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Lectures on Supersymmetric Gauge Theories

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I. Dynamics of Susy Gauge Theories

II. Nonabelian Superconductors and Confinement

III. Recent developments

Lectures on Supersymmetric Gauge Theories I:

Introduction: a Review

- **Symmetries and anomalies**
- **Nonrenormalization theorem**
- **NSVZ β functions**
- **Instantons and anomalies**
- **Seiberg's duality**
- **Gluini condensate**
- **Phases of SQCD**

Why Supersymmetry?

- $H = Q^\dagger Q$,

$$Q: \qquad |Boson\rangle \leftrightarrow |Fermion\rangle$$

$$\langle H \rangle \geq 0, \qquad \rightarrow \qquad \Lambda_{Cosm} \ll \Lambda_{QCD}$$

- **Hierarchy (naturalness) problem in the standard model**

$$M_{Higgs}, M_W \ll M_{Planck} \sim 10^{19}\text{GeV}$$

- **Susy GUTs: coupling constant unification at $\mu \sim 10^{16}$ GeV? MSSM $\rightarrow$ LHC ($\geq$ 2007)**
- **Deep results on details of nonperturbative dynamics**
- **Haag-Lopuszhanski-Sohnius: Susy algebra is the only possible nontrivial generalization involving Poincaré and internal symmetry algebra. (*cfr*. Coleman-Mandula)**
- **"A truely beautiful idea never really dies... " (*Y. Nambu*)**

Susy gauge theories

- **Susy algebra**

$$\{Q_\alpha, \bar{Q}_{\dot{\alpha}}\} = 2\sigma^\mu_{\alpha,\dot{\alpha}} P_\mu,$$

- **Superfields**

$$F(x,\theta,\bar{\theta}) = f(x) + \theta\psi(x) + \dots$$

$$Q_\alpha = \frac{\partial}{\partial\theta^\alpha} - i\sigma^\mu_{\alpha,\dot{\alpha}}\bar{\theta}^{\dot{\alpha}}\partial_\mu, \qquad \bar{Q}_{\dot{\alpha}} = \frac{\partial}{\partial\bar{\theta}^{\dot{\alpha}}} - i\theta^\alpha\sigma^\mu_{\alpha,\dot{\alpha}}\partial_\mu,$$

- **Chiral superfields:** $\bar{D}\Phi = 0$ ($D\Phi^\dagger = 0$)

$$\Phi(x,\theta,\bar{\theta}) = \phi(y) + \sqrt{2}\theta\,\psi(y) + \theta\theta\,F(y), \qquad y = x + i\theta\sigma\bar{\theta}$$

$$D_\alpha = \frac{\partial}{\partial\theta^\alpha} + i\sigma^\mu_{\alpha,\dot{\alpha}}\bar{\theta}^{\dot{\alpha}}\partial_\mu, \qquad \bar{D}_{\dot{\alpha}} = -\frac{\partial}{\partial\bar{\theta}^{\dot{\alpha}}} - i\theta^\alpha\sigma^\mu_{\alpha,\dot{\alpha}}\partial_\mu,$$

- **Verctor superfields** $V^\dagger = V$,

$$W_\alpha = -\frac{1}{4}\bar{D}^2 e^{-V} D_\alpha e^V = -i\lambda + \frac{i}{2}\,(\sigma^\mu\,\bar{\sigma}^\nu)_\alpha^\beta\,F_{\mu\nu}\,\theta_\beta + \dots$$

- **Supersymmetry transformation of fields:**

Chiral superfields

$$[Q_\alpha, \phi] = \sqrt{2}\psi_\alpha; \quad \{Q_\alpha, \psi_\beta\} = \sqrt{2}F; \quad [Q_\alpha, F] = 0,$$

$$[\bar{Q}_{\dot{\alpha}}, \phi] = 0; \quad \{\bar{Q}_{\dot{\alpha}}, \psi_\beta\} = i\sqrt{2}\sigma^\mu_{\beta\dot{\alpha}}\mathcal{D}_\mu A; \quad [\bar{Q}^{\dot{\alpha}}, F] = i\sqrt{2}(\bar{\sigma}^\mu)^{\dot{\alpha}\beta}\mathcal{D}_\mu\psi_\beta,$$

In particular, $\bar{D}\Phi = 0 \Rightarrow [\bar{Q}_{\dot{\alpha}}, \phi] = 0 : \phi$ **is a "chiral field";**

Vector superfields

$$[Q^\alpha, A^a_\mu] = -i\sqrt{2}\,\bar{\lambda}^a\,\bar{\sigma}; \qquad \{Q^\alpha, \lambda^a\} = \sigma^{\mu\nu}F^a_{\mu\nu} + iD^a; \qquad [Q^\alpha, D^a] = -\sigma^\mu\mathcal{D}_\mu\bar{\lambda}^a;$$

$$[\bar{Q}_{\dot{\alpha}}, A^a_\mu] = -i\sqrt{2}\,\bar{\sigma}\lambda^a\,; \quad \{\bar{Q}_{\dot{\alpha}}, \lambda^a\} = 0; \qquad [Q^\alpha, D^a] = -\mathcal{D}_\mu\lambda^a\sigma^\mu;$$

- **Lagrangian ($\int d\theta_1\, \theta_1 = 1$, etc)**

$$\mathcal{L} = \frac{1}{8\pi}\mathrm{Im}\,\tau_{cl}\left[\int d^4\theta\, \Phi^\dagger e^V \Phi + \int d^2\theta\, \frac{1}{2} W W\right] + \int d^2\theta\, \mathcal{W}(\Phi) \qquad (1)$$

- $\mathcal{W}(\Phi)$ = **superpotential;**

- $$\tau_{cl} = \frac{\theta}{2\pi} + \frac{4\,\pi\, i}{g^2}$$

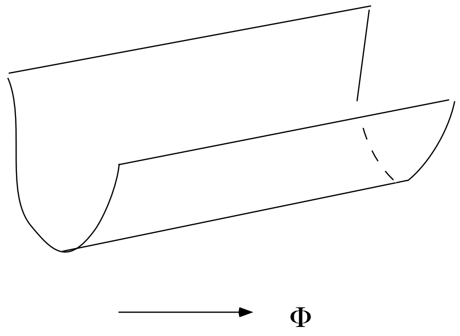
- **Scalar potential**

$$V_{sc} = \sum_{mat}\left|\frac{\partial\mathcal{W}}{\partial\phi}\right|^2 + \frac{1}{2}\sum_a\left|\sum_{mat}\phi^* t^a \phi\right|^2$$

- **For SQCD,** $\{\Phi\} \to Q \sim \underline{N},\ \tilde{Q} \sim \underline{N}^*$ **of** $SU(N)$

$$G_F = SU(n_f)\times SU(n_f)\times U_V(1)\times U_A(1)\times U_\lambda(1)$$

- **Flat directions (CMS)**



e.g., for $n_f < n_c$**,**

$$Q = \tilde{Q}^\dagger = \begin{pmatrix} a_1 & 0 & \dots & 0 \\ 0 & \ddots & & \\ 0 & \dots & & a_{n_f} \\ 0 & 0 & \dots & 0 \\ \dots & & & \dots \end{pmatrix}$$

Q:

Superpotential generated? CMS modified? Symmetry breaking?

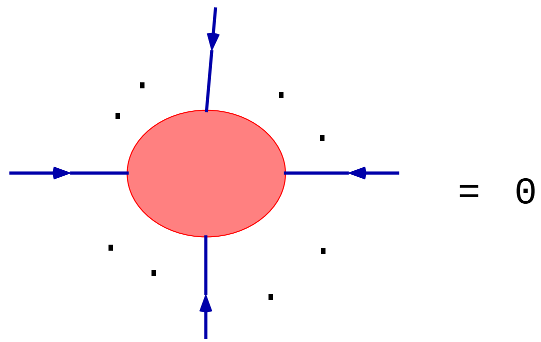
Nonrenormalization theorem

$$\mathcal{L} = \int d^2\theta d^2\bar{\theta}\, (\bar{\Phi}\Phi + \frac{1}{2}\Phi^2\delta^2(\bar{\theta}) + h.c.)$$

- **Perturbative N.R. theorem**

$$\langle T\Phi(x,\theta,\bar{\theta})\Phi(x',\theta',\bar{\theta}')\rangle$$

$$= -m\,\delta^2(\theta-\theta')\, e^{-i(\theta\sigma^\mu\bar{\theta}-\theta'\sigma^\mu\bar{\theta}')\partial_\mu}\Delta_c(x-x')$$



Only D terms $\propto \int d^2\theta\, d^2\bar{\theta}\,(\ldots)$ generated. No F terms

- If $\exists$ exact **non-anomalous symmetry** $G \rightarrow$ No terms violating G generated;

- **Perturbative anomaly** (West, Grisaru, et. al., SVZ)

$$\Delta L = \int d^2\theta \, d^2\bar{\theta} \; \Phi^2 \frac{D^2}{\Box} \Phi \sim \int d^2\theta \, \Phi^3$$

However, no such nonlocal term simulating F-term, in S_W

- Terms protected only by anomalous (e.g. $U_A(1)$) symmetries **can be generated by instantons**

- **Generalized non-renormalization theorem** (SVZ):

The gauge kinetic term

$$\int d^2\theta \, W_\alpha W^\alpha = \int d^2\theta \, d^2\bar{\theta} \; [\, (e^{-V} D_\alpha e^V) W^\alpha \,]$$

can be generated by 1 loop corrections - **only**.

→ **NSVZ exact β functions**:

- E.g. $SU(N)$ SQCD:

$$L = \frac{1}{4}\int d^2\theta \left(\frac{1}{g^2(M)} + \frac{b_0}{8\pi^2}\log\frac{M}{\mu}\right) W^a W^a + h.c. + \int d^4\theta \sum_i Z_i(\mu, M)\Phi_i^\dagger e^{2V_i}\Phi_i\,,$$

$$b_0 = -3N_c + \sum_i T_{Fi}; \qquad T_{Fi} = \frac{1}{2} \qquad \text{(quarks)}\,.$$

- Renormalize the fields $\Phi_i \longrightarrow Z_i^{-1/2}\Phi_i = e^{-\frac{1}{2}\log Z_i}\Phi_i \quad (\bar{D}(-\frac{1}{2}\log Z_i) = 0) \Rightarrow$ Anomaly $\propto \frac{1}{16\pi^2}(-\frac{1}{2}\log Z_i(\mu, M))\,WW$

- $$\frac{1}{g^2(\mu)} = \frac{1}{g^2(M)} + \frac{b_0}{8\pi^2}\log\frac{M}{\mu} - \frac{1}{8\pi^2}\log Z_i(\mu, M)$$

$$\beta_h(g) \equiv \mu\frac{d}{d\mu}\,g = -\frac{g^3}{16\pi^2}\left(3N_c - \sum_i T_{Fi}(1-\gamma_i(g))\right)\,,$$

where $\gamma_i(g(\mu)) = -\mu\frac{\partial}{\partial\mu}\log Z_i(\mu, M)|_{M,g(M)}$

- Actually by recaling $A_\mu = g_c A_{c\mu}, \quad \lambda = g_c\lambda_c$,

$$\frac{1}{g^2} = \frac{1}{g_c^2} + \frac{N_c}{8\pi^2}\log g_c^2, \qquad \beta(g_c) = -\frac{g_c^3}{16\pi^2}\frac{3N_c - \sum_i T_{Fi}(1-\gamma_i(g_c))}{1 - N_c g_c^2/8\pi^2}\,.$$

- $\gamma(g) = -\frac{g^2}{8\pi^2}\frac{N_c^2-1}{N_c} + O(g^4)$
- Zero of the beta function at g^* where

$$\gamma(g^*) = -\frac{3N_c - N_f}{N_f}$$

Susy Identities

- **Susy transf. of** $\Phi(x,\theta,\bar\theta) = \phi(y) + \sqrt{2}\theta\psi + \theta\theta F(y)$**:**

$$[\bar Q^{\dot\alpha}, \phi] = 0, \qquad \{\bar Q^{\dot\alpha}, \psi_\alpha\} = -\sqrt{2}\bar\sigma^\mu \partial_\mu \phi,$$

$$G = \langle T\phi_1(x_1)\phi_2(x_2)\ldots\phi_k(x_k)\rangle$$

$$\bar\sigma^\mu \partial_\mu^{x_1} G = \langle T[\,\bar Q^{\dot\alpha}, (\psi_1(x_1)\phi_2(x_2)\ldots)]\rangle = 0,$$

etc. G **indep. of** $x_i \to = \prod_i \langle\phi_i\rangle$

- **Analytic dep. on** g_i, m_i **etc (**$\mathcal{W}(\Phi) = m\,\Phi^2 + g\,\Phi^3 + \ldots$**)**

$$\frac{\partial G}{\partial m^*} = \langle T[\,\bar Q^{\dot\alpha}, (\bar\Phi^2|_{\bar\theta}\,\phi_1(x_1)\phi_2(x_2)\ldots)]\rangle = 0, \qquad \frac{\partial G}{\partial g_2 *} = 0$$

- **Symmetries**

Fields	Δ	q_V	q_λ	q_X
$Q, \tilde{Q}$	1	$1, -1$	1	$n_c - n_f$
$\psi_Q, \psi_{\tilde{Q}}$	$3/2$	$1, -1$	0	n_c
λ_α	$\frac{3}{2}$	0	1	$-n_f$
g_l	$2 - l$	$-(l+1)$	$1 - l$	2
Λ^{2N}	$2N$	$2N$	$\frac{4N}{3}$	0

Anomalies and Instantons

- $U_A(1)$ **anomaly** (**Steinberger, Schwinger, Adler, Bell, Jackiw**)

$$\partial_\mu J_5^\mu = \frac{e^2}{16\pi^2} F_{\mu\nu}\tilde{F}^{\mu\nu} \qquad (\pi_0 \to 2\gamma)$$

- **QCD:**

$$\partial_\mu J_L^\mu = \frac{g^2}{32\pi^2} G_{\mu\nu^a}\tilde{G}^{a,\mu\nu}$$

$$\Delta Q_5 = 2\, n_f \int d^4x \, \frac{g^2}{32\pi^2} G_{\mu\nu^a}\tilde{G}^{a,\mu\nu} \neq 0!$$

Axial $U_A(1)$ **broken: solution of "**$U(1)$**" problem (**$m_\eta \gg m_\pi$**? Why NO** $U_A(1)$ **Goldstone boson); But** $\frac{g^2}{32\pi^2} G_{\mu\nu^a}\tilde{G}^{a,\mu\nu} = \partial_\mu K^\mu$ **!?**

- **Finite energy config. classified by the Pontryagin number**

$$A_\mu \sim U^{-1}(x)\, \partial_\mu U(x), \qquad x \to \infty \qquad \Pi_3(SU(2)) = \mathbb{Z}$$

$$\int d^4x \, \frac{g^2}{32\pi^2} G_{\mu\nu^a}\tilde{G}^{a,\mu\nu} = n, \qquad n = 0, \pm 1, \pm 2, \ldots$$

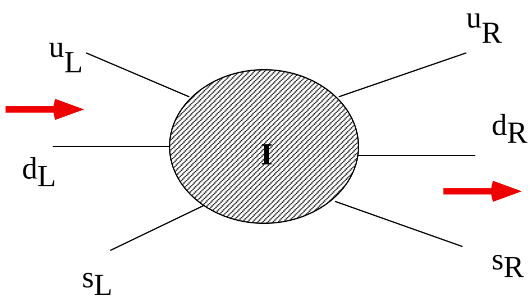
- **Config with $n = 1$: instanton** ([1])

$$A_\mu = -\frac{2i}{g^2}\frac{\tau_{\mu\nu}(x-x_0)_\nu}{(x-x_0)^2+\rho^2}, \qquad \tau_{\mu\nu} = \frac{\tau_\mu\bar{\tau}_\nu - \tau_\nu\bar{\tau}_\mu}{4}$$

- **Instanton effects in QCD ('t Hooft')**

$$\mathcal{L}_{eff} \sim \epsilon^{i_1\ldots i_{n_f}}\epsilon_{j_1\ldots j_{n_f}}\bar{\psi}_L^{j_1}(x)\ldots\bar{\psi}_L^{j_{n_f}}(x)\psi_{R,i_1}(x)\ldots\psi_{R,i_{n_f}}(x)$$

$U_A(1)$ **broken to** Z_{2n_f}**;** $SU_L(n_f)\times SU_R(n_f)$ **unbroken**



$$\langle\epsilon^{i_1\ldots i_{n_f}}\epsilon_{j_1\ldots j_{n_f}}\bar{\psi}_L^{j_1}(x_1)\ldots\bar{\psi}_L^{j_{n_f}}(x_{n_f})\psi_{R,i_1}(y_1)\ldots\psi_{R,i_{n_f}}(y_{n_f})\rangle \neq 0$$

[1] Belavin, Polyakov, Schwarz, 't Hooft

- **θ term**

$$\mathcal{L} = \theta \frac{g^2}{32\pi^2} G_{\mu\nu^a} \tilde{G}^{a,\mu\nu}$$

renormalizable. Experimentally ($d_n < 10^{-28}$ e cm $\rightarrow$

$$|\theta| < 10^{-9}$$

"Strong CP Problem" (Why?)

PQ symmetry (axions); $m_u = 0$, etc

- **$\Delta I = \frac{1}{2}$ problem (Why $\frac{A(K\to\pi\pi)^{\Delta I=1/2}}{A(K\to\pi\pi)^{\Delta I=3/2}} \sim 25$)**

Instanton Calculation in Susy QCD

- **Strong coulpling (standard) instanton method**

$$\langle \lambda\lambda(x_1)\lambda\lambda(x_2)\ldots\lambda\lambda(x_{n_c})\rangle = \text{const.}\,\Lambda^{3n_c}$$

$$L.H.S. = \text{const.} = \prod \langle\lambda\lambda\rangle = \langle\lambda\lambda\rangle^{n_c}$$

Require disentangle vac. sum (Z_{2n_c} **unbroken)**

- **Weak coupling instanton method (svz)**

(i) SQCD with massless $(Q, \tilde{Q})$**'s**

(ii) Flat direction $\rightarrow$ **Compute instanton corrections at large** $\langle Q\rangle \gg \Lambda$**;**

$$\Delta\mathcal{W}^{(ADS)} = (n_c - n_f)\frac{\Lambda^{(3n_c-n_f)/(n_c-n_f)}}{(\det Q\tilde{Q})^{1/(n_c-n_f)}} \qquad (\#)$$

(iii) Add $\mathcal{W}_{mass} = mQ\tilde{Q} \rightarrow$ **min. of the pot.**

(iv) Decouple the quarks $m \rightarrow \infty$**,** $\Lambda^*_{YM} = m\Lambda^*$

$$\langle\lambda\lambda\rangle = \Lambda^3$$

- **Numerical discrepancy (“$4/5$ puzzle”)**
- **Other methods (Compactification on $\mathbf{R}^3 \times S^1$; $\mathcal{N}=2$ SYM and decoupling the adjoint scalar) give WCI results**
- **For $SU(n_c)$:**

$$\langle \lambda\lambda \rangle = e^{2\pi i k/n_c} \Lambda^3, \qquad k = 1, 2, \ldots n_c$$

- $SU(r+1)$**,** $SO(2r+1)$**,** $USp(2r)$**,** $SO(2r)$ **SYM: (apart from $e^{2\pi i k/T_G}$)**

$$T_G = r+1,\ 2r-1,\ r+1,\ 2r-2,$$

$$\left\langle \frac{\mathrm{Tr}\lambda^2}{16\pi^2} \right\rangle_{SU(r+1)} = \Lambda^3 , \qquad \left\langle \frac{\mathrm{Tr}\lambda^2}{16\pi^2} \right\rangle_{SO(2r+1)} = 2^{\frac{4}{2r-1}-1} \Lambda^3 ,$$

$$\left\langle \frac{\mathrm{Tr}\lambda^2}{16\pi^2} \right\rangle_{USp(2r)} = 2^{1-\frac{2}{r+1}} \Lambda^3 , \qquad \left\langle \frac{\mathrm{Tr}\lambda^2}{16\pi^2} \right\rangle_{SO(2r)} = 2^{\frac{2}{r-1}-1} \Lambda^3 ,$$

$U(1)$-Related (Konishi) Anomaly

- $$-\frac{1}{4}\bar{D}^2(\,Q^\dagger e^V Q\,) = m\tilde{Q}Q + \frac{g^2}{16\pi^2}\mathrm{Tr}W_\alpha W^\alpha$$

  Im. part of the F-component = $U_A(1)$ anomaly

- **In SQCD**

  $$\{\bar{Q}_{\dot{\alpha}}, \bar{\psi}^{\dot{\alpha}} Q\} = m\,\tilde{Q}Q - \frac{g^2}{16\pi^2}\mathrm{Tr}\lambda_\alpha\lambda^\alpha$$

- **Vacuum alligned with mass perturbation**

  $$\langle m_i \tilde{Q}_i Q_i \rangle = \langle \frac{g^2}{16\pi^2}\mathrm{Tr}\lambda_\alpha\lambda^\alpha \rangle \qquad (\mathrm{no} \quad \mathrm{sum}) \quad i = 1, \ldots n_f$$

  cfr. **Dashen;** $\langle \bar{\psi}_i \psi_i \rangle = -\Lambda^2$ **(** $i = u, d, s$**)**

- **General chiral gauge th with** $\mathcal{W}(\Phi_i)$

  $$-\frac{1}{4}\bar{D}^2(\,\Phi_i^\dagger e^V \Phi_i\,) = \Phi_i \frac{\partial \mathcal{W}}{\partial \Phi_i} + C(\Phi_i)\frac{g^2}{16\pi^2}\mathrm{Tr}W_\alpha W^\alpha \qquad (\S)$$

- **Check of dynamical calculation (Instantons) and general argument**

• **Derivation:** $\delta\Phi_i = i\,A(z)\,\Phi_i$ **(** $A(z)$ **arbitrary)** $\rightarrow$ **Jacobian**

$$J = \det(\delta\Phi'_{z'}/\delta\Phi_z) = \det\langle z'|\,e^{iA(z)}(-\frac{\bar{D}^2}{4})\,|z\rangle = e^{\mathrm{Tr}\,iA(z)\frac{-\bar{D}^2}{4}}$$

• **Regularize the high eigenvalues by**

$$\mathrm{Tr}\,[iA(z)\frac{-\bar{D}^2}{4}] \rightarrow \lim_{M\rightarrow\infty} \mathrm{Tr}\,[iA(z)e^{L/M^2}(\frac{-\bar{D}^2}{4})]$$

$$L \equiv \bar{D}^2 e^{-V} D^2 e^V/16$$

• **Acting on** $\frac{-\bar{D}^2}{4}$

$$L = P^2 - \frac{1}{2}W^\alpha D_\alpha + C^\mu P_\mu + F,$$

where

$$W^\alpha = -\frac{1}{4}\,(\bar{D}^2 e^{-V} D^\alpha e^V),$$

$$C^\mu = -\frac{1}{2}\,\sigma^\mu_{\alpha\dot{\alpha}}(\bar{D}^{\dot{\alpha}} e^{-V} D^\alpha e^V),$$

$$F = (\bar{D}^2 e^{-V} D^2 e^V)/16.$$

- $M \to \infty$;

$$\int d^4p\, e^{-p^2/M^2} \sim M^4;$$

each power of L/M^2 from the exponent; also

$$\langle\theta\bar{\theta}|DD\bar{D}^2|\theta\bar{\theta}\rangle \neq 0,$$

$\therefore$ **only terms quadratic in $\frac{1}{2}W^\alpha D_\alpha$ contribute $\Rightarrow$ (§)**

- **Pauli-Villars, Supergraph $1-$loop calculation, Point-splitting, BPHZ, (Clark-Love, Gates-Grisaru-Rocek-Siegel, Piguet-Sibold, Konishi, Konishi-Shizuya); All these methods in Component formalism**

- **Functional-integral method particulary elegant for generalization**

Intrilligator, Leigh, Seiberg ('94)

- $\mathcal{N} = 1$ Gauge theory G with generic matter ϕ_i with

$$\mathcal{W}_{tree}(\phi_i) = \sum_r g_r X^r(\phi_i)$$

- Set $\mathcal{W}_{tree}(\phi_i) = 0$ first. $\rightarrow$ **Flat directions** along ϕ_i. Reinterpret in terms of gauge invariant composites (as (*) for SQCD).

- Turn on g_r and Λ_s. $\mathcal{W}_{eff}$ restricted by

  (i) **holomorphy** (i.e. holomorphic in g_r, X_r, Λ_s.)

  (ii) **invariance under various symmetries.** If some symmetry is broken by $\mathcal{W}$, it can be regarded as exact, by assigning appropriately the charges to g_r, Λ_s

  (iii) **Asymptotics**

- In many cases these are sufficient to determine $\mathcal{W}_{eff}$ exactly.

Phases of SQCD; Seiberg duality

- **Massless SQCD**

  → **Superpot.** (#); **Vacuum runaway** ($n_f < n_c$);

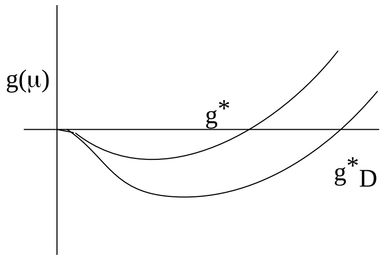
  → **No generation of superpotential for** $n_f > n_c$

- $n_f = n_c$:

$$(C.M.S.) \qquad \det M - B\,\tilde{B} = 0 \qquad\qquad (*)$$

$$(Q.M.S.) \qquad \det M - B\,\tilde{B} = \Lambda^{2n_f}$$

- $\frac{3n_c}{2} < n_f < 3n_c$ **(Conformal window), infrared fixed point (SCFT): described either as the original SQCD (with** Q, $\tilde{Q}$**) or as dual** $SU(\tilde{n}_c) = SU(n_f - n_c)$ **theory with dual quarks** $(q, \tilde{q}, M)$ **(Seiberg,** Kutasov, Schwimmer, ... **)**



N_f	**Deg.Freed.**	**Eff. Gauge Group**	**Phase**	**Symmetry**
0 **(SYM)**	**-**	**-**	**Confinement**	**-**
$1 \leq N_f < N_c$	**-**	**-**	**no vacua**	**-**
N_c	$M, B, \tilde{B}$	**-**	**Confinement**	$U(N_f)$
$N_c + 1$	$M, B, \tilde{B}$	**-**	**Confinement**	**Unbroken**
$N_c + 1 < N_f < \frac{3N_c}{2}$	$q, \tilde{q}, M$	$SU(\tilde{N}_c)$	**Free-magnetic**	**Unbroken**
$\frac{3N_c}{2} < N_f < 3N_c$	$q, \tilde{q}, M$ **or** $Q, \tilde{Q}$	$SU(\tilde{N}_c)$ **or** $SU(N_c)$	**SCFT**	**Unbroken**
$N_f = 3N_c$	$Q, \tilde{Q}$	$SU(N_c)$	**SCFT (finite)**	**Unbroken**
$N_f > 3N_c$	$Q, \tilde{Q}$	$SU(N_c)$	**Free Electric**	**Unbroken**