Introduzione alle teorie supersimmetriche

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Lay-out:

1. Why supersymmetry
2. General features of a supersymmetric theory
3. The minimal supersymmetric Standard Model
4. Breaking supersymmetry
5. Present experimental status
6. Conclusions and outlook

many respects

we are happy with the Standard Model

a unitary, renormalizable relativistic quantum field theory

amazingly consistent with data.

e problem of spontaneous gauge symmetry breaking (short nge of weak interactions) is solved in a simple way by the Higgs echanism.

is also allows breaking of the flavour symmetry in a enomenologically consistent way (flavour breaking confined in e charged current interaction sector).

Achieving the same results without the Higgs mechanism is extremely difficult.

At the same time,

we are unhappy with the Standard Model.

Many unsatisfactory aspects. Among others:

- What is the origin of flavour symmetry breaking?
- Is there a grand unification?
- Where is gravitation?

The last two points raise further problems:

- Hierarchy
- Naturalness

o facts:

- The masses of all known particles (including the minimal Standard Model Higgs boson) are not far from the weak scale, ~ 200 GeV.
- Much larger energy scales become relevant at some point

first question arises: why is the weak scale so much smaller than e Planck scale, $M_P \sim 10^{19}$ GeV, or the unification scale, $_{GUT} \sim 10^{16}$ GeV?

is is usually referred to as the hierarchy problem.

Even worse: The Higgs mass (masses of scalar particles, in general) is strongly sensitive to any large energy scale unless a *fine tuning* of parameters is performed.

This is the so-called naturalness problem.

There is a very simple reason for this: scalar masses are not naturally small, in the sense that no symmetry is recovered when they are let go to zero.

Fermion and vector boson masses *are* naturally small: radiative corrections are proportional to the masses themselves.

Naturalness and fine tuning in a simple example

nsider a theory of two real scalars fields:

$$\mathcal{L} = \frac{1}{2}\partial^\mu \phi \, \partial_\mu \phi + \frac{1}{2}\partial^\mu \Phi \, \partial_\mu \Phi - V(\phi, \Phi)$$

th

$$V(\phi, \Phi) = \frac{m^2}{2}\phi^2 + \frac{M^2}{2}\Phi^2 + \frac{\lambda}{4!}\phi^4 + \frac{\sigma}{4!}\Phi^4 + \frac{\delta}{4}\phi^2\Phi^2$$

sume λ, σ, δ are all positive, small and comparable in gnitude, and assume $M^2 \gg m^2 > 0$.

s the mass hierarchy $m^2 \ll M^2$ conserved at the quantum level?

Compute one-loop radiative corrections to m^2 by taking the second derivatives of the effective potential at the minimum $\phi = \Phi = 0$:

$$m^2_{\text{one loop}} = m^2(\mu^2) + \frac{\lambda m^2}{32\pi^2}\left(\log\frac{m^2}{\mu^2} - 1\right) + \frac{\delta M^2}{32\pi^2}\left(\log\frac{M^2}{\mu^2} - 1\right)$$

$$\mu^2 \frac{\partial m^2}{\partial \mu^2} = \frac{1}{32\pi^2}\left(\lambda m^2 + \delta M^2\right)$$

Corrections proportional to M^2 appear at one loop. One can choose $\mu^2 \sim M^2$ in order to get rid of them, but they reappear through the running of $m^2(\mu^2)$.

The mass hierarchy is preserved only if the parameters are such that

$$\lambda m^2 \sim \delta M^2 \to \frac{\delta}{\lambda} \sim \frac{m^2}{M^2}$$

This is what we usually call a *fine tuning* of the parameters.

e same thing happens if $m^2 < 0$, $M^2 \gg \left|m^2\right| > 0$. In this case the tree-level ential has a minimum at

$$\Phi = 0, \quad \phi^2 = -6m^2/\lambda \equiv v^2$$

l the symmetry $\phi \to -\phi$ is spontaneously broken. The degrees of freedom in s case are Φ and $\phi' \equiv \phi - v$, with

$$m^2_\Phi = M^2 \qquad m^2_{\phi'} = -2m^2 = \lambda v^2/3$$

one loop, the minimization condition $m^2 + \lambda v^2/6 = 0$ is replaced by

$$+\frac{\lambda v^2}{6} = -\frac{\lambda}{32\pi^2}\left(m^2+\frac{\lambda v^2}{2}\right)\left(\log\frac{m^2+\frac{\lambda v^2}{2}}{\mu^2}-1\right)-\frac{\delta}{32\pi^2}\left(M^2+\frac{\delta v^2}{2}\right)\left(\log\frac{M^2+\frac{\delta v^2}{2}}{\mu^2}-1\right)$$

llowing the same procedure as in the unbroken case one finds

$$m^2_{\phi'} = \frac{\lambda v^2}{3} + \frac{v^2}{32\pi^2}\left[\lambda^2 \log\frac{m^2+\frac{\lambda}{2}v^2}{\mu^2} + \delta^2 \log\frac{M^2+\frac{\delta}{2}v^2}{\mu^2}\right]$$

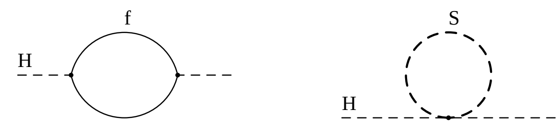
th $v \sim M$ without a suitable tuning of the parameters.

Naturalness: a closer look

The scalar potential in the Standard Model:

$$V(\phi) = m^2\left|\phi\right|^2 + \lambda\left|\phi\right|^4$$

One-loop corrections to m^2 due to fermionic (a) or bosonic (b) degrees of freedom:



(a) (b)

$$\left(\Delta m^2\right)_a = \frac{|\lambda_f|^2}{16\pi^2}\left[-2\Lambda^2 + 6m_f^2 \log\frac{\Lambda}{m_f} + \dots\right]$$

$$\left(\Delta m^2\right)_b = \frac{\lambda_S}{16\pi^2}\left[\Lambda^2 - 2m_S^2 \log\frac{\Lambda}{m_S} + \dots\right]$$

re Λ is an ultraviolet cut-off, to be identified with the energy le at which the SM is no longer reliable, and the dots stand for ms that do not grow with Λ.

dimensional regularization the Λ^2 term would be absent, but ntributions proportional to m_f^2, m_S^2 would still be there.

en if the heavy degrees of freedom are not directly coupled to e SM Higgs, it can be shown that similar contributions arise at gher orders.

In the absence of very special cancellations, the Higgs boson becomes as heavy as the heaviest degrees of freedom.

A symmetry that relates fermions to bosons would do the job, at least at one loop. Suppose there are two scalars for each fermion:

$$\left(\Delta m^2\right)_{a+b} = \frac{\lambda_S - |\lambda_f|^2}{8\pi^2}\Lambda^2 + \dots$$

For suitable values of the couplings the quadratic divergence disappears.

No surprise: with bosons and fermions in the same multiplet, scalar masses are protected by the same (chiral) symmetry that protects fermion masses from large radiative corrections.

Clearly, more restrictions will be needed in order to guarantee that the cancellation takes place at all orders.

ch a symmetry is called a supersymmetry:

$$Q|\text{boson}\rangle = |\text{fermion}\rangle \qquad Q|\text{fermion}\rangle = |\text{boson}\rangle$$

e symmetry generator Q (and its hermitian conjugate $Q^\dagger$) carry n $1/2$: it is a space-time symmetry.

e form of possible supersymmetry algebras is strongly nstrained on the basis of very general theorems in field theory. r example, it is impossible with ordinary symmetry generators ements of a commutator algebra).

torical remark: symmetries that relate particles with different spin first died in the context of approximate symmetries of hadrons. Non relativistic rk models have an approximate $SU(6)$ symmetry (3 quark flavors with spin), which is observed hadron spectrum. It relates hadrons with the same or content, but different spin (e.g. $K \leftrightarrow K^*$). Consequence of approximate n and flavor independence of quark-quark forces.

e Coleman-Mandula theorem tells us that this property cannot be sessed by a relativistic theory: with some reasonable assumptions, the most eral Lie algebra of symmetry operators that commute with the S matrix sists of Poincaré generators P_μ and $J_{\mu\nu}$, plus ordinary internal symmetry erators that act on one-particle states with matrices that are diagonal in, l independent of, momentum and spin. Crucial point: the Poincaré group is n compact, it has no non-trivial unitary representations.

les out $SU(6)$, but also Lie algebras of supersymmetry generators.

There is essentially one possibility:

$$\{Q, Q^\dagger\} = P^\mu$$
$$\{Q, Q\} = \{Q^\dagger, Q^\dagger\} = 0$$
$$[P^\mu, Q] = [P^\mu, Q^\dagger] = 0$$

(more on this later). Further specifications:

- $Q, Q^\dagger$ transform as spinors under the Lorentz group
- $Q, Q^\dagger$ commute with gauge symmetry generators.

In principle, we may have more than one Q: Q^i, $i = 1, \ldots, N$ (extended supersymmetry).

A few basic properties of a supersymmetric theory can already be recognized:

- particles in the same supersymmetric multiplet (which we will call a supermultiplet) have equal masses and equal gauge transformation properties (electric charge, weak isospin and color)
- within the same supermultiplet, there is an equal number of bosonic and fermionic degrees of freedom (a proof on the next slide)

A proof (taken from S. Martin):

$$\left.\begin{array}{l}(-1)^{2s}|\text{boson}\rangle = +|\text{boson}\rangle \\ (-1)^{2s}|\text{fermion}\rangle = -|\text{fermion}\rangle\end{array}\right\} \Rightarrow \left\{(-1)^{2s}, Q\right\} = \left\{(-1)^{2s}, Q^\dagger\right\} = 0$$

nsider the subspace of states $|i\rangle$ within a supermultiplet with the same envalue p^μ of the four-momentum operator P^μ. $\sum_i |i\rangle\langle i| = 1$ within this space of states.

$$\begin{aligned}\sum_i \langle i|(-1)^{2s}P^\mu|i\rangle &= \sum_i \langle i|(-1)^{2s}QQ^\dagger|i\rangle + \sum_i \langle i|(-1)^{2s}Q^\dagger Q|i\rangle \\ &= \sum_i \langle i|(-1)^{2s}QQ^\dagger|i\rangle + \sum_i\sum_j \langle i|(-1)^{2s}Q^\dagger|j\rangle\langle j|Q|i\rangle \\ &= \sum_i \langle i|(-1)^{2s}QQ^\dagger|i\rangle + \sum_j \langle j|Q(-1)^{2s}Q^\dagger|j\rangle \\ &= \sum_i \langle i|(-1)^{2s}QQ^\dagger|i\rangle - \sum_j \langle j|(-1)^{2s}QQ^\dagger|j\rangle \\ &= 0.\end{aligned}$$

Fermionic fields: a reminder

We use the Weyl representation of the Dirac matrices:

$$\gamma^\mu = \begin{bmatrix} 0 & \sigma^\mu \\ \bar\sigma^\mu & 0 \end{bmatrix} \qquad \sigma^\mu = (\sigma^0, \vec\sigma) \qquad \bar\sigma^\mu = (\sigma^0, -\vec\sigma) \qquad \gamma_5 = \begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix}$$

where $\vec\sigma$ are the three Pauli matrices

$$\sigma^1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \quad \sigma^2 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix} \quad \sigma^3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

and

$$\sigma^0 = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

e familiar lagrangian for a free, massive Dirac spinor is

$$\mathcal{L} = \bar\psi\,(i\partial\!\!\!/ - m)\,\psi \quad \rightarrow \quad (i\partial\!\!\!/ - m)\psi = 0$$

ur-component Dirac spinors realize a reducible representation of e Lorentz group:

$$= \begin{pmatrix} \xi_L \\ \xi_R \end{pmatrix} \quad \psi_L = \begin{pmatrix} \xi_L \\ 0 \end{pmatrix} = \frac{1}{2}(1-\gamma_5)\psi \quad \psi_R = \begin{pmatrix} 0 \\ \xi_R \end{pmatrix} = \frac{1}{2}(1+\gamma_5)\psi$$

e two-component spinors ξ_L, ξ_R transform independently under rentz transformations.

terms of ξ_L, ξ_R we have

$$\mathcal{L} = i\xi_L^\dagger\,\bar\sigma^\mu\partial_\mu\,\xi_L + i\xi_R^\dagger\,\sigma^\mu\,\partial_\mu\xi_R - m(\xi_L^\dagger\xi_R + \xi^\dagger$$