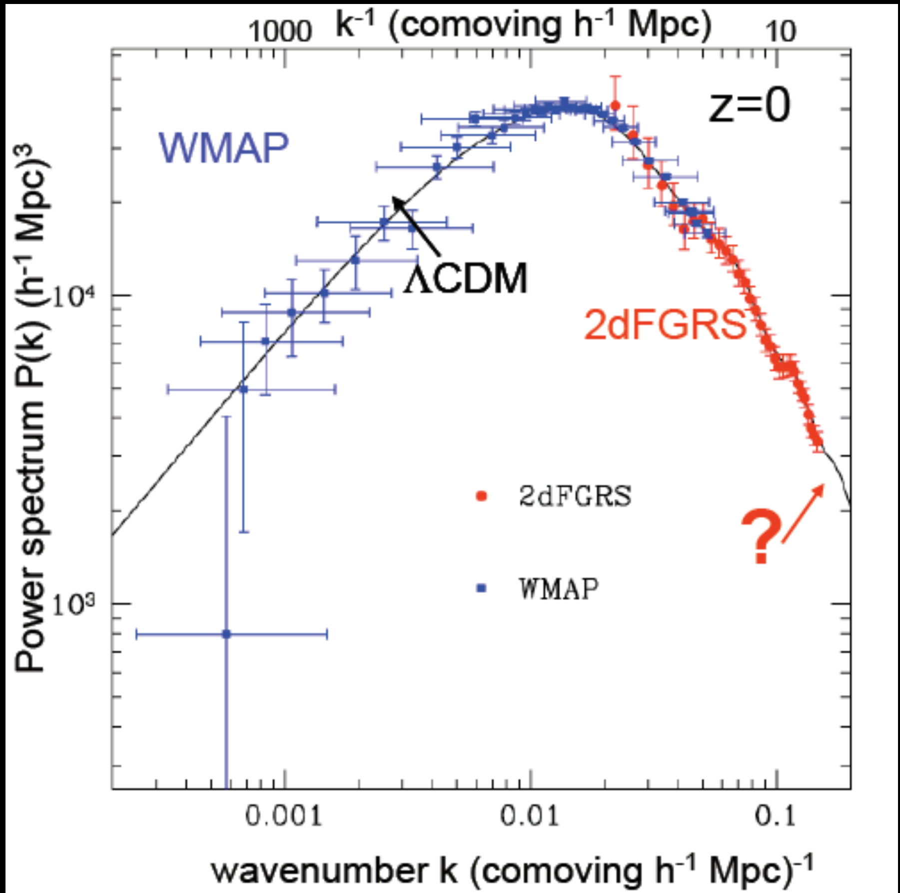
$$\nabla^2\Phi = -4\pi G_N a^2 \delta\rho_m = -\frac{3}{2}H^2 a^2 \frac{\rho_m}{\rho_\gamma}\frac{\delta\rho_m}{\rho_m}$$

$$\frac{\delta\rho_m}{\rho_m} \propto (\rho_\gamma/\rho_m)(Ha)^{-2}\Phi \propto a^{-1} \times a^2 \times a^{-1} = a^0$$

$$\ddot{\delta}_{\mathbf{k}} + 2H\dot{\delta}_{\mathbf{k}} = 4\pi G\,\overline{\rho}\,\delta_{\mathbf{k}} \Rightarrow \delta_{\mathbf{k}} \propto a$$

$$\delta_{\mathbf{k}} = -\frac{2}{3}\frac{k^2}{\Omega_m H^2}\,\Phi_{\mathbf{k}}$$

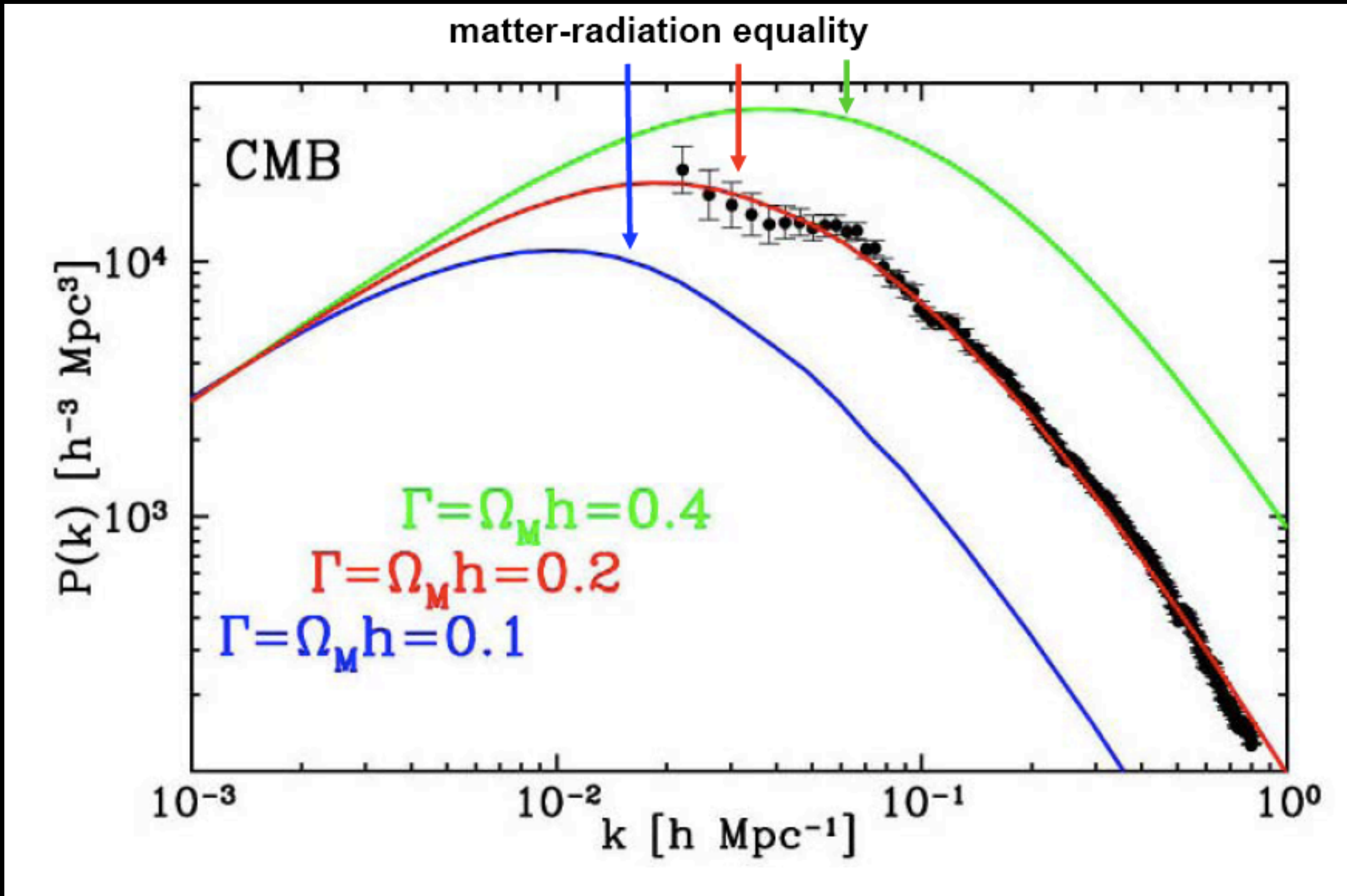


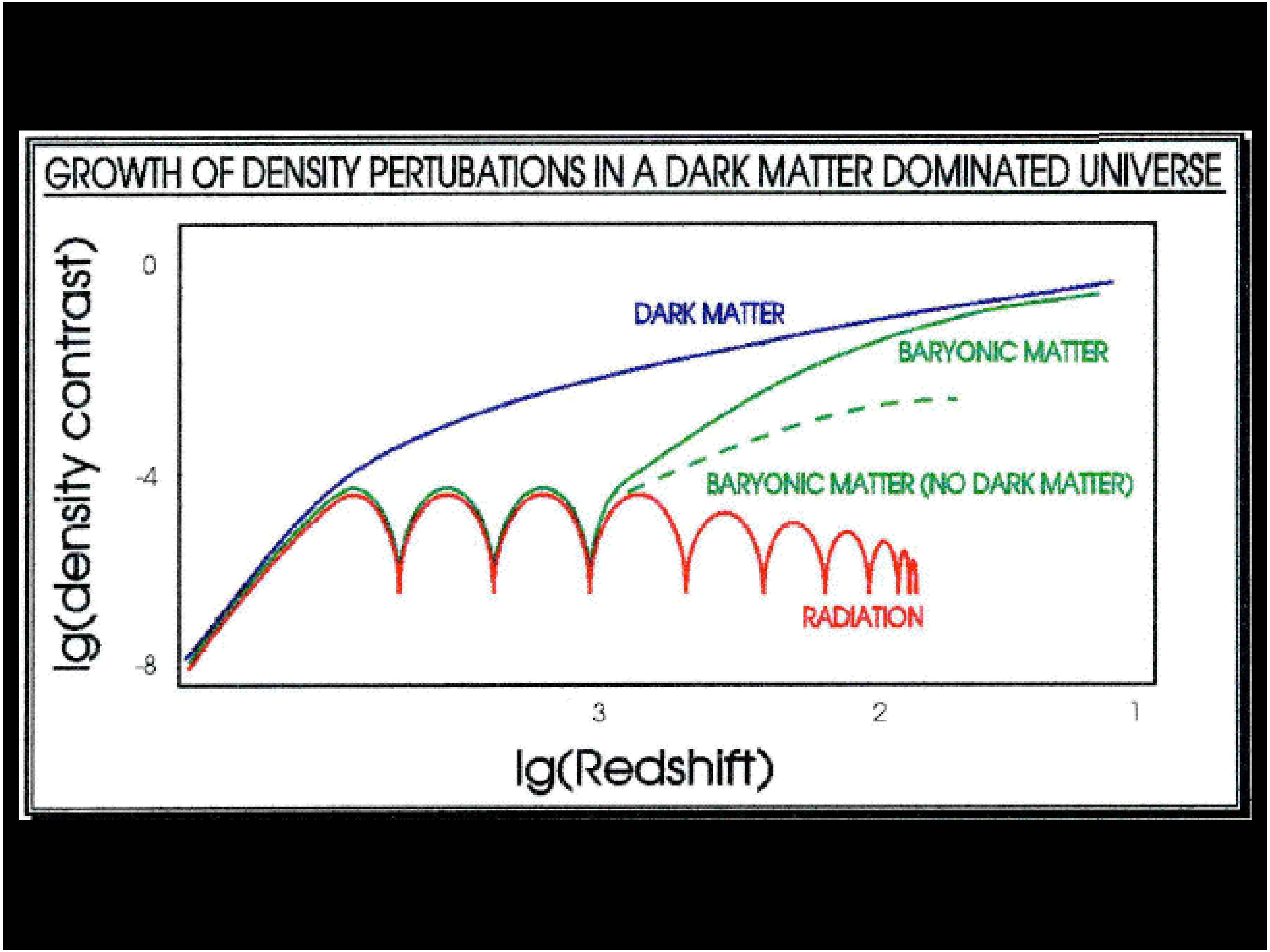


$$\delta_{\mathbf{k}} = -\frac{2}{3}\frac{k^2}{\Omega_m H^2}\Phi_{\mathbf{k}} \Rightarrow P_\delta \sim k^4 P_\Phi$$

$$P_\delta = \begin{cases} k & \text{as } P_\Phi \sim k^{-3} \\ k^{-3} & \text{as } P_\Phi \sim k^{-3} \times k^{-4} \end{cases}$$

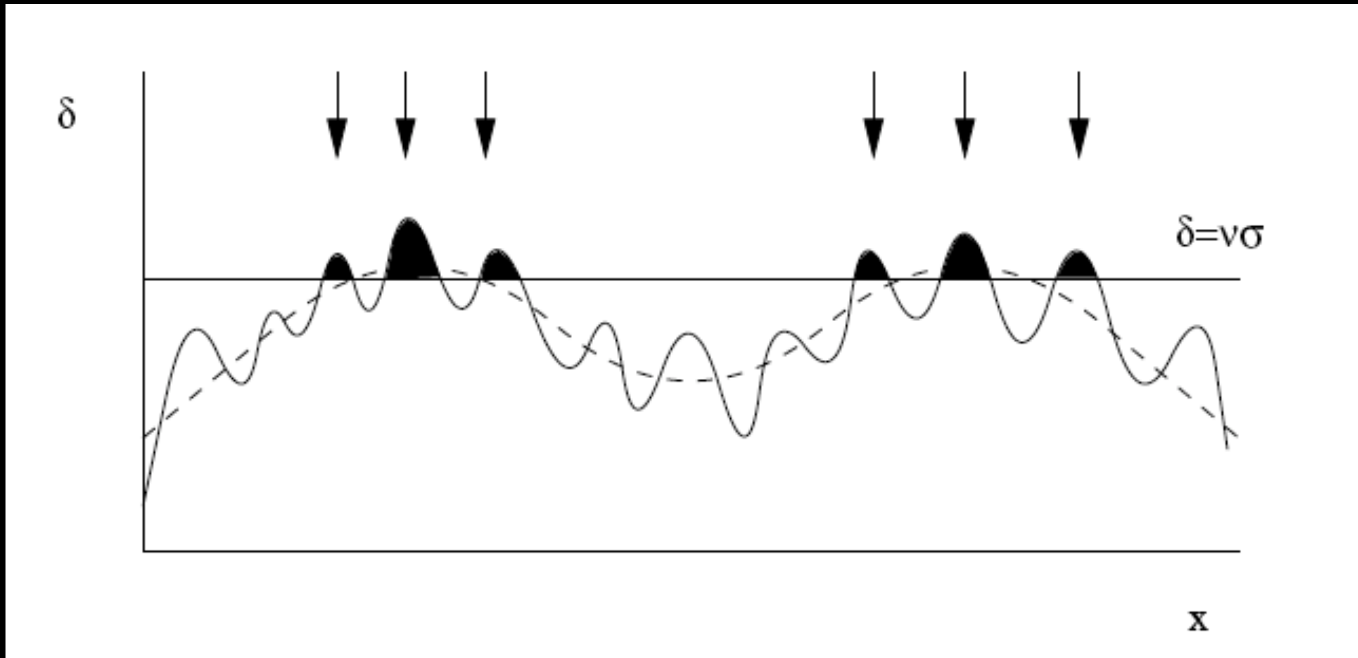
Power spectrum for CDM



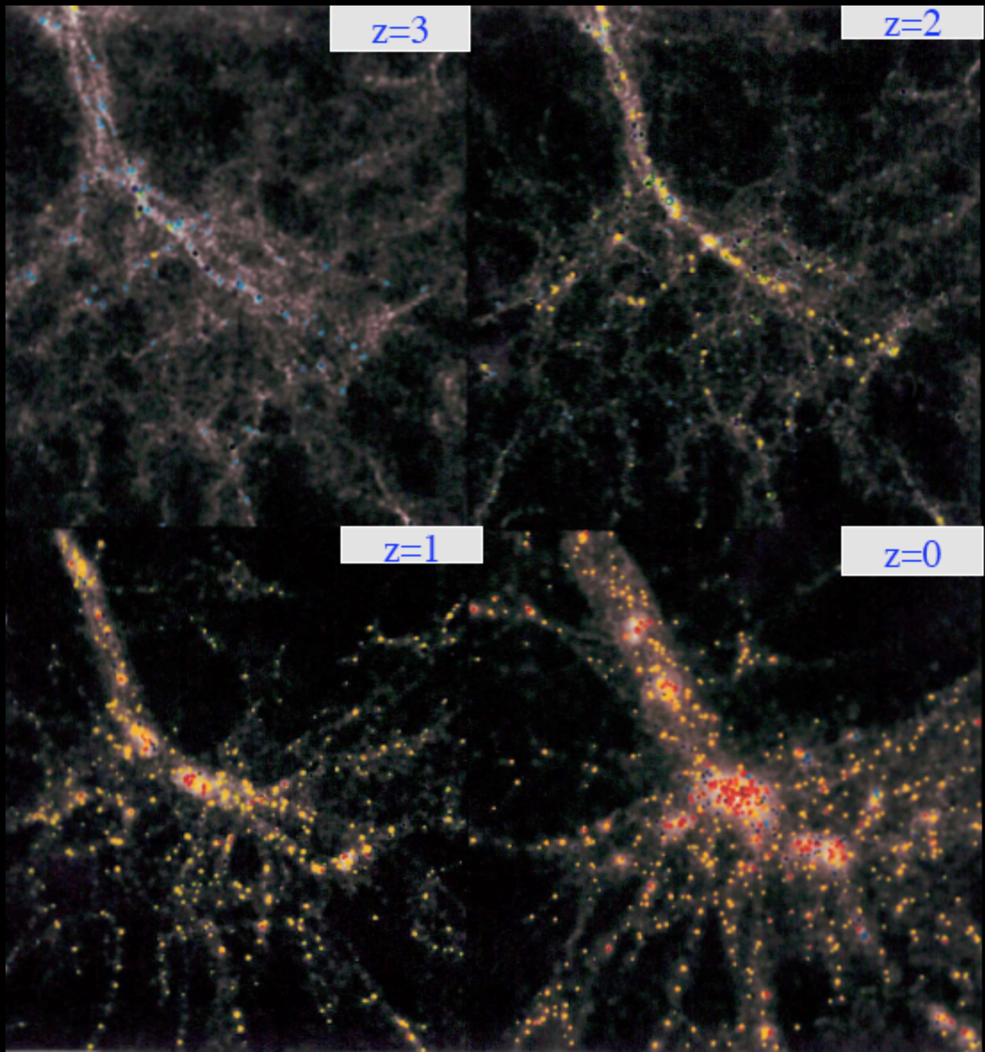


GROWTH OF DENSITY PERTUBATIONS IN A DARK MATTER DOMINATED UNIVERSE
lg(density contrast)
0
-4
-8
DARK MATTER
BARYONIC MATTER
BARYONIC MATTER (NO DARK MATTER)
RADIATION
3
2
1
lg(Redshift)

Bias



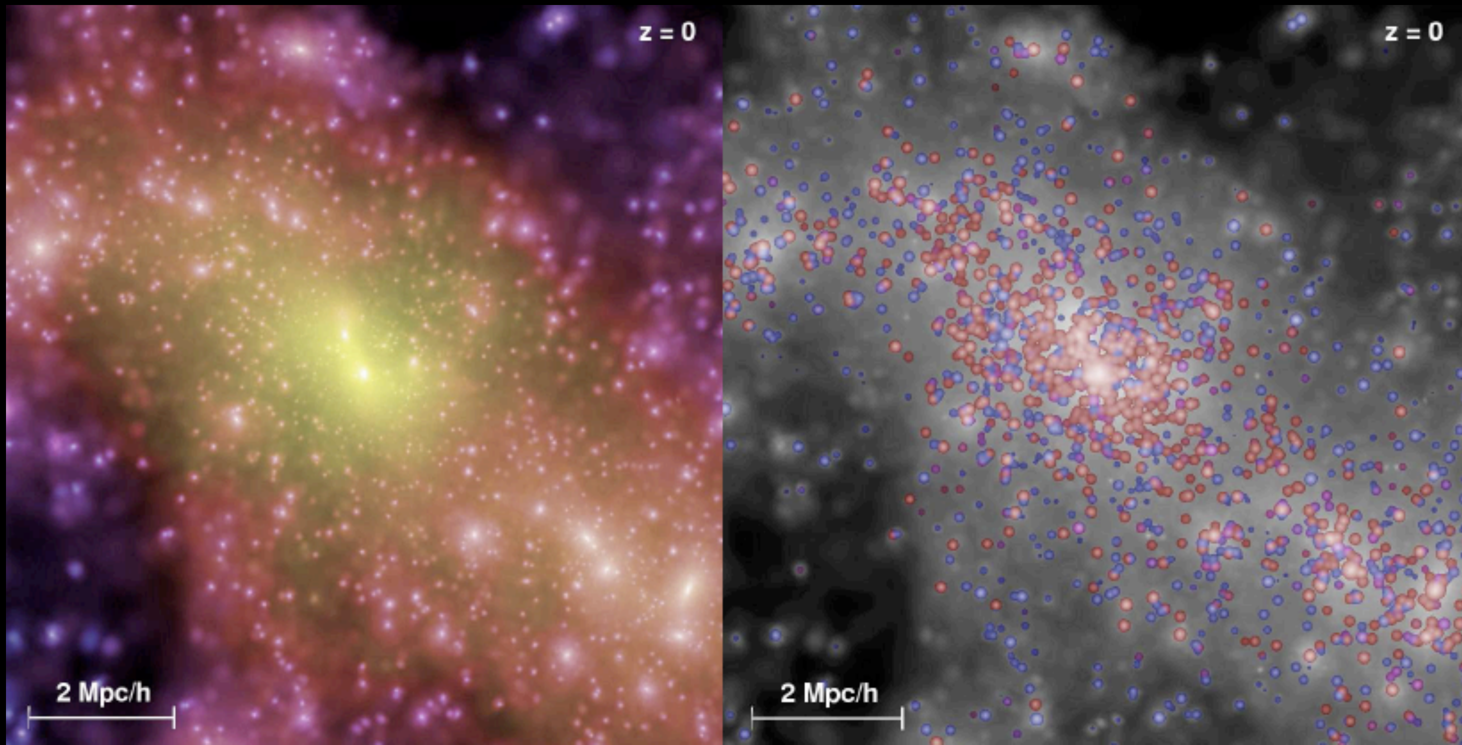
$$\left(\frac{\delta\rho}{\rho}\right)_{\text{galaxies}} = b\left(\frac{\delta\rho}{\rho}\right)_{\text{mass}}$$



Galaxies form early in
rare peaks

Dark Matter halo: $10^{14}\,M_\odot$

Dark Matter | Galaxies



The Millenium Simulation Project:

The dark matter halo mass function

A successfull theory of structure formation must be able to predict the number density of dark matter haloes as a function of their mass (systems with ~ 200 mean density)

$$\frac{dn}{dM} dM = \frac{\bar{\rho}}{M} \left| \frac{dF}{dM} \right| dM$$

$|dF/dM|$ is the fraction of volume occupied by virialized object of mass between M and $M + dM$

Informations on the DM halo mass function from

- Optical detection of their member galaxies
- X-ray emission from hot electrons confined by the gravitational potential wells
- SZ effect whereby hot electrons up-scatter the CMB photons leaving an apparent deficit of low-frequency CMB flux in their direction
- Weak lensing (clusters selected as peaks in a smoothed two-dimensional shear map)
- Systematic, not statistical uncertainties, provide the limiting factor in cosmological measurements: none of these technique measure mass directly, but some proxy quantity as galaxy counts, X-ray flux and/or temperature or the SZ decrement (e.g. X-ray selection requires the intra-cluster gas to be heated to a detectable level, bias effects; weak lensing techniques may miss a fraction of the real mass)

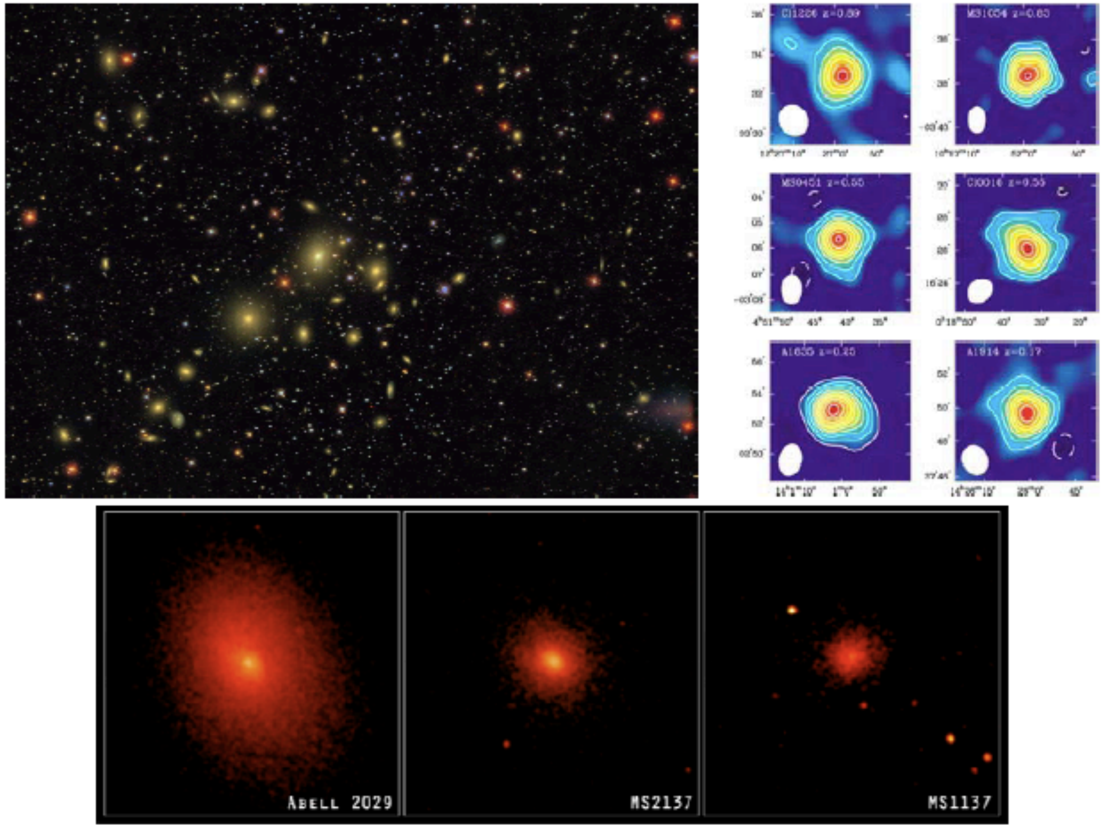
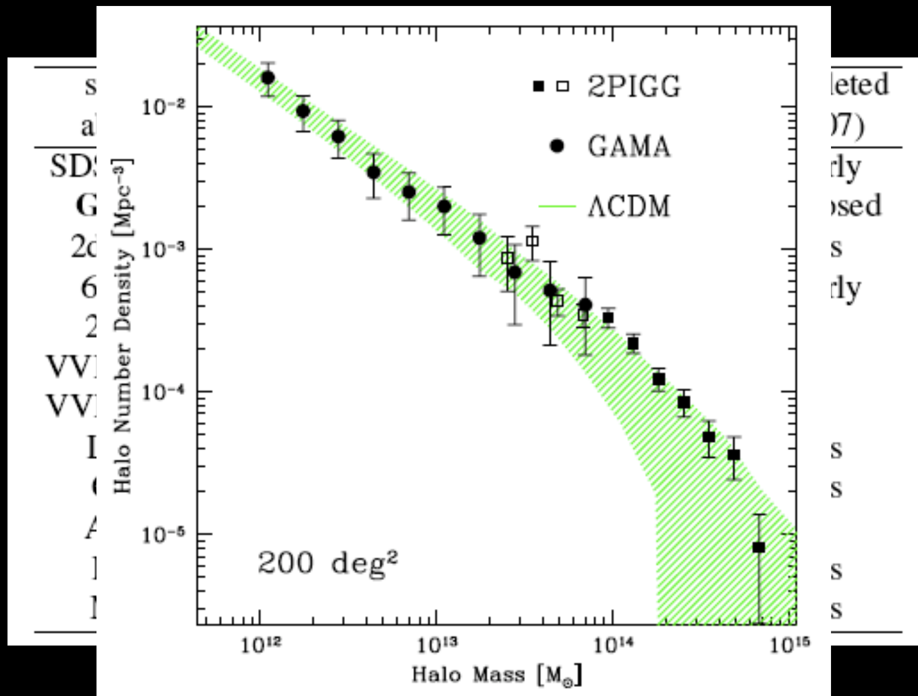


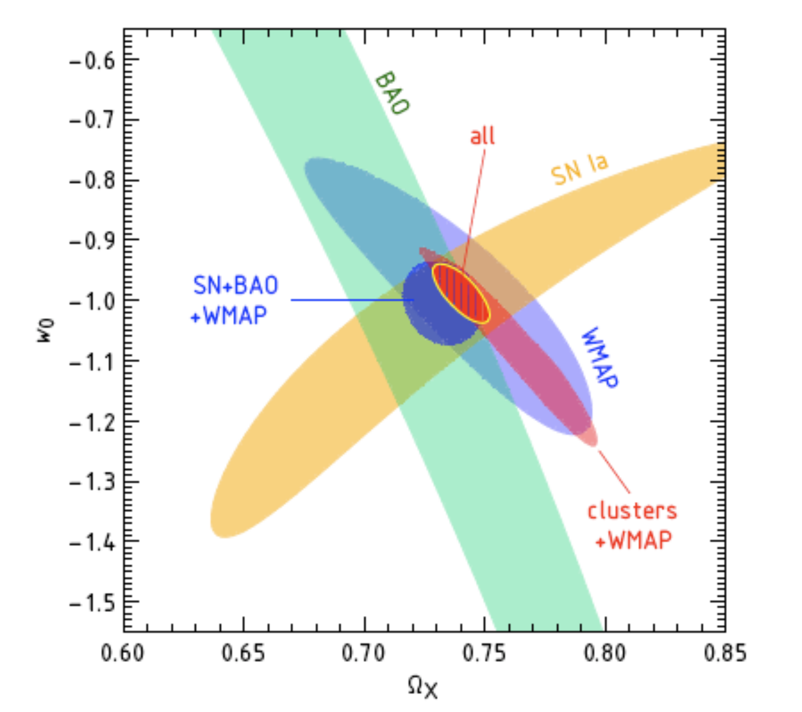
Fig. VI-5: Galaxy clusters as viewed in three different spectral regimes: top left, an optical view showing the concentration of yellowish member galaxies (SDSS); top right, Sunyaev, Zel'dovich flux decrements at 30 GHz (Carlstrom, et al. 2001); bottom, x-ray emission (Chandra Science Center). These images are not at a common scale.

- The DM halo mass function will be accurately tested by planned large-scale galaxy surveys, both ground (e.g. Large Synoptic Survay Telescope, Galaxy And Mass Assembly, volume comparable to horizon size) and satellite (e.g. the ESA EUCLID) based (optical, weak lensing, X-ray emission, SZ effect are complementary)



Why the halo mass function is so relevant ?

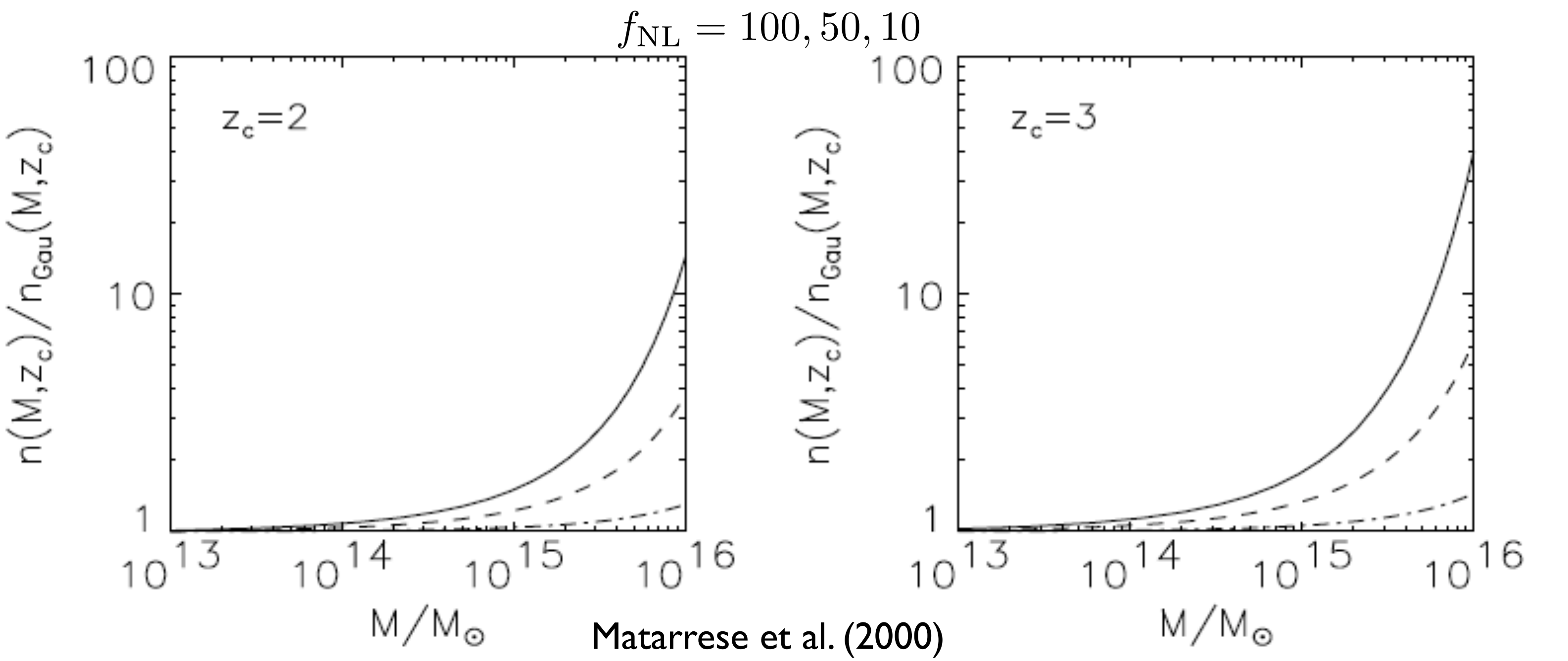
$dn/dV\,dz$ is exponentially sensitive to the Dark Energy through the growth function



X-ray cluster cosmology white paper, arXiv: 0903.5320

Why the halo mass function is so relevant ?

Rare events are an excellent probe of non-Gaussianity in the primordial power spectrum: $\Phi(x) = \Phi_g(x) + f_{\rm NL}\Phi_g^2$

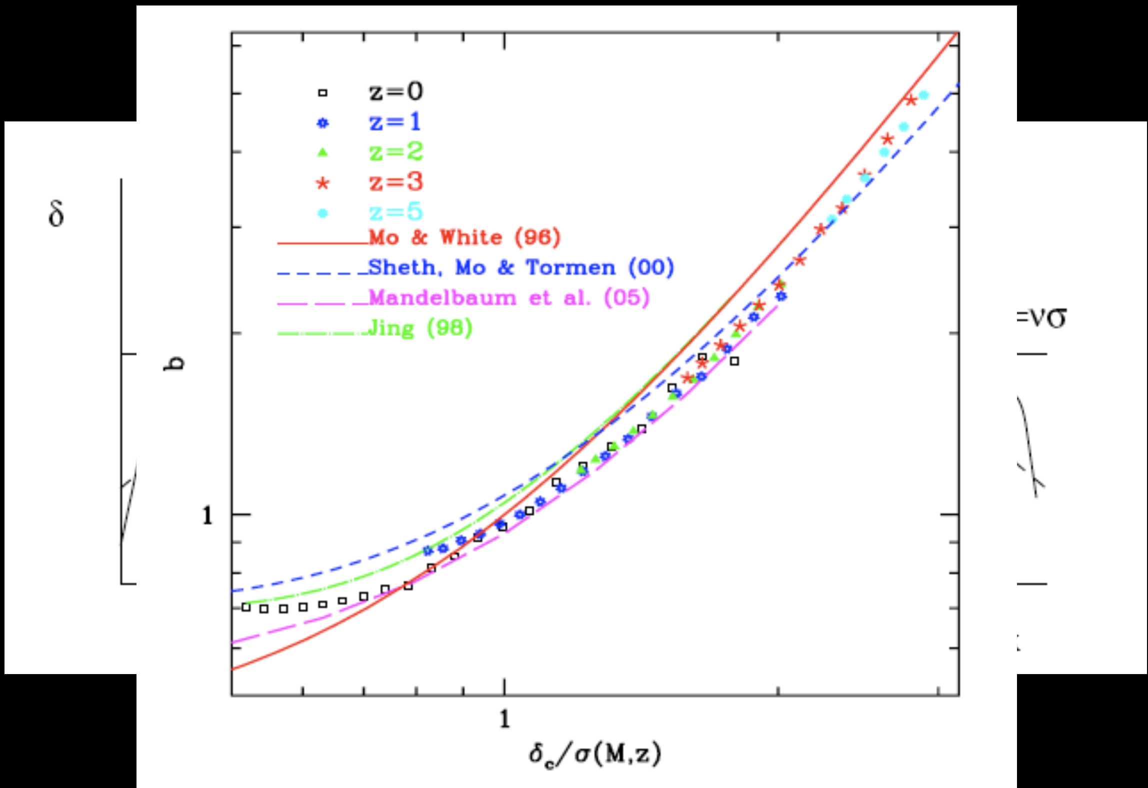


Matarrese et al. (2000)

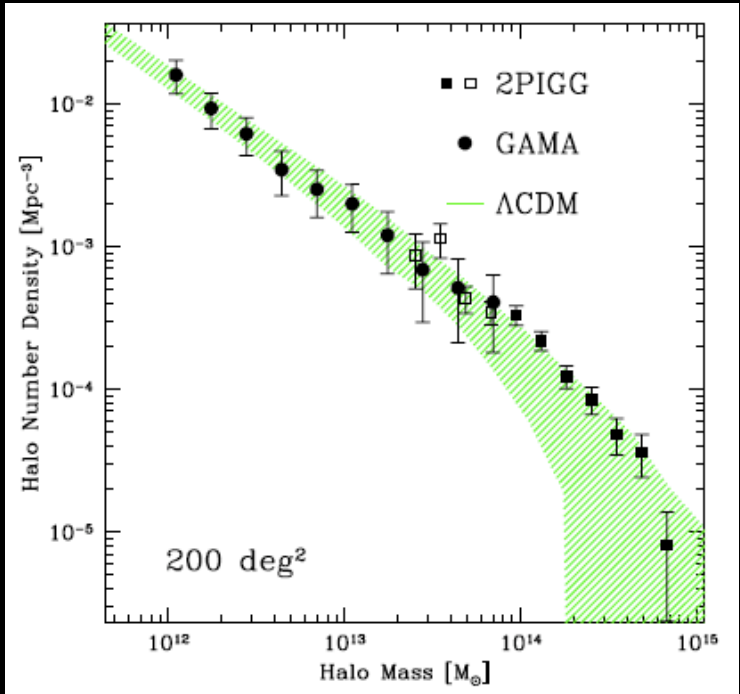
Cosmic Inflation Probe (*CIP*), a galaxy survey measuring 10 million galaxies at 3<z<6, would offer an opportunity to use this formula to constrain ***f_{NL}~5*** (note that the scale measured by CIP is smaller than that measured by CMB by a factor of ~10!)

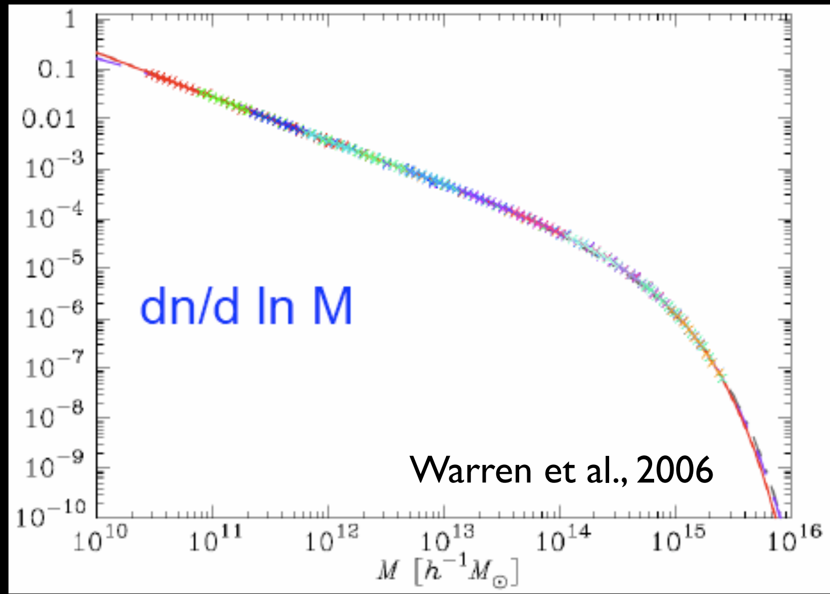
Why the halo mass function is so relevant

The High-peak bias model, based on the knowledge of the halo mass function, correctly predicts that high-mass haloes are positivey biased: $\left(\frac{\delta\rho}{\rho}\right)_{\rm galaxies} = b \left(\frac{\delta\rho}{\rho}\right)_{\rm mass}$



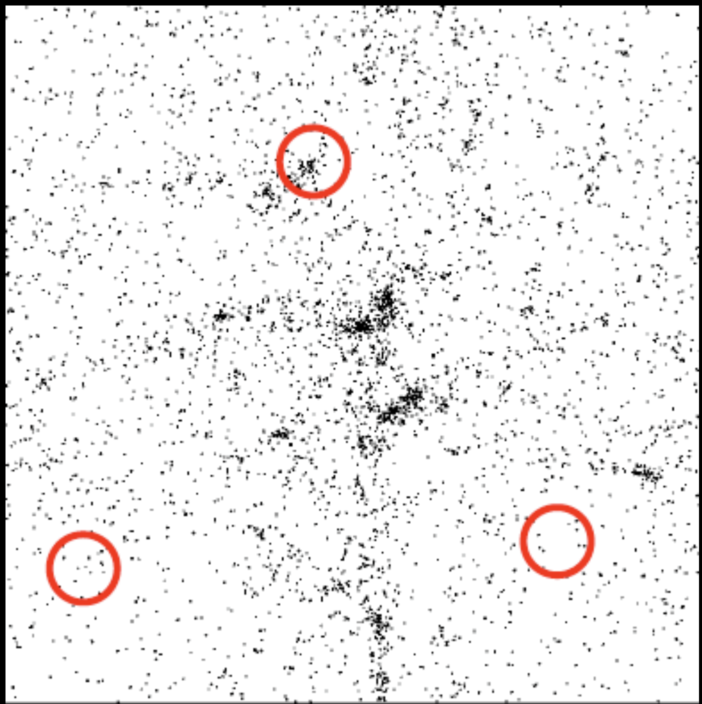
How dark matter mass is distributed





At present the knowledge of the halo mass function comes mainly from N-body simulations

The smoothing procedure



$$\delta(\mathbf{x}, R) = \int d^3x' \, W(|\mathbf{x} - \mathbf{x}'|, R)\, \delta(\mathbf{x}')$$

Smooth out the perturbation on a sphere of radius R

$$S \equiv \sigma^2(R) \equiv \langle \delta^2(\mathbf{x}, R) \rangle = \int_{-\infty}^{\infty} d\ln k \, \Delta_\delta^2(k) \, |W(k, R)|^2$$

Window function / filter

Top-hat in momentum space

$$W(k,R) = \theta(k_f - k),\ \ k_f = R^{-1}$$

One may not identify a well-defined mass

$$V = 12\pi R^3 \int_0^\infty dx \left(\frac{\sin x}{x} - \cos x \right)$$ is not defined

Window function / filter

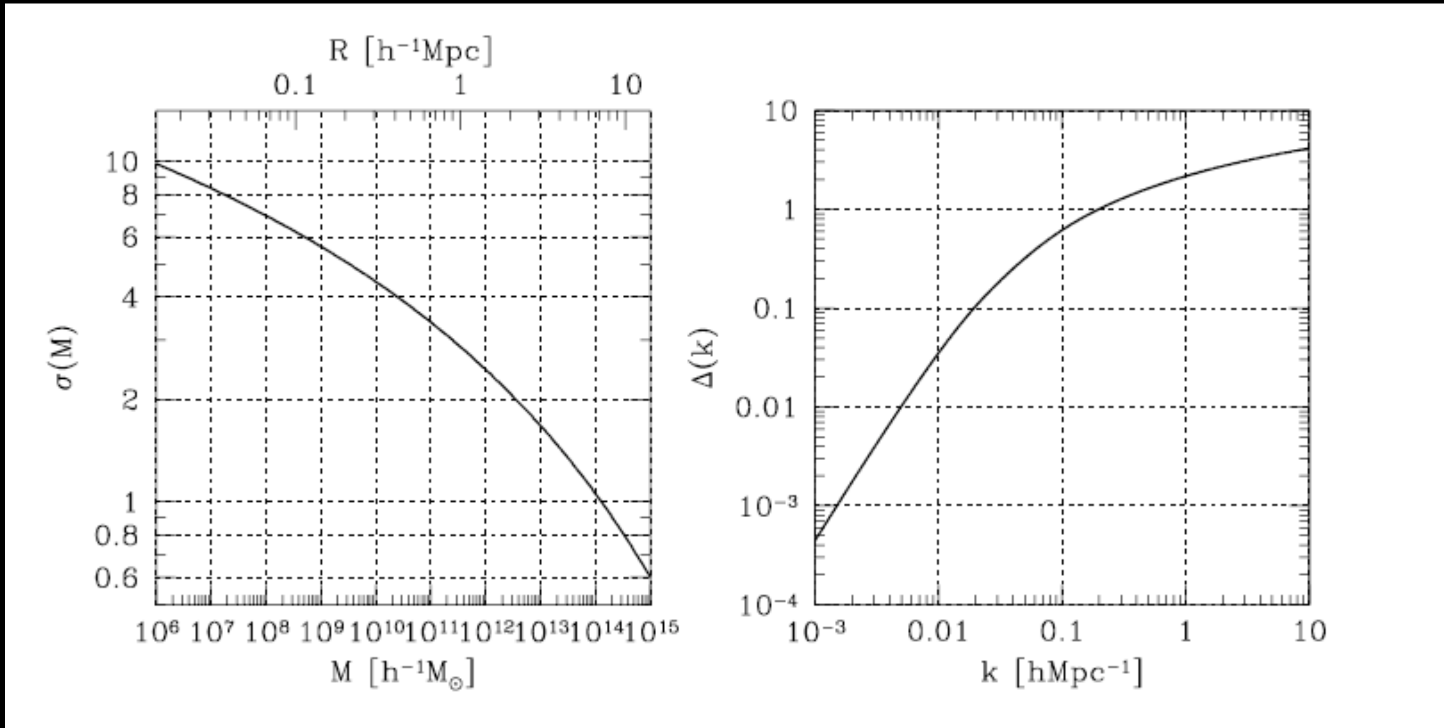
Top-hat in real space

$$W(\mathbf{x}, R) = \frac{3}{4\pi R^3}\,\theta(R - r)$$

$$W(k, R) = 3\,\frac{(\sin(kR) - kR\cos(kR))}{(kR)^3}$$

$$M = \overline{\rho}\,V,\ \ V = \frac{4\pi R^3}{3}$$

→ N-body simulations use this window function

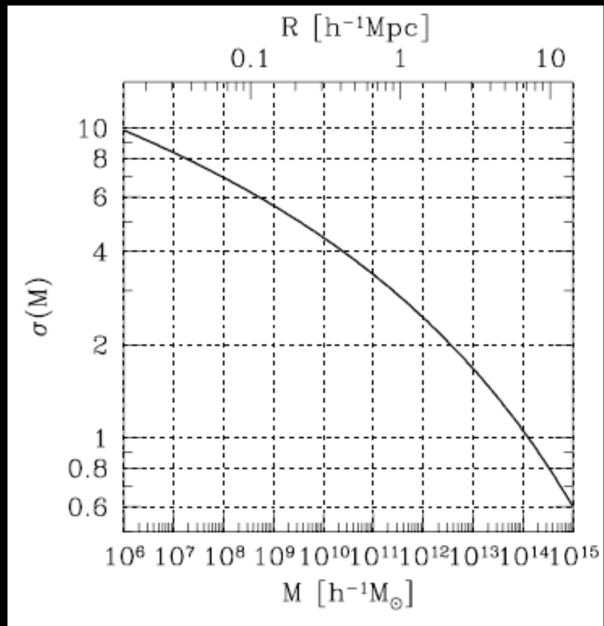


$$\frac{dn}{dM} = 2\,\frac{\overline{\rho}}{M^2}\,f(\sigma)\,\frac{d\ln\sigma^{-1}}{d\ln M}$$

$$S = \sigma^2(M),\;\; f(\sigma) = 2\,\sigma^2\,\frac{dF}{dS}$$

Dictionary

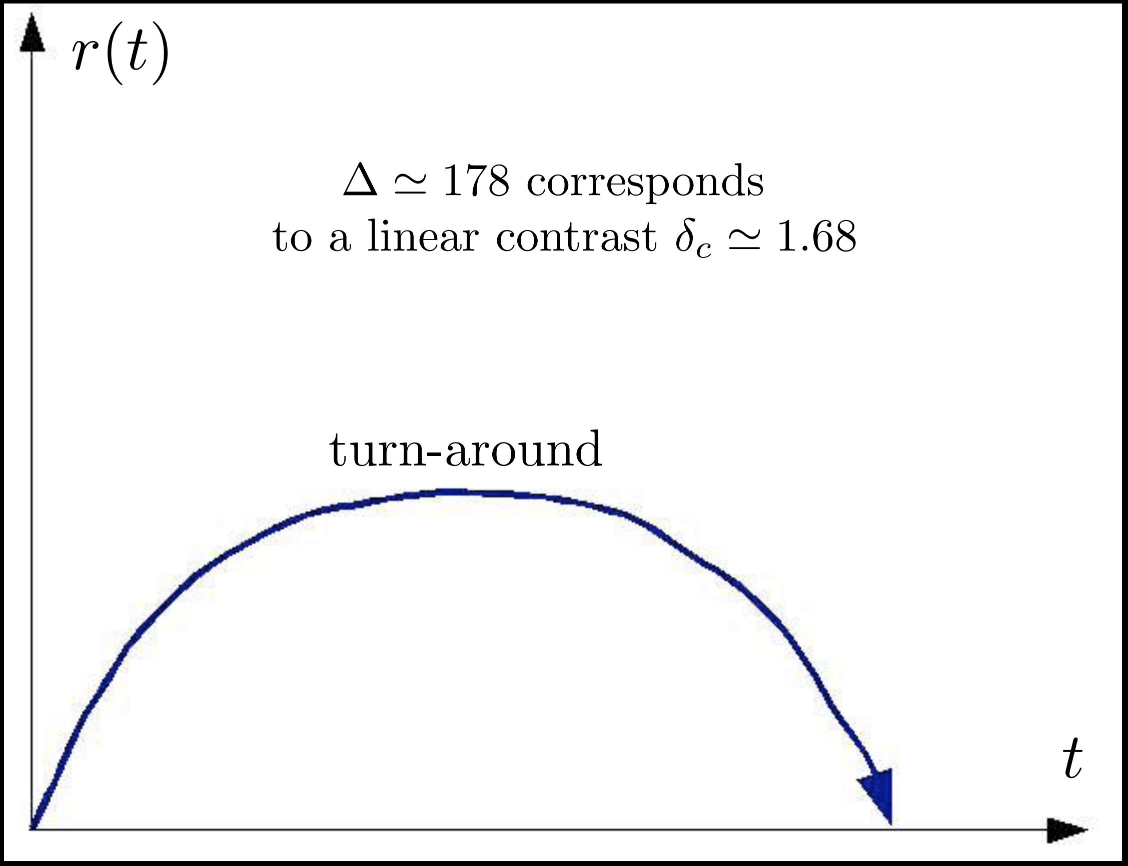
R ⟷ $M(R)$



S ⟷ $\sigma^2(R)$

The spherical collapse model

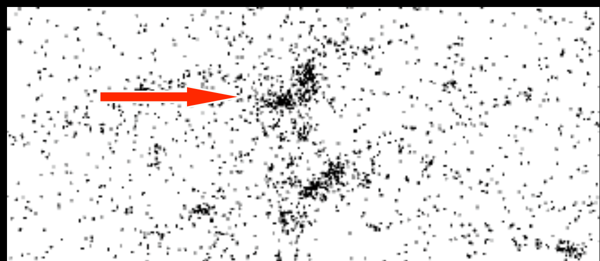
According to Birkhoff's theorem, a spherical density overdense perturbation departs from the background evolution and behaves in exactly the same way as part of of a closed Universe



Press-Schecther theory (1974)

It deals with the clustering problem
in the following way:

- Identify the preferential sites for halo formation in Lagrangian space: at any given cosmic time haloes will form preferably in those regions where the initial linear density field is larger than some critical value



Press-Schecther theory and the collapse barrier

- It is assumed that initial linear perturbations are gaussian distributed:

$$\Pi_{\rm PS} = \frac{1}{\sqrt{2\pi S}} e^{-\delta^2/(2S)}$$

- Virialized objects at a given radius form if the density contrast is larger than the collapse barrier δ_c

$$\begin{aligned} F_{\rm PS}(R) &= \int_{\delta_c}^{\infty} d\delta\, \Pi_{\rm PS}\left(\delta, S(R)\right) = \frac{1}{2}\,{\rm Erfc}\left(\frac{\nu(R)}{\sqrt{2}}\right) \\ \delta_c &= 1.68(1+z) \rightarrow 1.68\, D(z) \text{ for } \Lambda\text{CDM} \\ \nu &= \delta_c/\sigma(R) \end{aligned}$$

Cloud-in-cloud problem

In the hierarchical models, the variance $\sigma^2(R)$ diverges at small radii: all mass in the Universe must be finally contained in virlialized objects:

$$F(R=0)=1$$

instead

$$F_{\rm PS}(R=0)=1/2$$

The PS procedure misses the cases in which, one a given smoothing scale R , the smoothed density contrast $\delta(R)$ is below threshold, but still it happened to be above threshold at some scale $R' > R$ .The missing factor of two is put by hand

The excursion set method

Bond, Cole, Efstathiou, Kaiser (1991)

The smoothed density contrast performs a random walk

$$\delta(R) = \int \frac{d^3k}{(2\pi)^3}\,\delta_{\mathbf{k}}\,W(k,R)\,e^{-i\mathbf{k}\cdot\mathbf{x}}$$

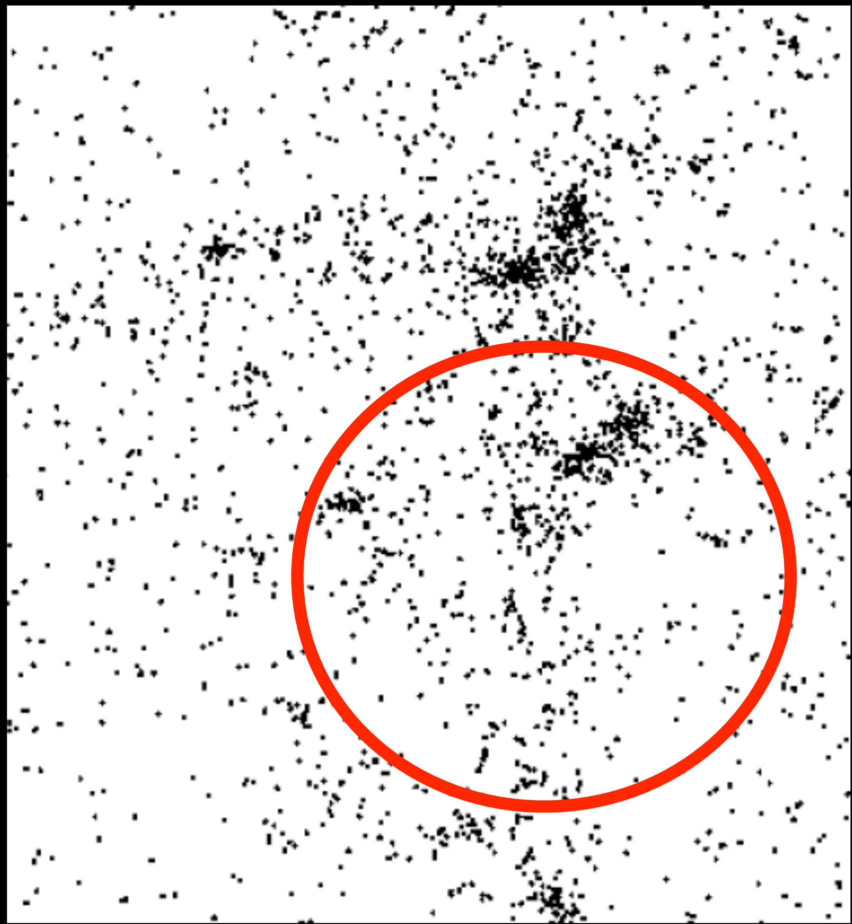
$$W(k,R) = \theta(R^{-1}-k)$$

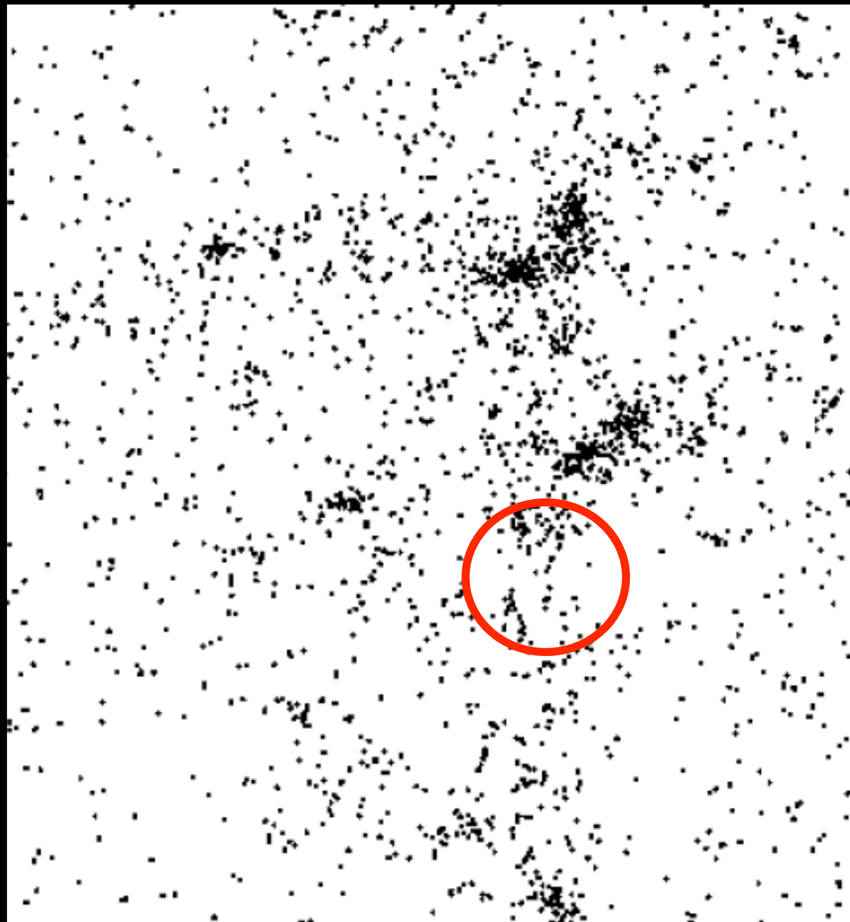
$$\frac{\partial\delta(R)}{\partial R} = \int \frac{d^3k}{(2\pi)^3}\;\delta_{\mathbf{k}}\,\frac{\partial W(k,R)}{\partial R}\,e^{-i\mathbf{k}\cdot\mathbf{x}}$$

$$= R^2 \int \frac{d^3k}{(2\pi)^3}\;\delta_{\mathbf{k}}\,\delta_D(R-k^{-1})\,e^{-i\mathbf{k}\cdot\mathbf{x}}$$

$$\langle \frac{\partial\delta(R_1)}{\partial R_1}\frac{\partial\delta(R_2)}{\partial R_2}\rangle = f(R_1)\delta_D(R_1-R_2)$$

$$\langle \delta_{\mathbf{k}}\,\delta_{\mathbf{k}'}\rangle = (2\pi)^3\,P_\delta(k)\,\delta(\mathbf{k}-\mathbf{k}')$$





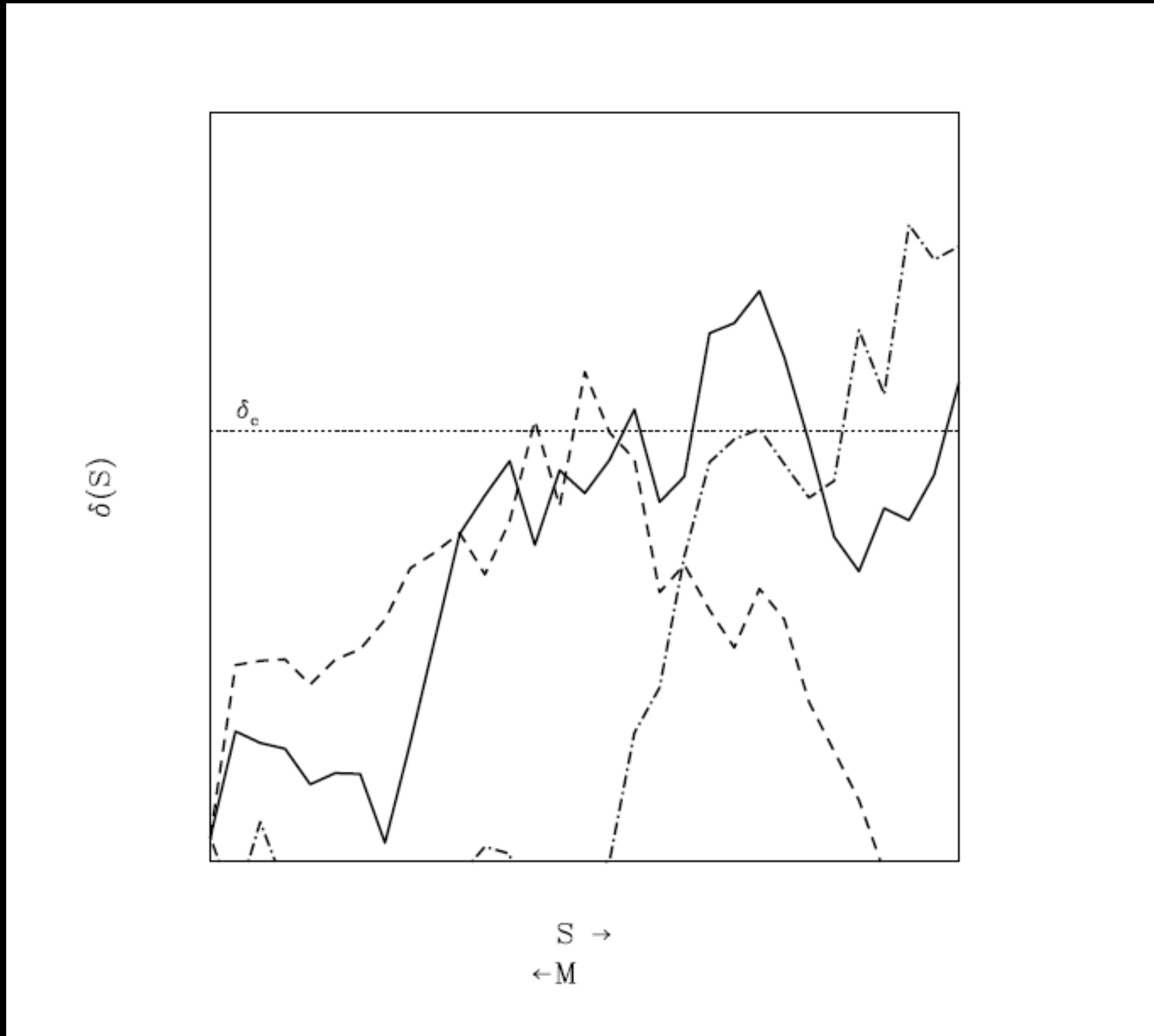
The smoothed density contrast performs a random walk as a function of the pseudo-time S

MARKOVIAN DYNAMICS & NO MEMORY EFFECTS:
the conditional probability depends on the latest step

$$\frac{\partial\delta(S)}{\partial S} = \eta(S)$$

$$\langle\eta(S_1)\eta(S_2)\rangle = \delta(S_1 - S_2)$$

$$S \equiv \sigma^2(R) \equiv \langle\delta^2(\mathbf{x}, R)\rangle = \int_{-\infty}^{\infty} d\ln k\, \Delta_\delta^2(k)\, |W(k,R)|^2$$



The normalization of the PS theory is not correct because it does not discard multiple crossings

The problem of finding the probability of halo formation can be matched into the so-called

FIRST-PASSAGE TIME PROBLEM

find the probability that a particle subject to a random walk passes for the first time through a given point

Very well-known problem for markovian dynamics;
application in chemical kinetics, biology, etc.
(for a textbook, see Redner, 2001)

A markovian random walk
with diffusion coefficient D:

$$\langle \delta^2(S) \rangle = D\,S$$

satisfies a Fokker-Planck (diffusion) equation

$$\frac{\partial \Pi}{\partial S} = \frac{D}{2}\,\frac{\partial^2 \Pi}{\partial \delta^2}$$

with boundary conditions:

$$\begin{aligned} \delta(S=0) &= \delta_D(\delta) \\ \Pi(\delta_c, S) &= 0 \ \text{(absorbing barrier)} \end{aligned}$$

The probability is given by

$$\Pi(\delta, S) = \frac{1}{\sqrt{2\pi S}}\left(e^{-\delta^2/(2DS)} - e^{-(2\delta_c-\delta)^2/(2DS)}\right)$$

The first-passage time probability is inferred from the survival probability

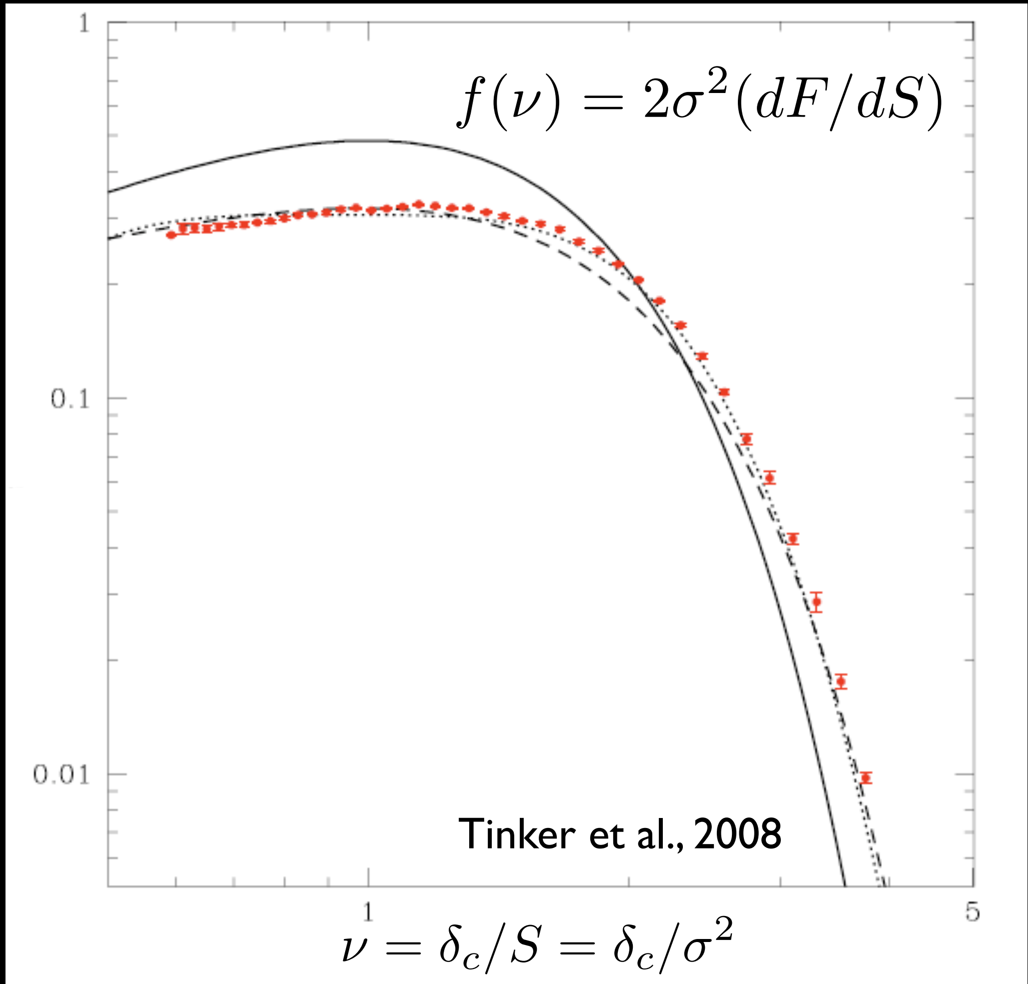
$$\int_{-\infty}^{\delta_c} d\delta\, \Pi(\delta, S) = 1 - F(S)$$

$$\Downarrow$$

$D = 1$

$$\frac{dF}{dS} = -\int_{-\infty}^{\delta_c} d\delta\, \frac{\partial \Pi}{\partial S} = \frac{2}{\sqrt{2\pi}S^{3/2}}\, e^{-\delta_c^2/(2S)}$$

The PS prediction is recovered with the missing factor of two



At large masses, the PS theory underestimates the dark matter halo mass function by a factor ~ 10;
at small halo masses it overestimates it by a factor ~ 2

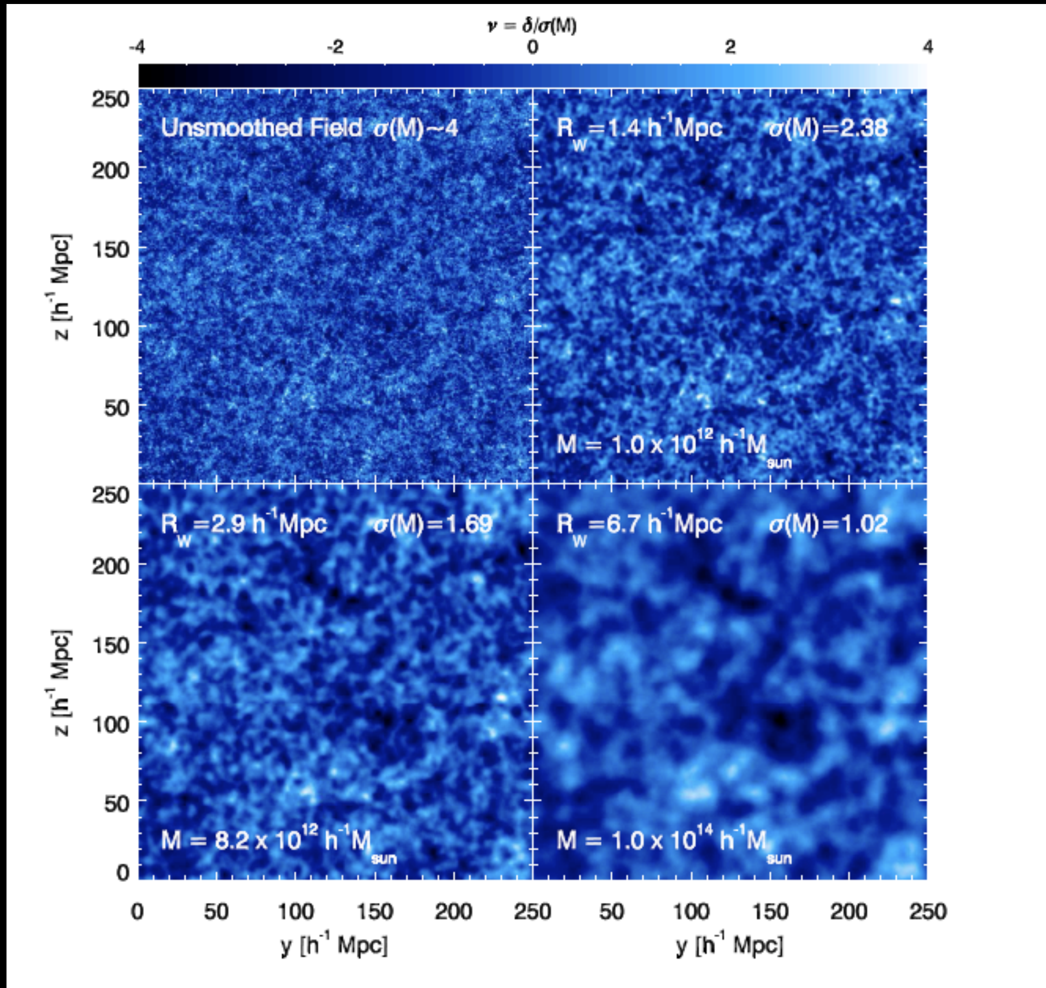
The diffusing barrier

The collapse is not spherical

In fact, the formation of dark matter haloes does not take place through a spherical collapse, but through an ellipsoidal collapse along each of the principal ellipsoidal axes under the action of external tides

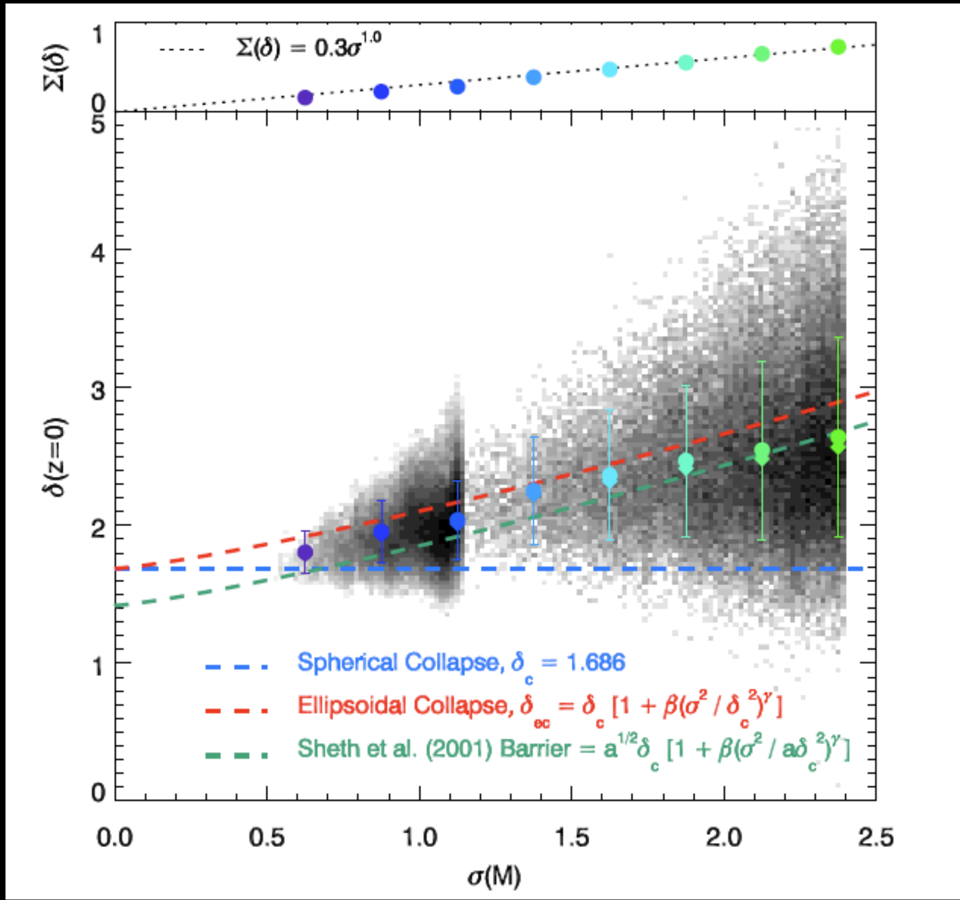
$$\nabla_i \nabla_j \Phi \Rightarrow \{\lambda_i\} \ (i = 1, 2, 3) \text{ such that } \delta = (\lambda_1 + \lambda_2 + \lambda_3)$$

The collapse barrier must be fuzzy to encode the randomness of the initial conditions



Robertson et al., 2009

For each halo identified, the center-of-mass of the halo particles is computed from their positions in the linear density field at z ~ 100 and use the window-smoothed field to compute the overdensity within the lagrangian radius R about this location. This overdensity is then linearly extrapolated to z=0



Robertson et al., 2009

The distribution of the smoothed linear overdensity is approximately log-normal in shape with a width

$$\Sigma_B \simeq 0.3\,\sigma(M)$$

The scatter in the collapse barrier reflects the intrinsic scatter in the linear overdensity of collapsed regions introduced by the smoothing process

$$\begin{aligned} \langle (B - \langle B \rangle)^2 \rangle^{1/2} &= \left(e^{\Sigma_B^2} - 1 \right) \langle B \rangle \\ &\simeq \Sigma_B \langle B \rangle \\ &\simeq 0.3\, \delta_c\, \sigma(M) = 0.3\, \delta_c \sqrt{S} \end{aligned}$$

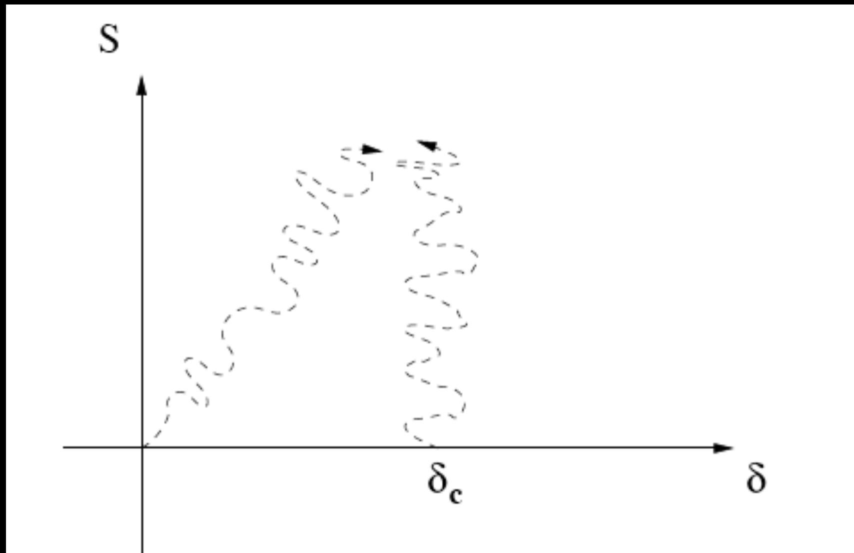
The collapse barrier moves stochastically with a diffusion coefficient

$$D_B \simeq (0.3\, \delta_c)^2 \simeq 0.25$$

It encodes in an effective way the properties of the ellipsoidal collapse model (like the shear)

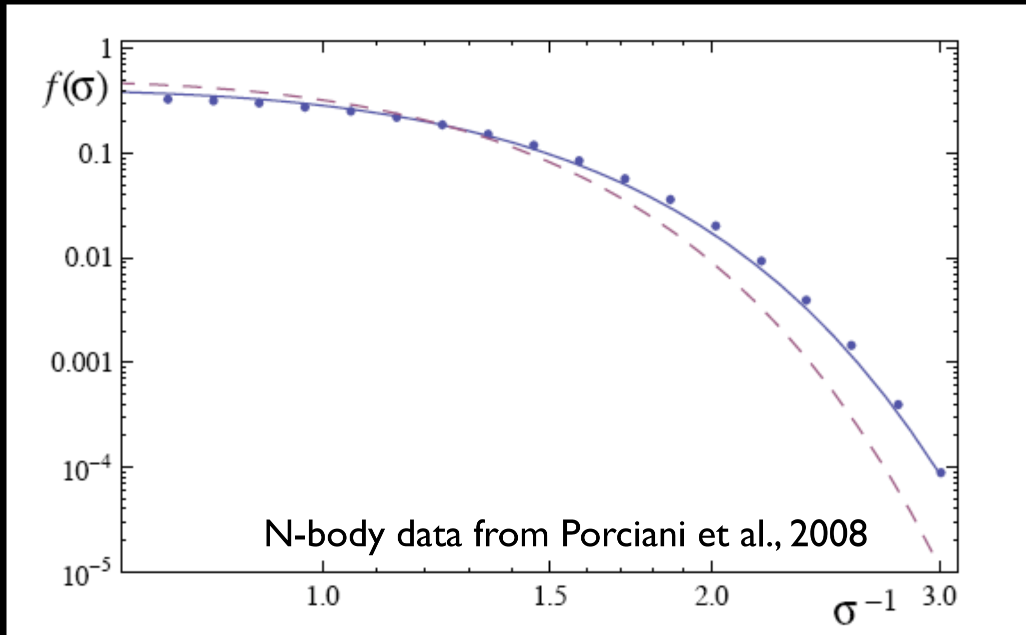
M. Maggiore, C. Porciani, R. Sheth and A.R., in prep.

The first-passage time problem becomes the well-known problem of the ``diffusing cliff”



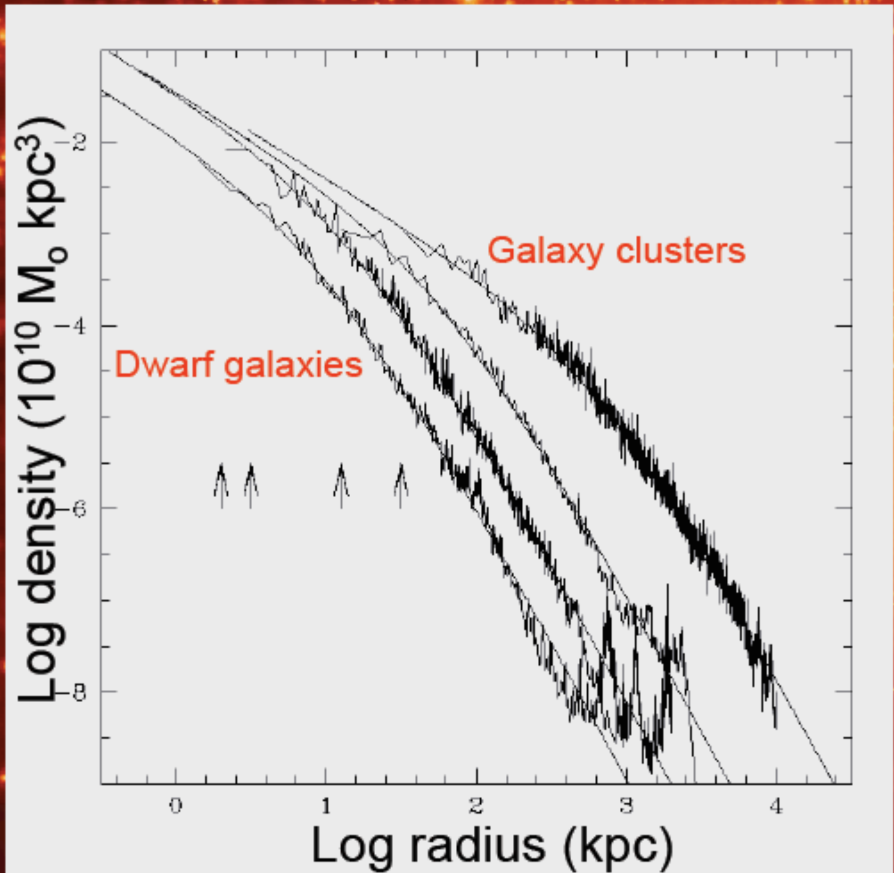
The first-passage time problem of two particles with diffusion coefficients $D = 1$ and $D_B = 0.25$ is mapped into a one-degree problem of a stochastic particle with effective coefficient

$$D_{\text{eff}} = 1 + D_B = 1.25$$



f(σ)
1
0.1
0.01
0.001
10^-4
10^-5
1.0
1.5
2.0
3.0
σ^-1
N-body data from Porciani et al., 2008

The Density Profile of Cold Dark Matter Halos



Halo density profiles are independent of halo mass & cosmological parameters

There is no obvious density plateau or `core' near the centre.

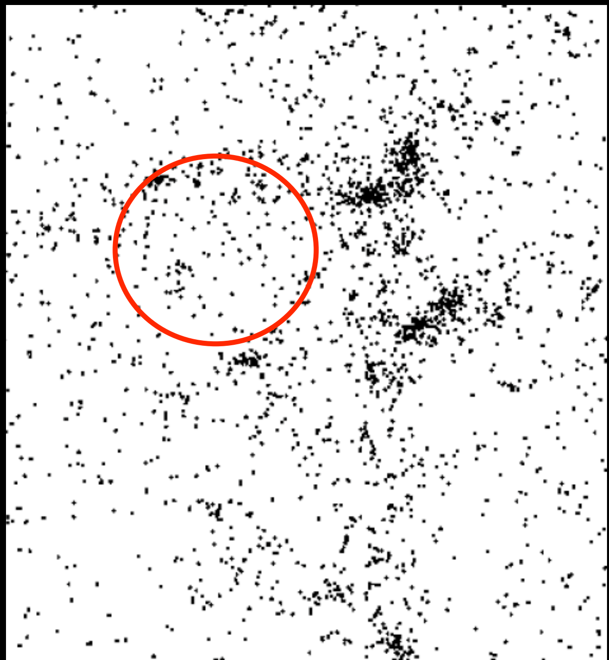
(Navarro, Frenk & White '97)

$$\frac{\rho(r)}{\rho_{crit}} = \frac{\delta_c}{(r/r_s)(1+r/r_s)^2}$$

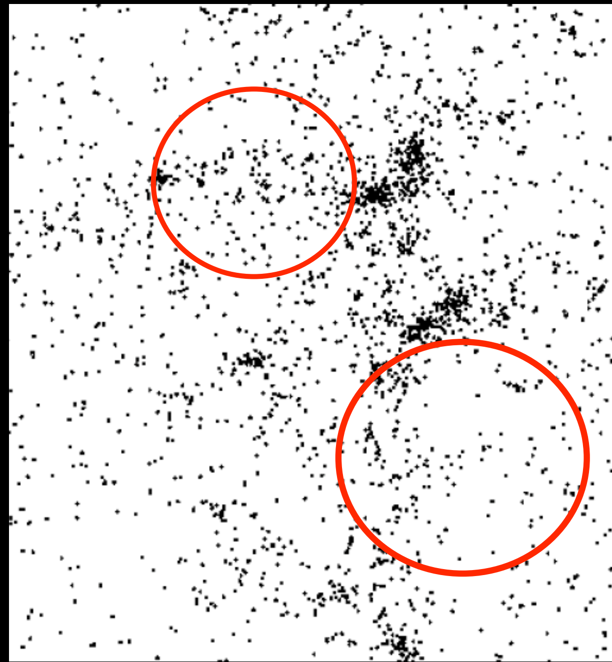
More massive halos and halos that form earlier have higher densities (bigger δ)

C. Frenk, ENTAap DM meeting, CERN, Feb. 2009

The Halo Model



1-halo



2-haloes

$$
\begin{aligned}
P(k) &= P_{1\mathrm{h}}(k) + P_{2\mathrm{h}}(k) \\
P_{1\mathrm{h}}(k) &= \int dM \frac{dn}{dM} \left[R^3 \bar{\delta} \rho(kR) \right]^2 \\
P_{2\mathrm{h}}(k) &= \left[\int dM \frac{dn}{dM} R^3 \bar{\delta} \rho(kR) b(M) \right]^2 P_{\mathrm{lin}}(k)
\end{aligned}
$$

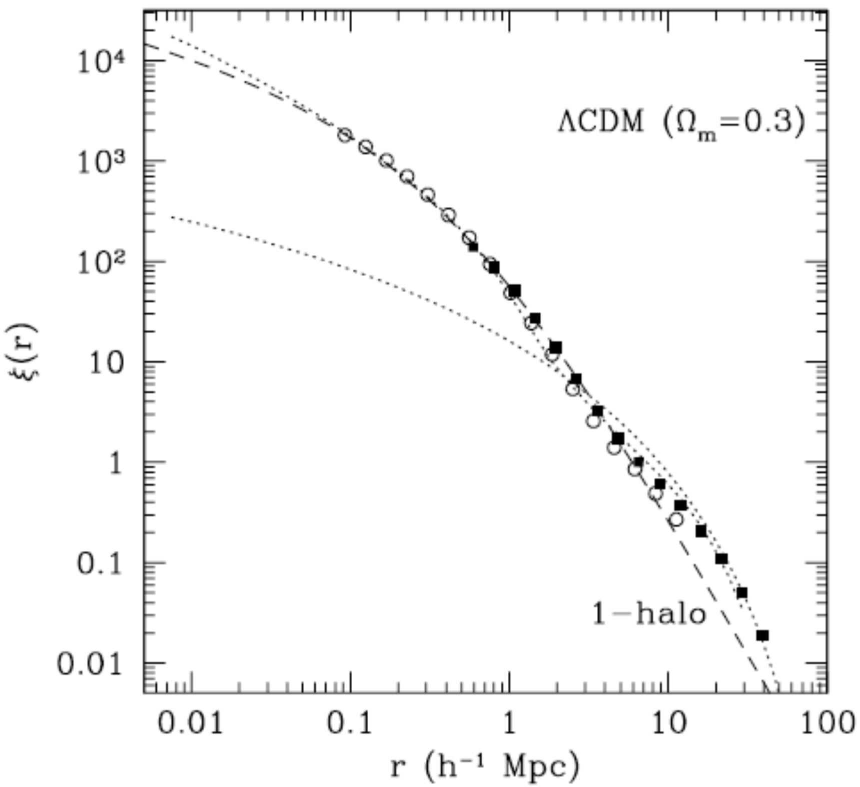
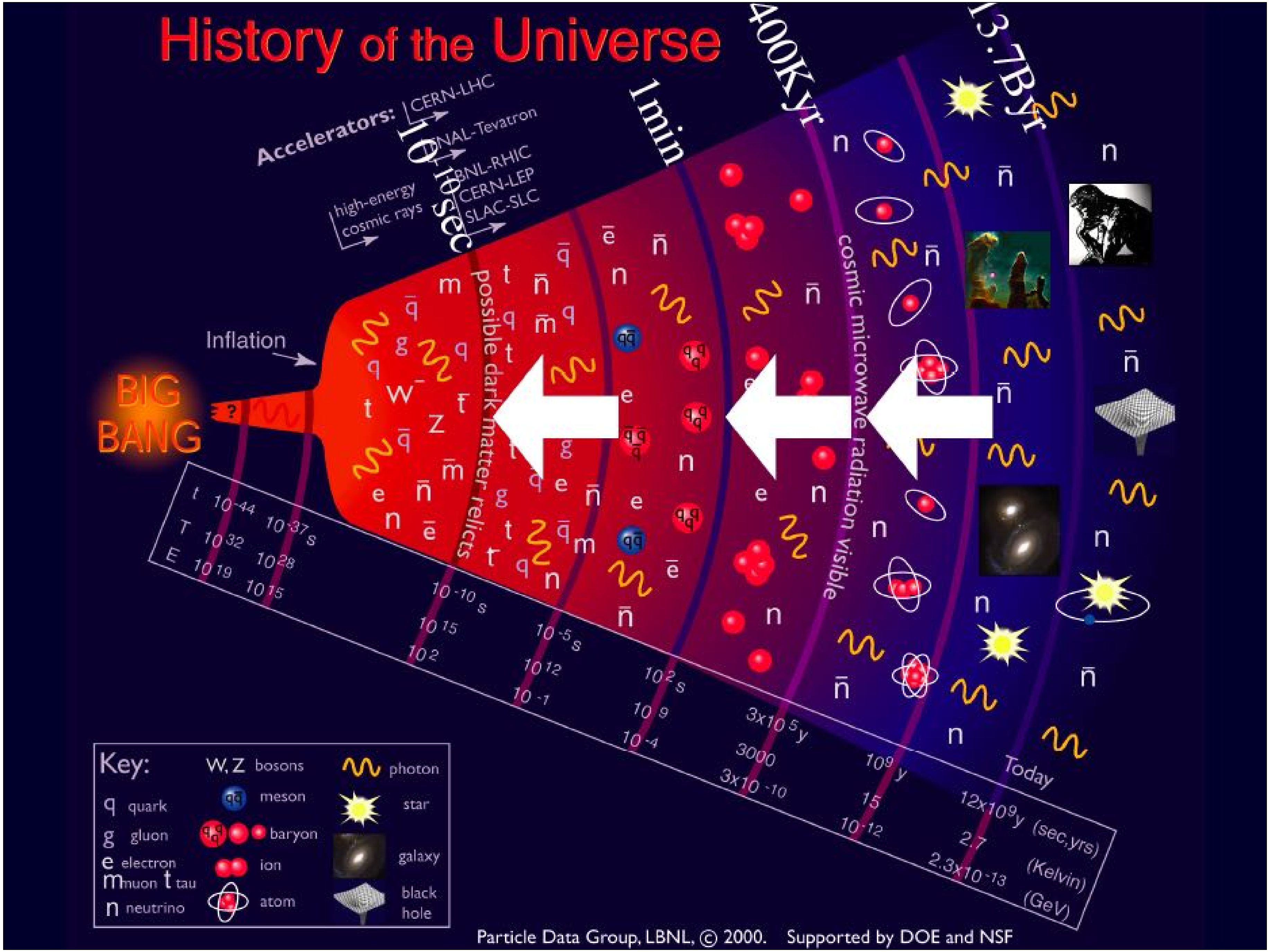


FIG. 6.—Same as Fig. 5, but for the ΛCDM model. The symbols show $\xi(r)$ computed from a (100 Mpc)^3 (*open circles*) and a (640 Mpc)^3 (*filled squares*) N-body simulation. The dotted curves show $\xi_{\rm lin}(r)$ from the linear theory (*bottom curve*) and the nonlinear $\xi(r)$ (*top curve*) given by the fitting formula of Ma (1998)

.


History of the Universe
Accelerators:
CERN-LHC
FNAL-Tevatron
BNL-RHIC
CERN-LEP
SLAC-SLC
high-energy cosmic rays
10-10 sec
1min
400Kyr
13.7Byr
Inflation
BIG BANG
possible dark matter relicts
cosmic microwave radiation visible
t 10-44 10-37 s
T 1032 1028
E 1019 1015
10-10 s
1015
102
10-5 s
1012
10-1
102 s
109
10-4
3x105 y
3000
3x10-10
109 y
15
10-12
Today
12x109 y (sec,yrs)
2.7 (Kelvin)
2.3x10-13 (GeV)
Key:
q quark
g gluon
e electron
m muon t tau
n neutrino
W, Z bosons
meson
baryon
ion
atom
photon
star
galaxy
black hole
Particle Data Group, LBNL, © 2000. Supported by DOE and NSF

Despite the
Dark Puzzles,
the future is
brighter