Inflaton, curvaton, and other light field perturbations

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and in prep.

OUTLINE:

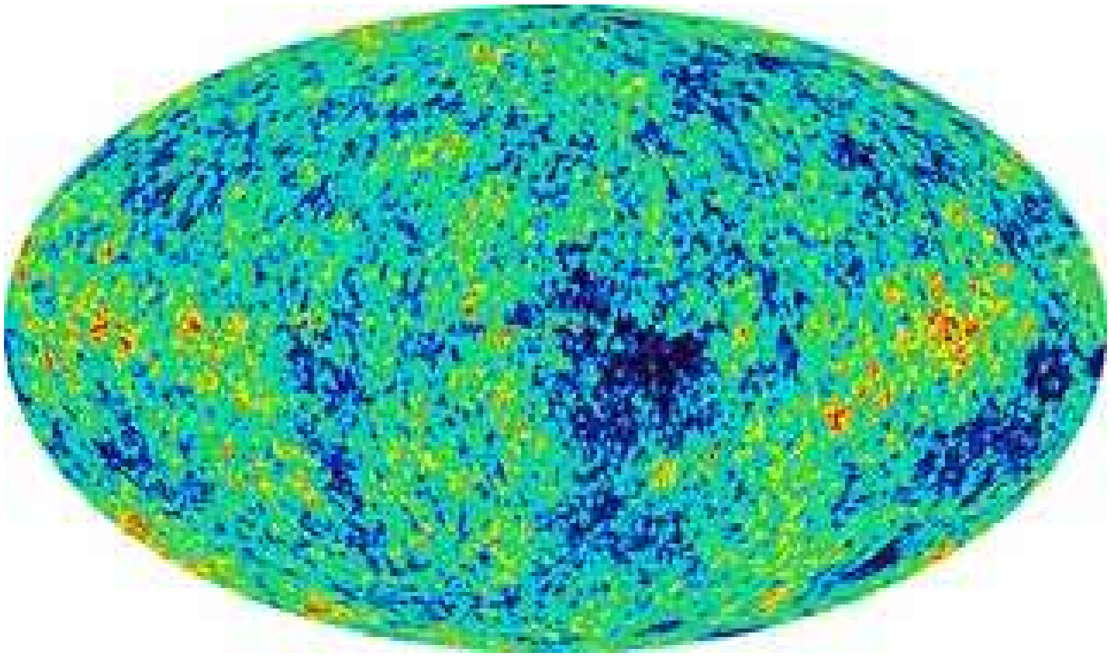
- Inflation
- Light fields (m<<H)
- Curvaton
- Cosmological perturbations and observations
- From heaviness to lightness

Concise Oxford dictionary:

Curvaton / 'ku:vaton/ *noun*

Light scalar field partially or totally responsible for the primordial density perturbations

COSMIC MICROWAVE BACKGROUND

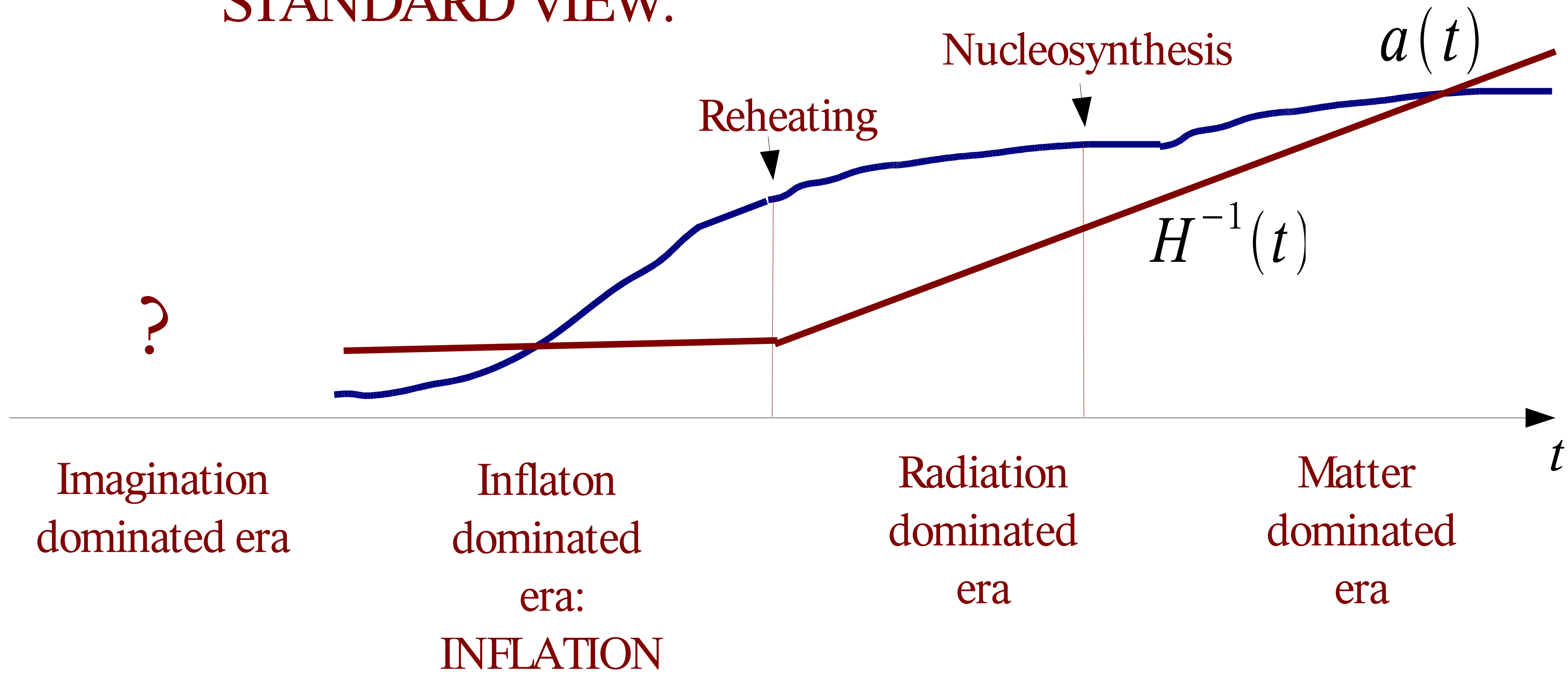


[WMAP: Bennet et al., '03]

Perturbations are approximately:

- adiabatic
- Gaussian
- scale-invariant
- and no gravity waves (background) have been observed

STANDARD VIEW:

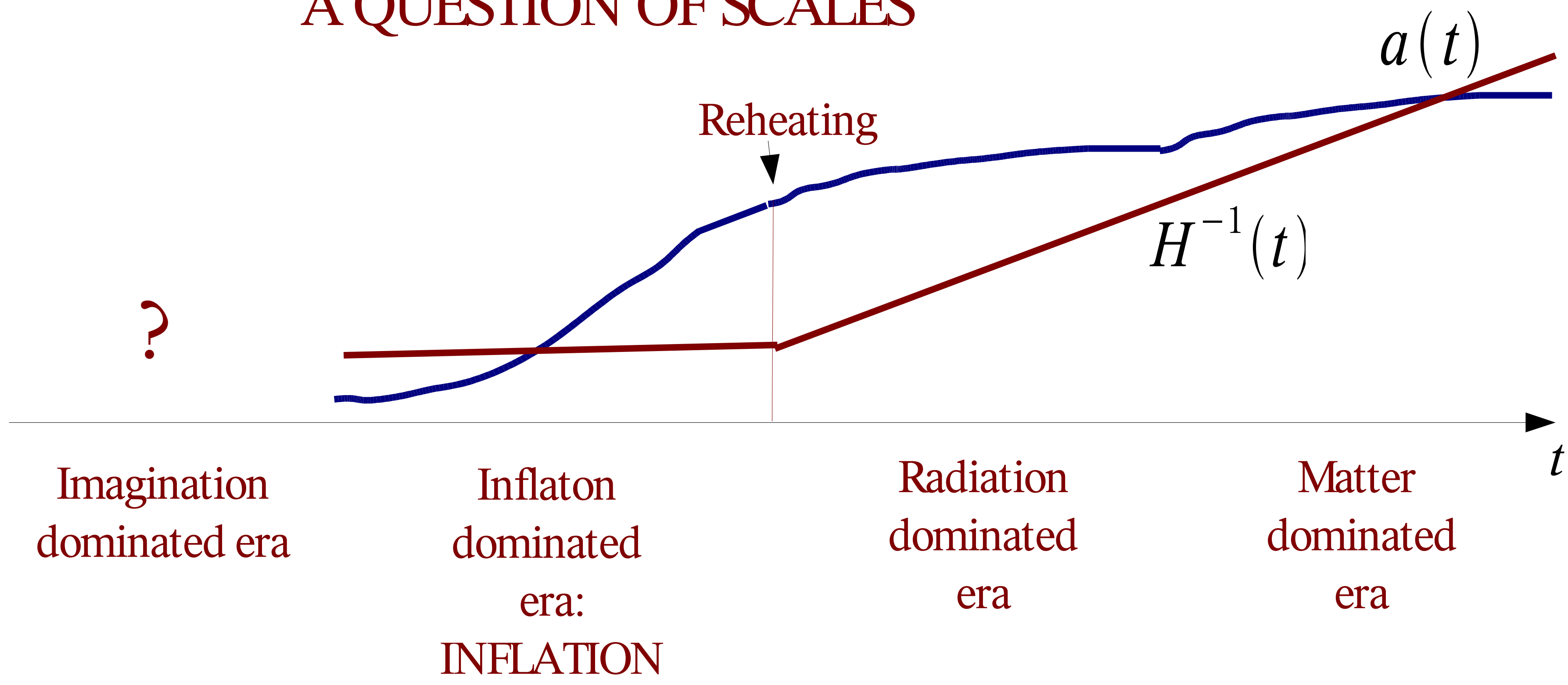


INFLATION provides us with three things:

Superluminal expansion	INFLATON
Origin of matter: reheating	INFLATON
Density perturbations	INFLATON

quantum fluctuations: $\delta\phi \sim H$ ← Hubble parameter during inflation

A QUESTION OF SCALES



$\lambda = \dfrac{2\pi}{k}$ λ : comoving length k : comoving wavenumber

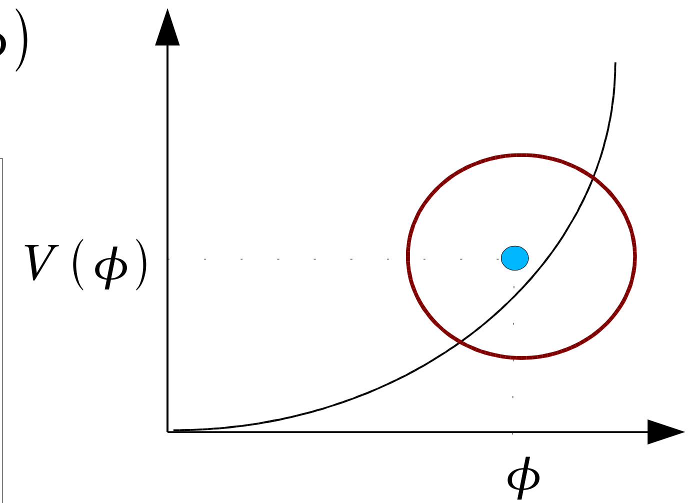
$H = \dfrac{\dot{a}}{a}$ H^{-1} : Hubble radius H^{-1}/a : comoving Hubble radius

$\lambda \gg H^{-1}/a \Rightarrow k \ll aH$ large scale limit

$\lambda \ll H^{-1}/a \Rightarrow k \gg aH$ small scale limit

INFLATION (in a nut-shell)

Inflaton field $$L = -\frac{1}{2}(\nabla\phi)^2 - V(\phi) = \frac{1}{2}\dot{\phi}^2 - V(\phi)$$



Energy density: $$\rho_\phi = \frac{1}{2}\dot{\phi}^2 + V(\phi$$

Pressure: $$P_\phi = \frac{1}{2}\dot{\phi}^2 - V(\phi$$

Klein-Gordon equation: $$\ddot{\phi} + 3H\dot{\phi} + V'(\phi) =$$

FRIEDMANN EQUATIONS

$$H^2 \equiv \left(\frac{\dot{a}}{a}\right)^2 = \frac{1}{3m_{Pl}}\rho_\phi = \frac{1}{3m_{Pl}}\left[\frac{1}{2}\dot{\phi}^2 + V(\phi)\right] \simeq \frac{1}{3m_{Pl}}V(\phi)$$

$$\frac{\ddot{a}}{a} = -\frac{1}{6m_{Pl}}(\rho_\phi + 3P_\phi) = -\frac{1}{3m_{Pl}}\left[\frac{1}{2}\dot{\phi}^2 - V(\phi)\right] > 0$$ Accelerated expansion

Slow-roll parameters: $$\epsilon \equiv \frac{m_{Pl}^2}{2}\left(\frac{V'}{V}\right)^2 \sim \frac{\dot{\phi}^2}{V(\phi)} \qquad \eta \equiv m_{Pl}^2\frac{V''}{V} \sim \frac{\ddot{\phi}}{3H\dot{\phi}}$$

FIELD PERTURBATIONS

$$\phi(t)+\delta\phi(t,\boldsymbol{x})$$

Evolution equation: $\ddot{\phi}+3H\dot{\phi}+V'\phi=0$

Using Fourier modes

$$\delta\ddot{\phi}_k+3H\delta\dot{\phi}_k+\left(\frac{k^2}{a^2}+m_\phi^2\right)\delta\phi_k=0,$$

$$m_\phi^2\equiv V''(\phi)$$

$$\delta\phi(t,\boldsymbol{x})=\int\frac{dk^3}{(2\pi)^{3/2}}\left(\hat{a}_k\delta\phi_k(t)e^{-i\boldsymbol{k}\boldsymbol{x}}+\hat{a}_k^*\delta\phi_k^*(t)e^{i\boldsymbol{k}\boldsymbol{x}}\right)$$

Canonical variable: $v_k=a\,\delta\phi_k$

Conformal time: $ds^2=-dt^2+a^2dx^2=a^2(-d\eta^2+dx^2)$

$$\boxed{\frac{d^2v_k}{d\eta^2}+\left(k^2+m_\phi^2a^2-\frac{a''}{a}\right)v_k=0}$$

Canonical equation

[Mukhanov, Brandenberger and Feldman., Phys Rep.]

PERTURBATIONS IN DE SITTER

$$\frac{d^2 v_k}{d\eta^2} + \left(k^2 + m_\phi^2 a^2 - \frac{a''}{a}\right) v_k = 0 \qquad v_k = a\,\delta\phi_k$$

Canonical equation

De Sitter: $H = const$ and $a = e^{Ht} = \frac{-1}{H\eta}$

$$\frac{d^2 v_k}{d\eta^2} + \left(k^2 + \frac{m_\phi^2 / H^2 - 2}{\eta^2}\right) v_k = 0$$

Heavy $m_\phi >> H$

Light $m_\phi << H$

HEAVY FIELD $m_\phi >> H$

$$\frac{d^2 v_k}{d\eta^2} + \left(k^2 + \frac{m_\phi^2}{H^2 \eta^2} \right) v_k = 0$$

On large scales $k \ll aH$

$$\delta\phi_k = \frac{v_k}{a} \propto e^{-\frac{3}{2}Ht} \left(A_k \sin(m_\phi t) + B_k \cos(m_\phi t) \right) \rightarrow 0$$

Fluctuations of a heavy scalar field are diluted by inflation

$$\delta\phi \sim 0$$

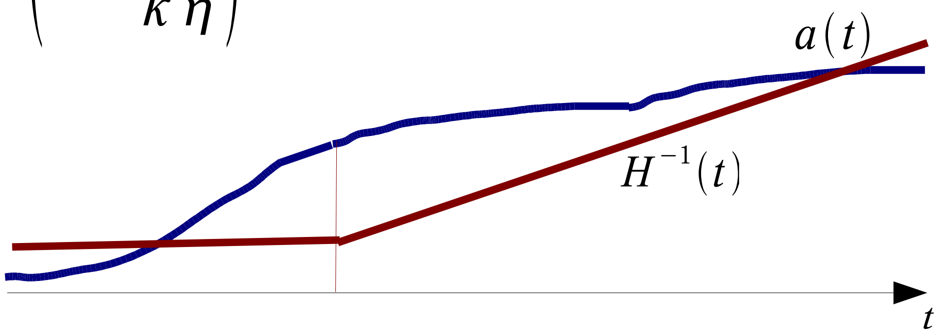
LIGHT FIELD $m_\phi \ll H$

$$\frac{d^2 v_k}{d\eta^2} + \left(k^2 - \frac{2}{\eta^2}\right) v_k = 0 \qquad a = e^{Ht} = \frac{-1}{H\eta}$$

$$v_k = \frac{\alpha_k}{\sqrt{k}} e^{-ik\eta}\left(1 - \frac{i}{k\eta}\right) + \frac{\beta_k}{\sqrt{k}} e^{ik\eta}\left(1 + \frac{i}{k\eta}\right)$$

Wavelengths smaller than the Hubble radius, k>>aH (| kη| >>1) are in the Minkowski vacuum state:

$$v_k = \frac{1}{\sqrt{k}} e^{-ik\eta}$$



For de Sitter spacetime we can choose the Bunch-Davis vacuum: $\alpha_k = 1, \beta_k = 0$

$$\delta\phi_k = \frac{v_k}{a} = \frac{1}{a\sqrt{k}} e^{\frac{-ik}{aH}}\left(1 + \frac{iaH}{k}\right) \longrightarrow \frac{H}{k^{3/2}}$$

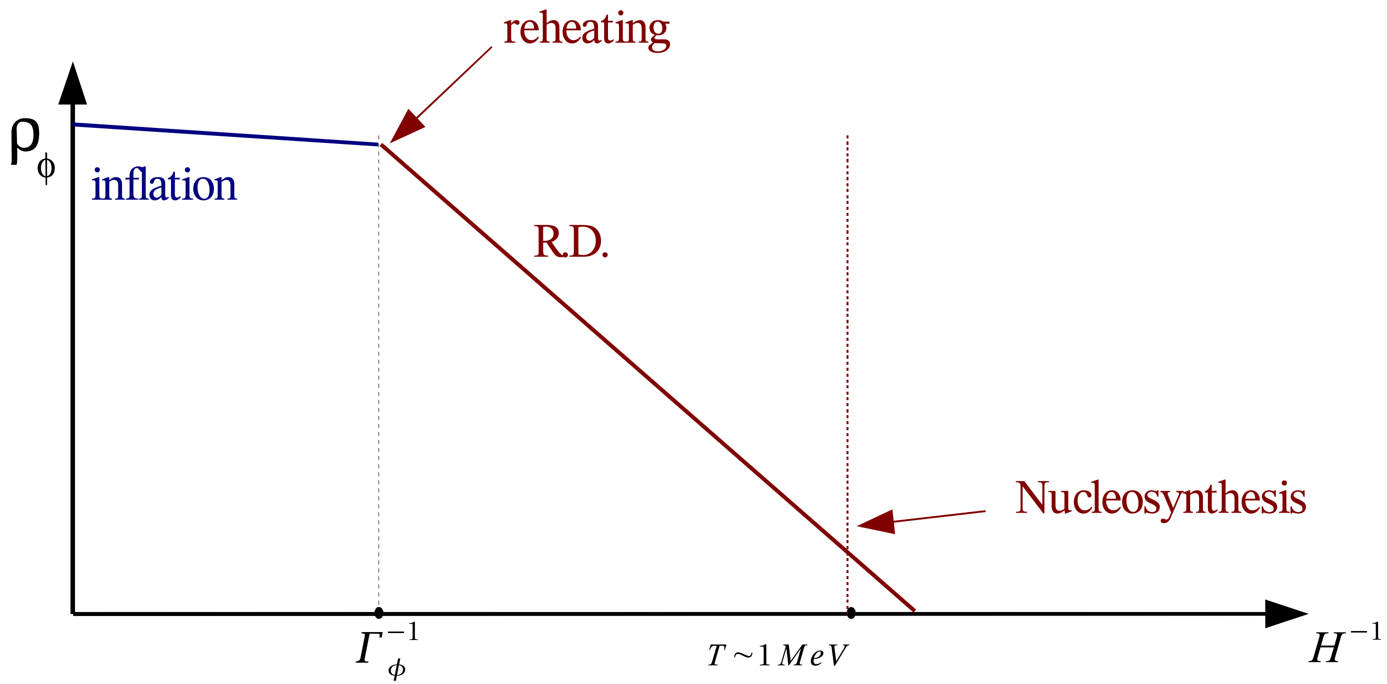
in the large scales limit $k \ll aH$

$$\boxed{\delta\phi \sim H}$$

Quantum fluctuations of size H

NOTE: since $\eta \equiv m_{Pl}^2 \frac{V''}{V} = \frac{m_\phi^2}{3H^2} \ll 1$ the inflaton is light!

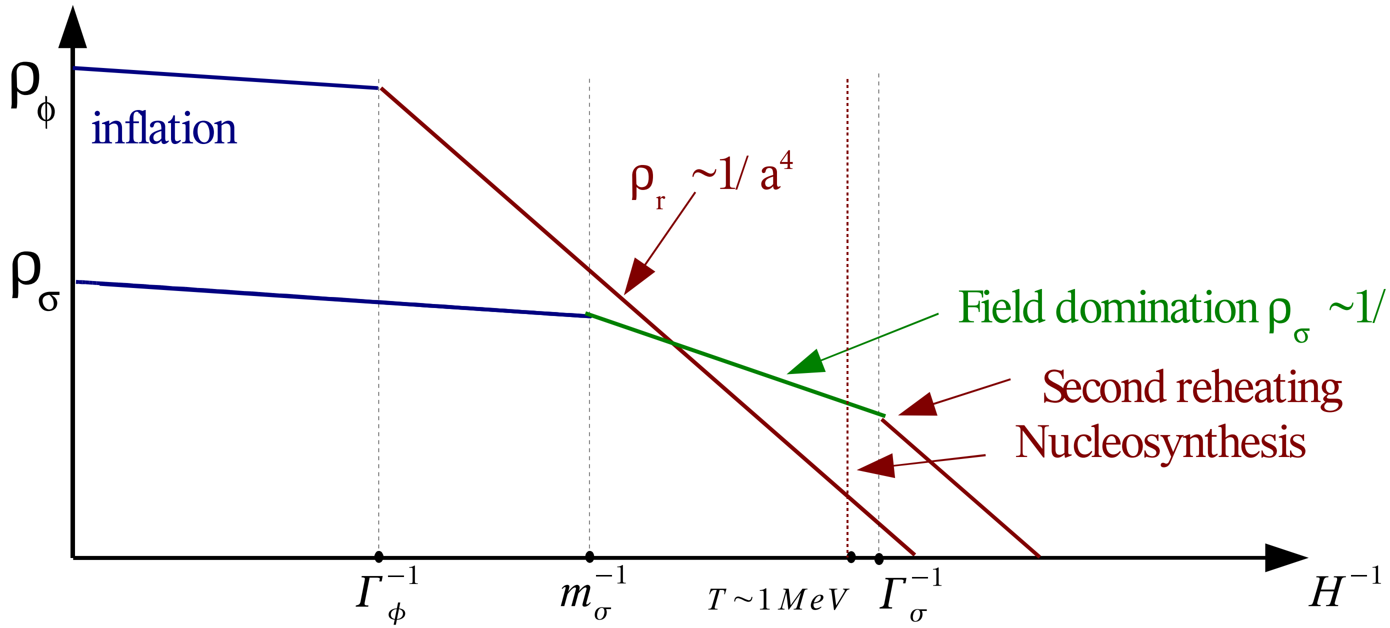
LIGHT FIELDS



Moduli problem: [Coughlan et al., '83]

Weakly coupled light scalar fields (m<<H) are not diluted during inflation and can dominate the universe and decay during or after nucleosynthesis

LIGHT FIELDS

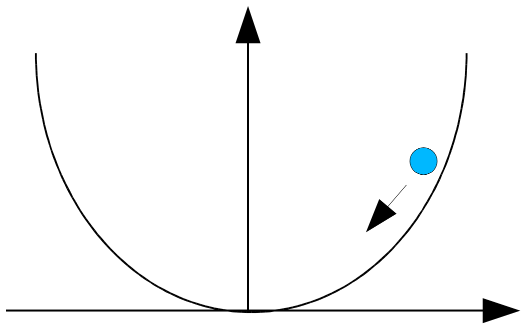


- Scalar field σ negligible during inflation, $\rho_\sigma << \rho_\phi$
- Light field, $L=\frac{1}{2}\dot{\sigma}-\frac{1}{2}m_\sigma^2\sigma^2$, $m_\sigma \ll H$

$$\ddot{\sigma}+3H\dot{\sigma}+m_\sigma^2=0$$

$$m_\sigma < H \;\Rightarrow\; \sigma \simeq const$$

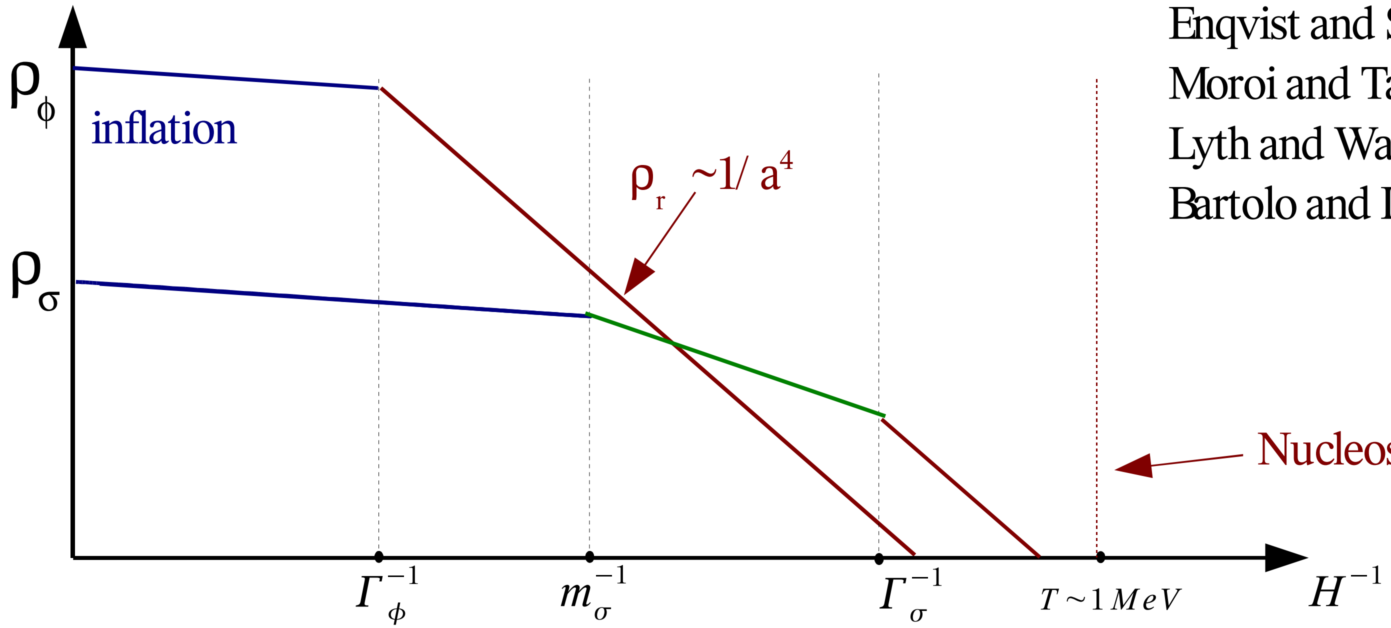
$$m_\sigma \geqslant H \;\Rightarrow\; \sigma \simeq a^{-3/2}\sin(m_\sigma t)$$



Non-relativistic fluid, $\rho_\sigma \sim 1/a^3$

CURVATON

[Linde and Mukhanov;
Mollerach, '90;
Enqvist and Sloth, '02;
Moroi and Takahashi, '01;
Lyth and Wands, '02;
Bartolo and Liddle, 03]



QUESTION: WHY???

CURVATURE PERTURBATIONS

$$\delta\phi \sim H \longrightarrow \zeta = \frac{\delta\rho}{3(\rho+P)}$$

Curvature perturbation
on the uniform density
hypersurface in the flat gauge

$$\zeta_X = \frac{\delta\rho_X}{3(\rho_X+P_X)}, \zeta_Y = \frac{\delta\rho_Y}{3(\rho_Y+P_Y)} \qquad \zeta = \frac{(\rho_X+P_X)\zeta_X + (\rho_Y+P_Y)\zeta_Y}{\rho+P}$$

$$\zeta = \zeta_\phi = -H\frac{\delta\rho_\phi}{\dot{\rho}_\phi} \simeq -H\frac{\delta\phi}{\dot{\phi}} = 3H^2\frac{\delta\phi}{V'} = \frac{V}{V'}\frac{\delta\phi}{m_{Pl}^2} \simeq \frac{1}{\sqrt{\epsilon}}$$

$$\rho_\phi \simeq V(\phi) \qquad \cancel{\ddot{\phi}} + 3H\dot{\phi} + V' = 0,$$

$$\dot{\phi} = \frac{-V'}{3H}$$

$$H^2 \simeq \frac{1}{3m_{Pl}^2}V \qquad \epsilon \equiv \frac{m_{Pl}^2}{2}\left(\frac{V'}{V}\right)^2$$

$$\boxed{\zeta \simeq \frac{1}{\sqrt{\epsilon}}\frac{\delta\phi}{m_{Pl}}}$$

INFLATON PERTURBATIONS

$$\zeta \simeq \frac{1}{\sqrt{\epsilon}} \frac{\delta \phi}{m_{Pl}} \simeq \frac{H}{\sqrt{\epsilon}\, m_{Pl}} \simeq \frac{V^{1/2}}{\sqrt{\epsilon}\, m_{Pl}^2}$$

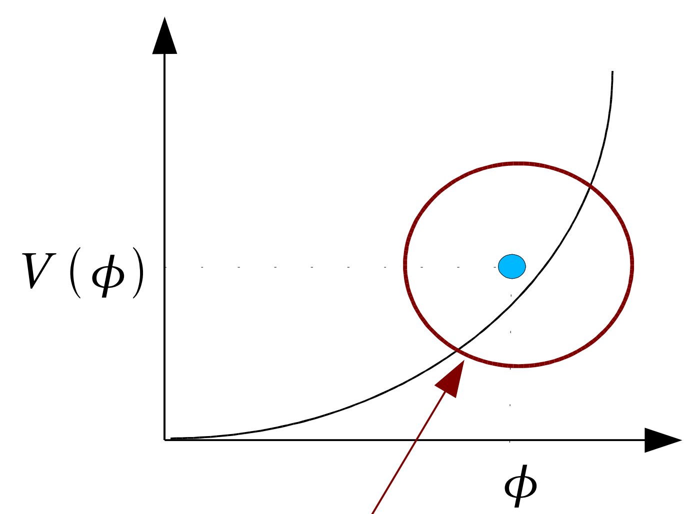
OBSERVABLES:

$$P_\zeta, n_S, r$$

$P_\zeta \equiv \frac{V}{\epsilon\, m_{Pl}^4}$ Power spectrum
$r \equiv \frac{P_T}{P_\zeta} = 16\epsilon$ Tensor/ scalar ratio
$n_S \equiv 1 + d\frac{\ln P_\zeta}{d \ln k} = 1 + 2\eta - 6\epsilon$ Scalar spectral index

Relation between the inflaton potential and the density perturbations

Constraints on the inflaton potential



$$\epsilon \equiv \frac{m_{Pl}^2}{2}\left(\frac{V'}{V}\right)^2 \qquad \eta \equiv m_{Pl}^2 \frac{V''}{V}$$

Slow-roll parameters

INFLATON PERTURBATIONS

$$\zeta \simeq \frac{1}{\sqrt{\epsilon}} \frac{\delta \phi}{m_{Pl}} \simeq \frac{H}{\sqrt{\epsilon}\, m_{Pl}} \simeq \frac{V^{1/2}}{\sqrt{\epsilon}\, m_{Pl}^2}$$

OBSERVABLES:

$$P_\zeta ,\, n_S ,\, r$$

$$P_\zeta \equiv \frac{V}{\epsilon\, m_{Pl}^4}$$

Power spectrum

$$r \equiv \frac{P_T}{P_\zeta} = 16\,\epsilon$$

Tensor/ scalar ratio

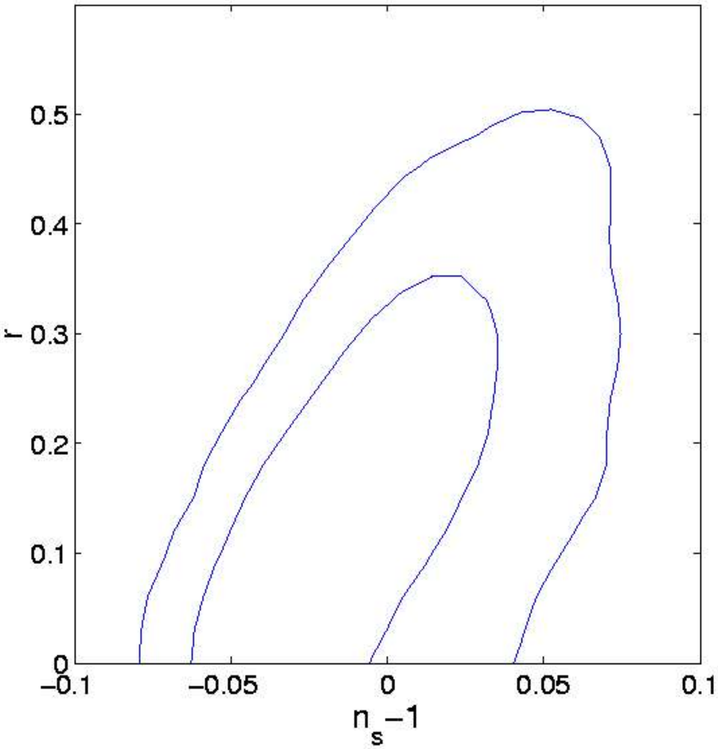
$$n_S \equiv 1 + d\frac{\ln P_\zeta}{d \ln k} = 1 + 2\eta - 6\epsilon$$

Scalar spectral index

Data constraints

[Leach and Liddle, 2002]

$$r = 16\,\epsilon$$



$$n_S - 1 = 2\eta - 6\epsilon$$

INFLATON CONSTRAINTS

$$\zeta \simeq \frac{1}{\sqrt{\epsilon}} \frac{\delta \phi}{m_{Pl}} \simeq \frac{H}{\sqrt{\epsilon}\, m_{Pl}} \simeq \frac{V^{1/2}}{\sqrt{\epsilon}\, m_{Pl}^2}$$

OBSERVABLES: P_ζ, n_S, r		CONSTRAINTS: V, V', V''
$P_\zeta \equiv \frac{V}{\epsilon\, m_{Pl}^4}$ Power spectrum	$\rightarrow$	$V = 10^{-7} \epsilon^2 m_{Pl}^4$ COBE normalization
$r \equiv \frac{P_T}{P_\zeta} = 16\epsilon$ Tensor/ scalar ratio	$\rightarrow$	$\epsilon \ll 1 \;\Rightarrow\; m_{Pl} V'/V \ll 1$ No gravity waves observed
$n_S \equiv 1 + d\frac{\ln P_\zeta}{d\ln k} = 1 + 2\eta - 6\epsilon$ Scalar spectral index	$\rightarrow$	$\eta \ll 1 \;\Rightarrow\; m_{Pl}^2 V''/V \ll 1$ Scale invariance

INFLATON CONSTRAINTS

- Inflation is very economical but tightly constrained:
severe constraints on inflaton potential

- Some inflationary models motivated by particle physics (supersymmetry) require the violation of some of these constraints

[Dimopoulos and Lyth, 2002] [Dvali and Kachru, 2003]

$$V(\phi) \ll (10^{16}\,GeV)^4 \quad and \quad m_\phi \sim H$$

CONSTRAINTS:

$$V, V', V''$$

$V = 10^{-7}\epsilon^2 m_{Pl}^4$ COBE normalization
$\epsilon \ll 1 \Rightarrow m_{Pl} V'/V \ll 1$ No gravity waves observed
$\eta \ll 1 \Rightarrow m_{Pl}^2 V''/V \ll 1$ Scale invariance

Superluminal expansion	INFLATON
Origin of matter: reheating	INFLATON
Density perturbations	CURVATON

CONSTRAINTS:

$$V\ ,\ V'\ ,\ V''$$

$V < 10^{-7}\epsilon^2 m_{Pl}^4 \sim (10^{16} GeV)^4$ COBE bound
$\epsilon \ll 1 \quad \Rightarrow \quad m_{Pl} V'/V \ll 1$
$\eta \sim 1 \quad \Rightarrow \quad \cancel{m_\phi \ll H}$ $m_\phi \sim H$

- The curvaton can generate perturbations and liberate the inflaton relaxing the constraints on inflaton potential: **division of labour**

Drawback:

- more difficult to directly test inflation

CURVATON GENERATED PERTURBATIONS

- Any light field (overdamped during inflation, m<<H) inherits the same quantum fluctuation (flat spectrum) as the inflaton

Curvaton σ :

$$\delta\sigma \simeq H \qquad \zeta_\sigma = \frac{\delta\rho_\sigma}{3\rho_\sigma} \simeq \frac{\delta\sigma}{\sigma}$$

- By dominating the universe and decaying before nucleosynthesis the curvaton imprints its perturbations: generation of curvature perturbations

$$\boxed{\zeta = \frac{4\rho_r\zeta_r + \rho_\sigma\zeta_\sigma}{3(\rho + P)} \simeq \left(\frac{\rho_\sigma}{\rho}\right)_{,dec.} \frac{\delta\sigma}{\sigma} \simeq \Omega_{\sigma,dec.}\frac{H}{\sigma}}$$

$$if\ \rho_\sigma \gg \rho_r$$

- These may be much larger than the inflaton perturbations

$$\zeta_r = \zeta_\phi \simeq \frac{H}{\sqrt{\epsilon}\, m_{Pl}} \ll \zeta_\sigma \simeq \Omega_{\sigma,dec.}\frac{H}{\sigma} \qquad if\ \sigma \ll \sqrt{\epsilon}\, m_{Pl}, \quad and \quad \Omega_{\sigma,dec.} \simeq 1$$

New extra parameter: σ expectation value during inflation

PURE CURVATON PERTURBATIONS

$$\zeta \simeq \Omega_{\sigma,dec} \frac{\delta\sigma}{\sigma} \simeq \Omega_{\sigma,dec} \frac{H}{\sigma} \qquad \sigma \ll m_{Pl}$$

OBSERVABLES: P_ζ, n_S, r		CONSTRAINTS with $\sigma \ll m_{Pl}$: V, V', V''
$P_\zeta \simeq \Omega_{\sigma,dec} \frac{V}{\sigma^2 m_{Pl}^2}$	$\rightarrow$	$V \ll 10^{-7} \epsilon^2 m_{Pl}^4$ COBE bounds
$r \equiv \frac{P_T}{P_\zeta} \simeq \frac{16}{\Omega_{\sigma,dec}} \left(\frac{\sigma}{m_{Pl}}\right)^2 \ll 1$	$\rightarrow$	No gravity waves observable
$n_S \simeq 1 - 2\epsilon$	$\rightarrow$	$\epsilon \ll 1 \qquad \eta \sim 1$

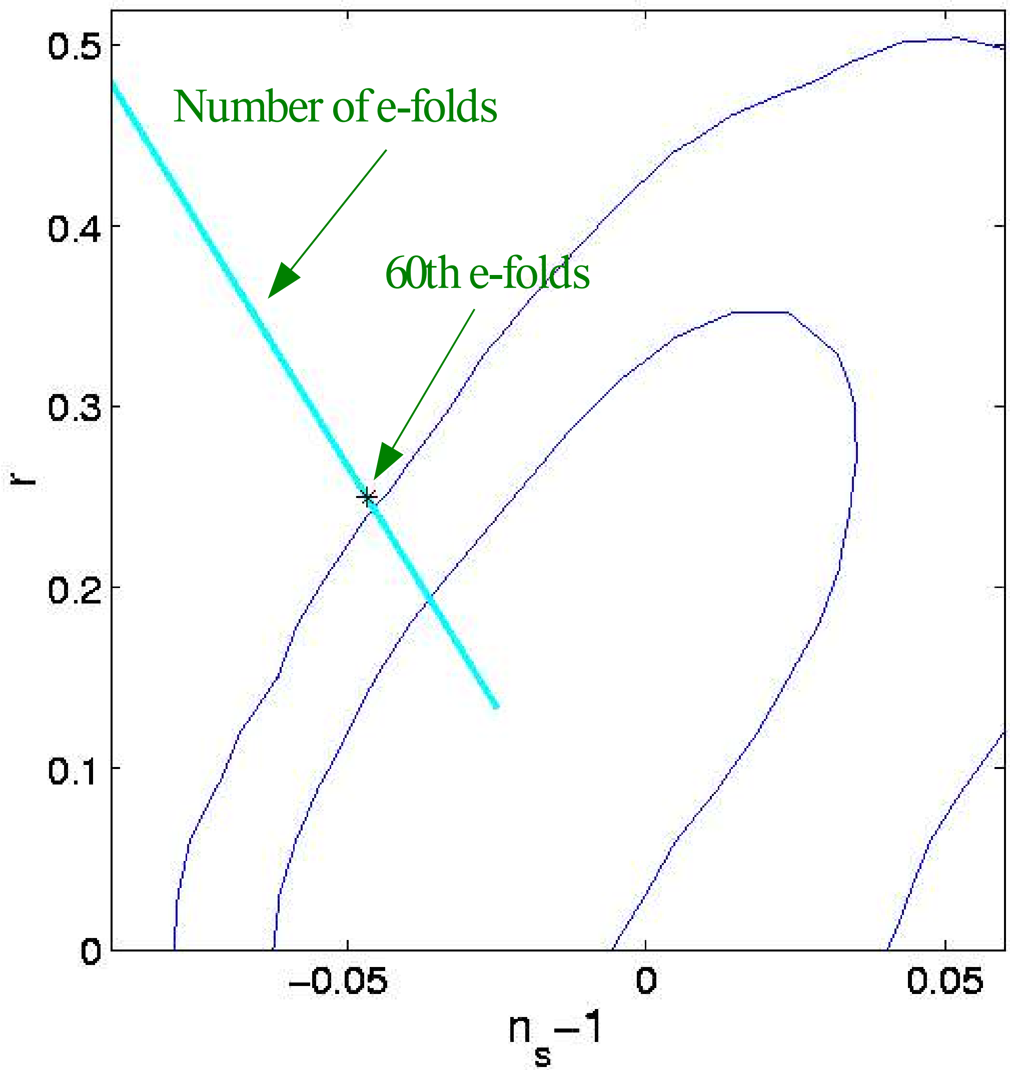
QUARTIC INFLATION

$$V(\phi) = \lambda \phi^4$$

Quartic inflation is excluded at 95% C.L. by combined WMAP data

[Peiris et al., '03]

[Leach and Liddle, '03]

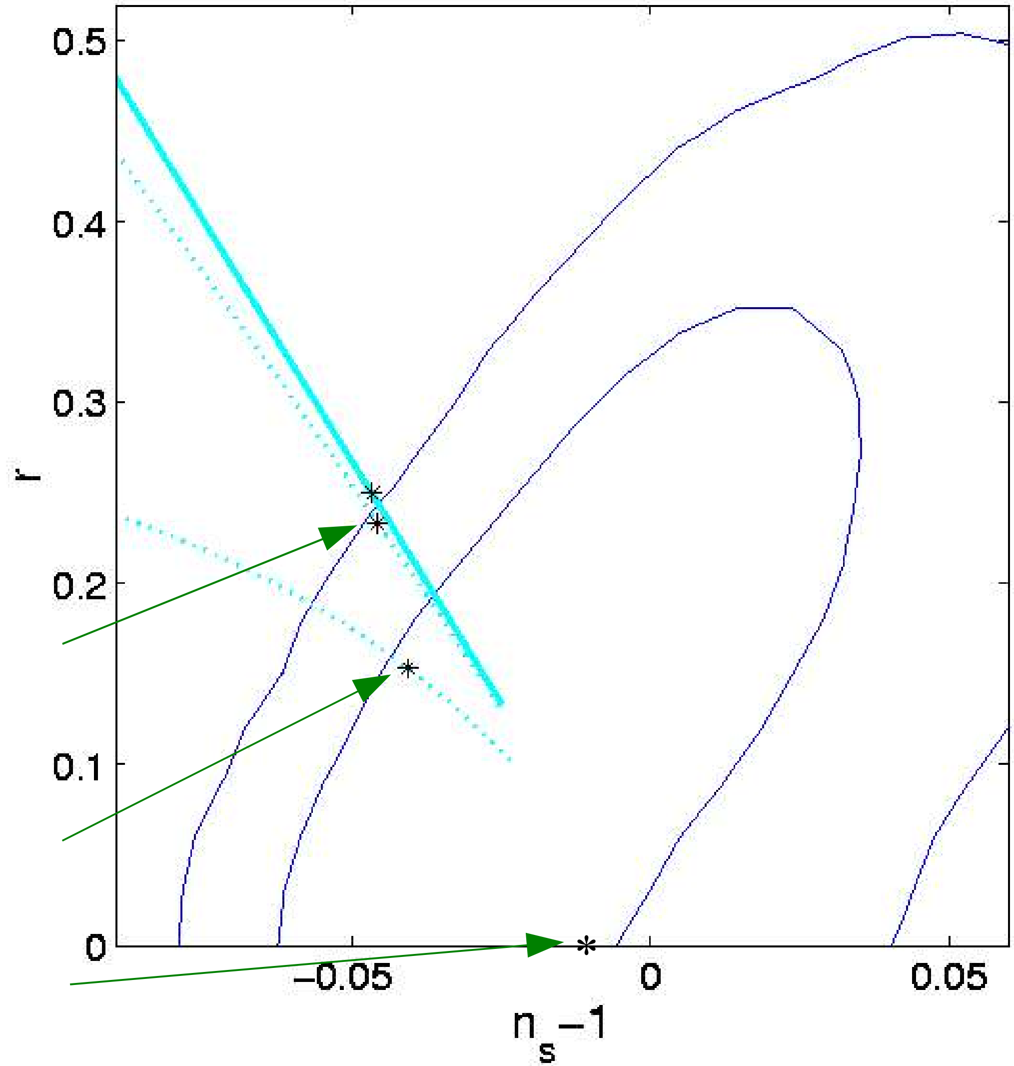


QUARTIC INFLATION + CURVATON

$$V(\phi) = \lambda\,\phi^4 \quad with \quad \sigma$$

Assume: $\Omega_{\sigma,dec.} = 1$

[Langlois and F.V., '04]



Mixed perturbations with $\sigma \sim 0.5\, m_{Pl}$

Mixed perturbations with $\sigma \sim 0.1\, m_{Pl}$

Pure curvaton perturbations $\sigma \ll m_{Pl}$

NON-GAUSSIANITIES

INFLATION: Non-Gaussianities generated from inflation are small:

[Maldacena, '03; Acquaviva et al., '02; Bernardeau and Uzan, '02]

$\delta\phi \sim H$ is a Gaussian field. And $\zeta = \dfrac{\delta\rho_\phi}{3(\rho_\phi + P_\phi)}$

$$\rho_\phi \simeq V(\phi),\ \delta\rho_\phi = V'(\phi)\delta\phi + \frac{1}{2}V''(\phi)\delta\phi^2$$

We need the non-linear term to be large

$$\frac{V''\delta\phi^2}{V'\delta\phi} = \eta \times \frac{1}{\sqrt{\epsilon}}\frac{\delta\phi}{m_{Pl}} \simeq \eta \times 10^{-5}!$$

Need to break slow-roll!

CURVATON:

$$\rho_\sigma \simeq m_\sigma^2\sigma^2 \quad \Rightarrow \quad \zeta \simeq -\frac{1}{2}\Omega_{\sigma,dec.}\frac{\delta\rho_\sigma}{\rho_\sigma} = -\Omega_{\sigma,dec.}\left[\frac{\delta\sigma}{\sigma} + \frac{1}{2}\left(\frac{\delta\sigma}{\sigma}\right)^2\right]$$

Simple characterization of non-Gaussianities:

[Verde et al., '00; Komatzu and Spergel, '01]

$$\zeta = \zeta_L - \frac{3}{5}f_{NL}\zeta_L^2 \quad \Rightarrow \quad f_{NL} \simeq \frac{5}{4\,\Omega_{\sigma,dec.}} \quad for \quad \Omega_{\sigma,dec.} \ll 1$$

[Lyth, Ungarelli and Wands, 03]

Compare with: $f_{NL} \sim 100$ of WMAP and $f_{NL} \sim 5$ of Planck

REHEATING AND THE CURVATON IN THE LAB

• The Minimal Supersymmetric Standard Model contains many flat directions (directions in the field space where V ~ 0): curvaton as flat direction of the MSSM. [Mazumdar and Enqvist, '03; Enqvist, '04]

• Possibility to see the curvaton in the laboratory if LHC sees SUSY

Superluminal expansion	INFLATON
Origin of matter: reheating	CURVATON
Density perturbations	CURVATON

• Baryons and leptons may have been generated by the curvaton (Affleck-Dine field) [Hebecker, March-Russel, Yanagida, '02; Moroi and Murayama, '02; MacDonald, '03]

(Small) Isocurvature perturbations

ADIABATIC vs ISOCURVATURE

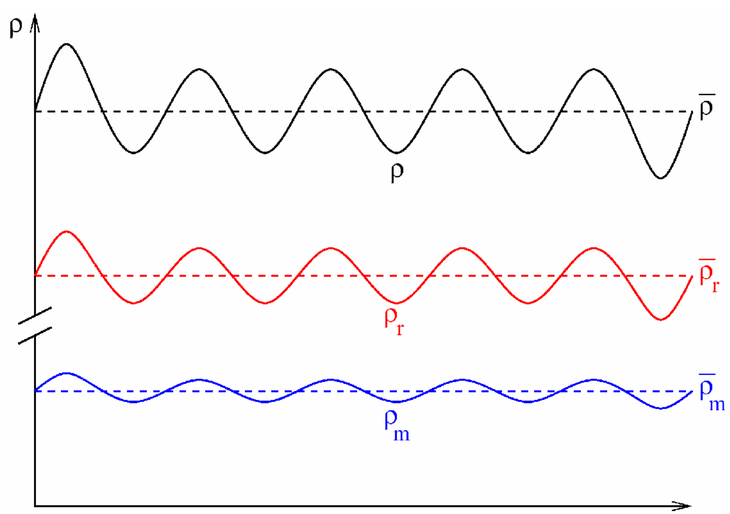
ADIABATIC: Perturbation affecting all the cosmological species such that

$$\delta\left(n_X/n_Y\right)=0$$

$$\frac{\delta\rho_X}{(1+w_X)\rho_X}=\frac{\delta\rho_Y}{(1+w_Y)\rho_Y}$$

It is thus associated with a curvature perturbation:

$$\zeta=\frac{\delta\rho_X+\delta\rho_Y}{3(\rho+P)}\neq 0$$



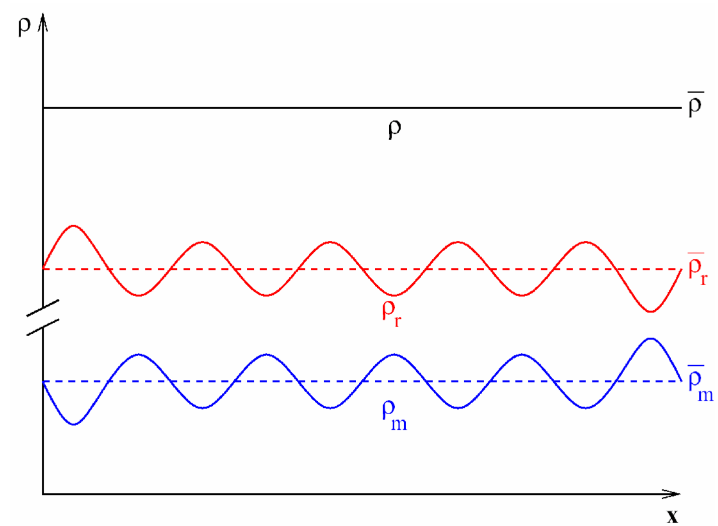
ISOCURVATURE: Perturbations in the matter components that does not perturb the geometry

$$\delta\rho_X+\delta\rho_Y=0$$

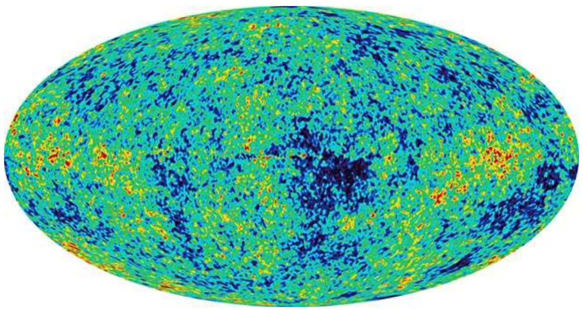
It is thus associated with a relative entropy perturbation:

$$\delta\left(n_X/n_Y\right)\neq 0$$

$$\frac{\delta\rho_X}{(1+w_X)\rho_X}-\frac{\delta\rho_Y}{(1+w_Y)\rho_Y}\neq 0$$



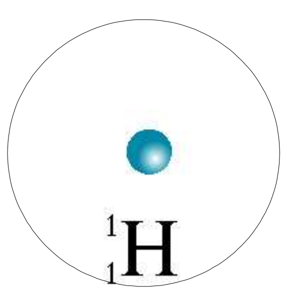
SUMMARY

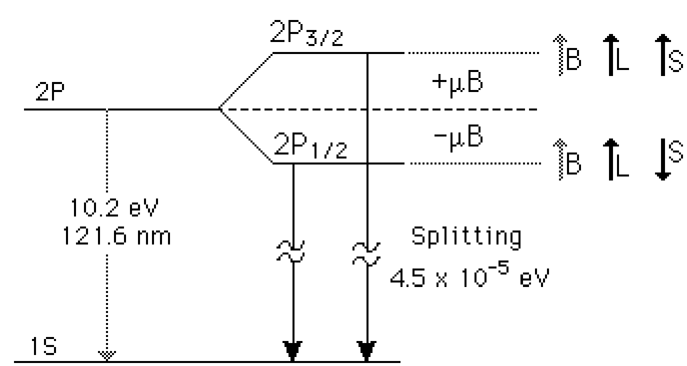


Perturbations:
- Adiabatic
- Gaussian
- Scale-invariant

Fine structure:
- $\longrightarrow$ Small isocurvature perts
- $\longrightarrow$ Small non-Gaussianities
- $\longrightarrow$ Small deviation from scale-invariance

Observables	Values	INFLATION	CURVATON
P_ζ	$(2\times10^{-5})^2$	Y	Y
n_S	$\simeq 0$	Y	N
r	$\simeq 0$	?	N
f_{NL}	$\simeq 0$	N	Y
Isocurvature	$\simeq 0$	N	Y





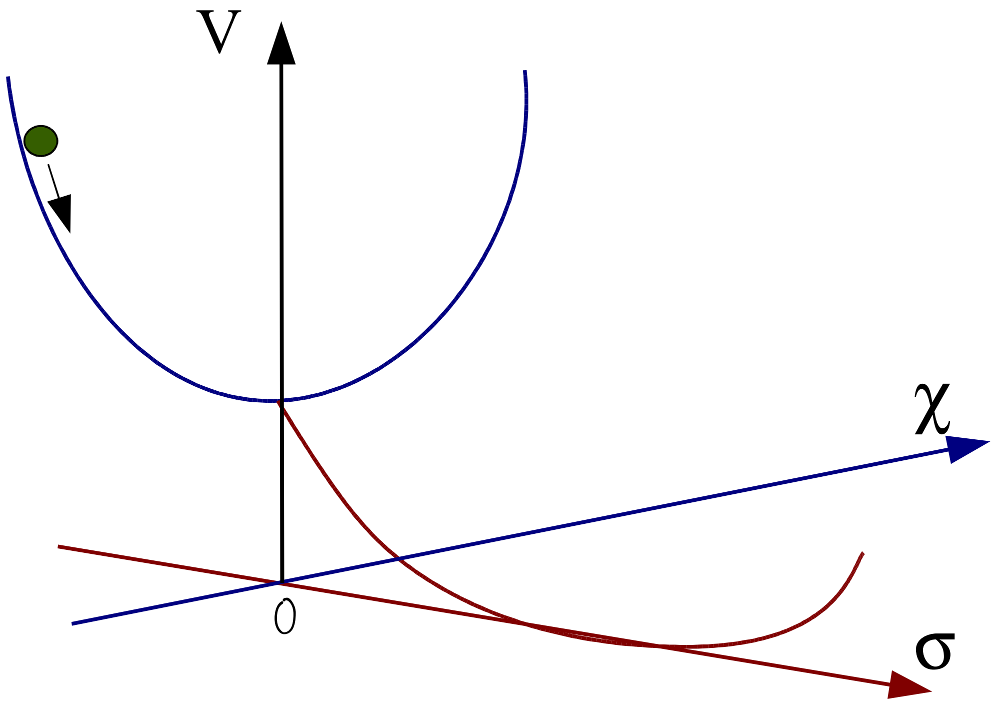
FROM HEAVINESS TO LIGHTNESS

[Langlois and F.V., in prep.]

During inflation, H slowly decreases: a field χ which is initially light, $m_\chi \ll H$, becomes heavy when $m_\chi \gg H$, before the end of inflation.

What happens if the curvaton couples to χ?

$$V(\sigma,\chi) = \frac{1}{2} m_\sigma^2 \sigma^2 + \frac{1}{2} m_\chi^2 \chi^2 + \frac{1}{2} g^2 \sigma^2 \chi^2 \quad with \quad m_\sigma \ll m_\chi$$



$$\chi \simeq a^{3/2} \sin(m_\chi t)$$

$$m_\sigma^{(eff)2} = m_\sigma^2 + g^2 X_0^2 e^{-3Ht} \sin^2(m_\chi t)$$

Time dependent mass:

σ-particle production

and change in σ-spectrum

PERTURBATION EQUATION: POTENTIAL

$$\frac{d^2 v_k}{d\eta^2} + \left(k^2 + \frac{m_\phi^{(eff)2}/H^2 - 2}{\eta^2} \right) v_k = 0$$

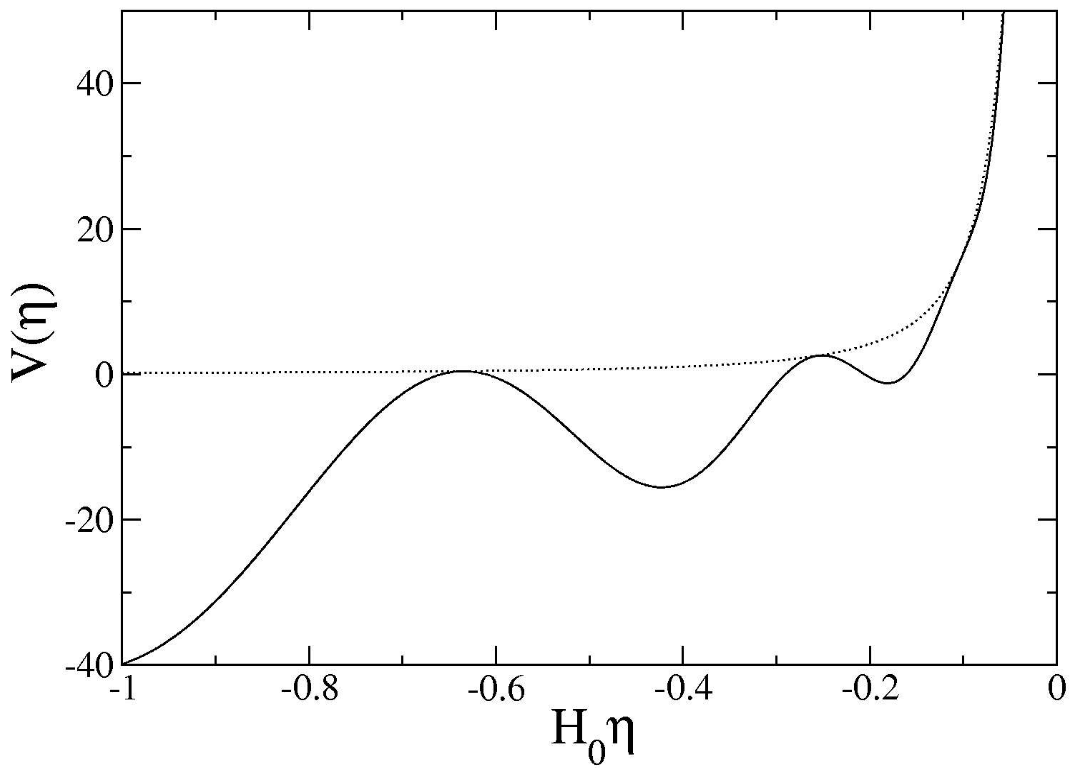
Canonical equation in de Sitter

Time dependent potential

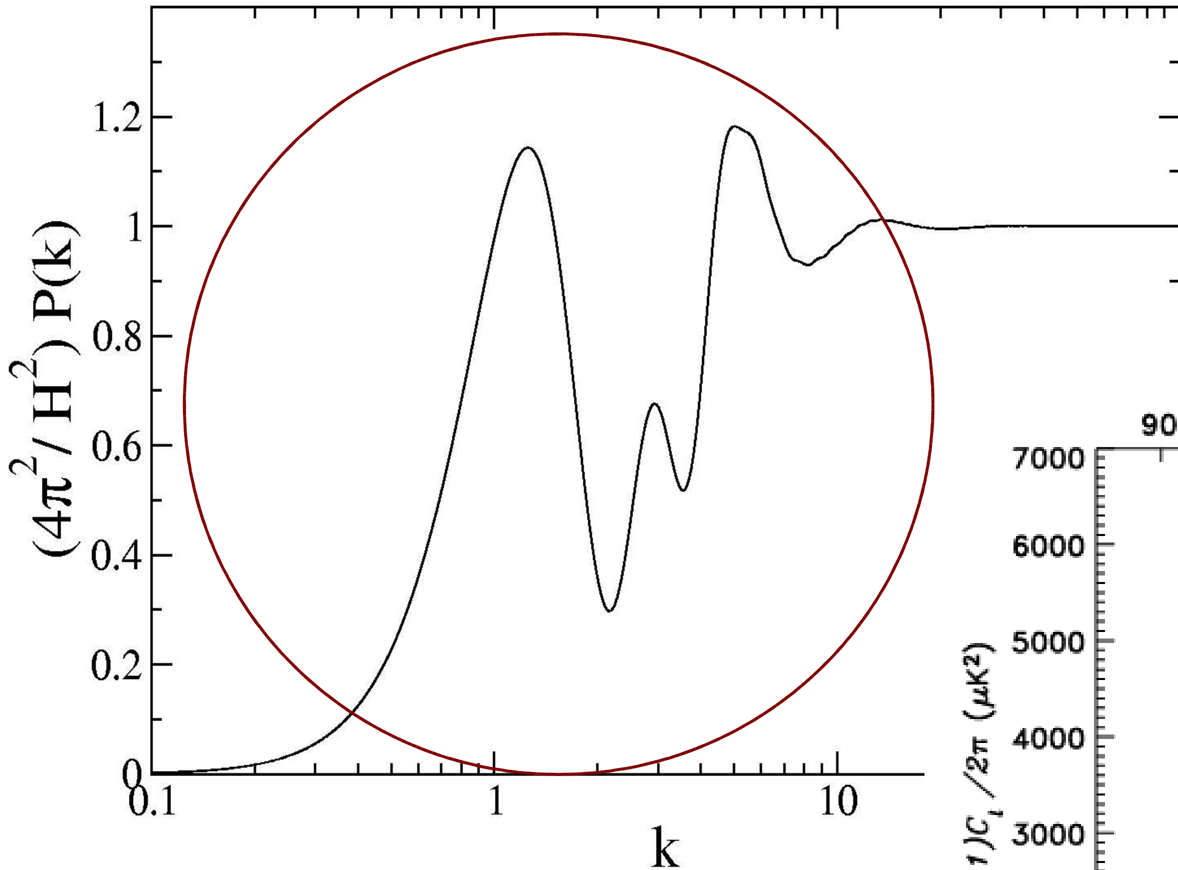
$$V(\eta) = \frac{-m_\phi^{(eff)2}/H^2 + 2}{\eta^2}$$

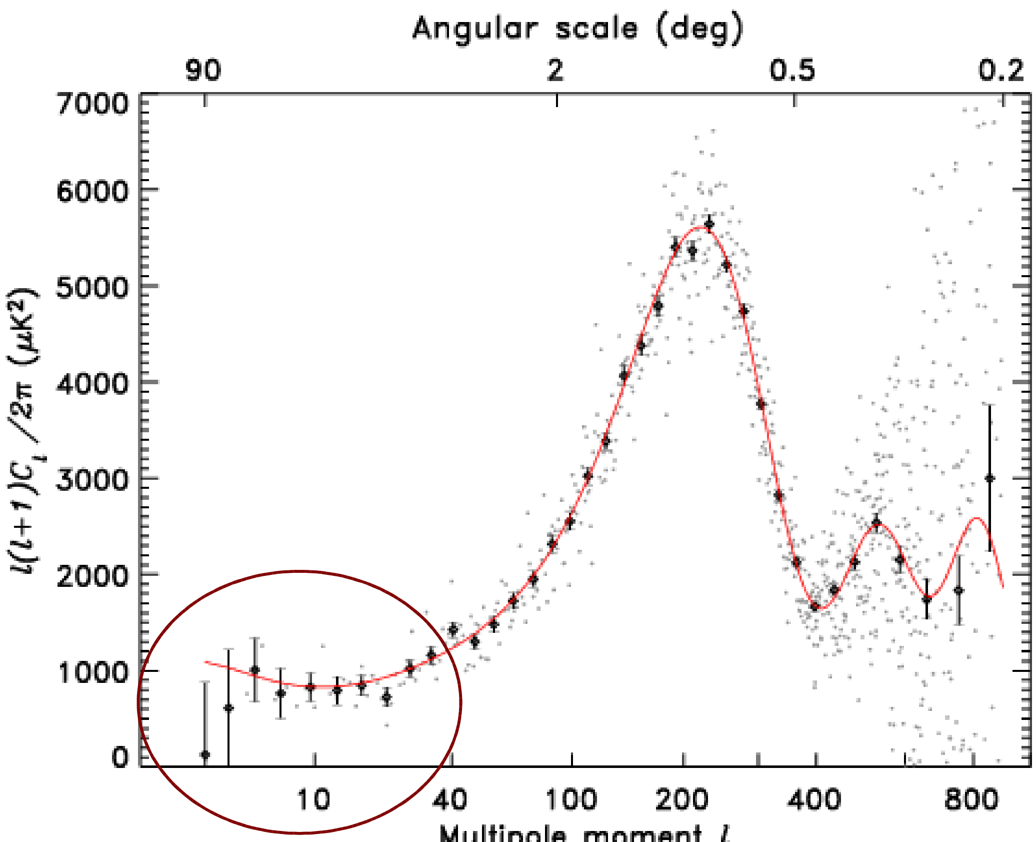
$$m_\sigma^{(eff)2} = m_\sigma^2 + g^2 X_0^2 e^{-3Ht} \sin^2(m_\chi t)$$

Time dependent mass



FEATURES IN THE SPECTRUM





CONCLUSIONS

Why the curvaton?

- Light fields are generically predicted by supersymmetric models
- Separating the field responsible for superluminal expansion (inflaton) from the field responsible for density perturbations (curvaton) and
relax the constraints on the inflaton potential
- Contact with particle physics

Observational consequences:

- Inflation is more difficult to be tested
- The curvaton changes the predictions in the (n_s,r)-plane and introduces a new degeneracy (σ parameter)
- Features in the spectrum of perturbations may be present
- Small non-Gaussianities