- $\langle\mu\rangle$ IN THE MAXIMUM ABELIAN GAUGE

$$\langle\mu\rangle = \frac{Z[S+\Delta S]}{Z[S]}$$

IF ΔS IS KNOWN

$$\rho = \frac{\partial \ln\langle\mu\rangle}{\partial\beta} = \langle S\rangle_S - \langle S+\Delta S\rangle_{S+\Delta S}$$

$$\langle\mu\rangle = \exp\left(\int_0^\beta \rho(\beta')\, d\beta'\right)$$

IN THE MAX ABELIAN GAUGE

$$S_V[\Phi] = \sum_{\mu\nu} \mathrm{Tr}\left\{U_\mu(n)\sigma_3 U_\mu^\dagger(n)\sigma_3\right\}$$

IS MAXIMIZED WITH RESPECT TO GAUGE $U_\mu(n) = V(n)\,\dot{U}_\mu(n)\,V^\dagger(n+\hat{\mu})$

AND THE 'OPERATOR Φ IS DETERMINED BY MAXIMIZATION.

$\Longrightarrow$ DIRECT DETERMINATION OF $\langle\mu\rangle$

PROCEDURE

- DETERMINE PROBABILITY DISTRIBUTION OF $\ln\langle\mu\rangle = -\beta\Delta S$
- RECONSTRUCT $\langle\mu\rangle$ BY CUMULANTS EXPANSION

$$\int dX\, p(X)\, e^{\beta(X-\langle X\rangle)} = e^{\sum c_n(\beta^n/n!)}$$

$$c_1 = 0 \qquad c_2 = \langle\Delta^2\rangle \qquad c_3 = \langle\Delta^3\rangle, \; c_4 = \langle\Delta^4\rangle - 3\langle\Delta^2\rangle^2 \quad \ldots$$

$\Rightarrow$ fig. 3

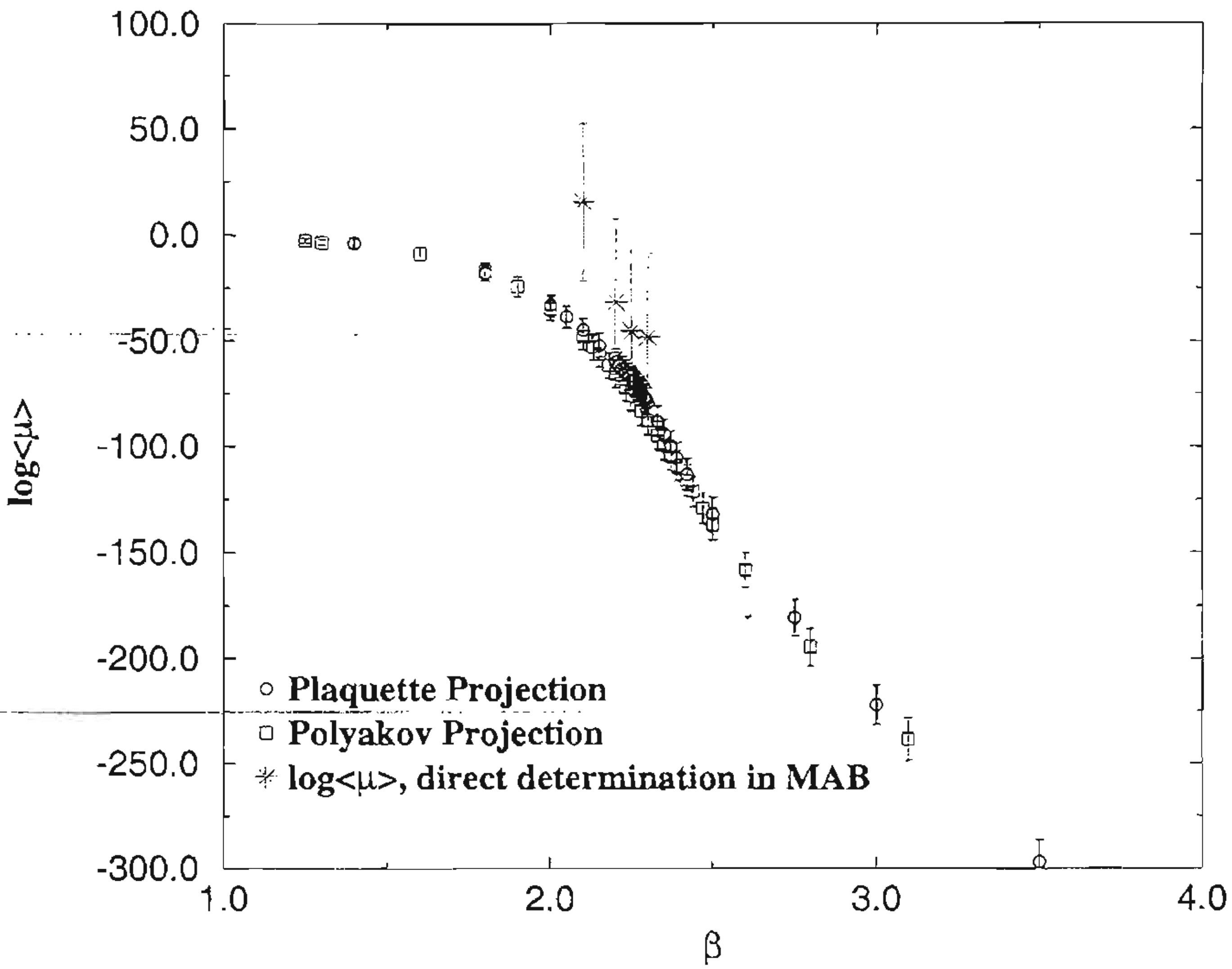
(SEE CHERNODUB et al Phys Lett B399, 267, 1997)

COMPUTE $\langle\mu\rangle$ as $\exp\int_0^\beta d\beta' \rho(\beta')$ FOR THE GAUGES IN WHICH ΔS IS KNOWN. COMPUTE BY CLUSTER EXPANSION [h_γ]

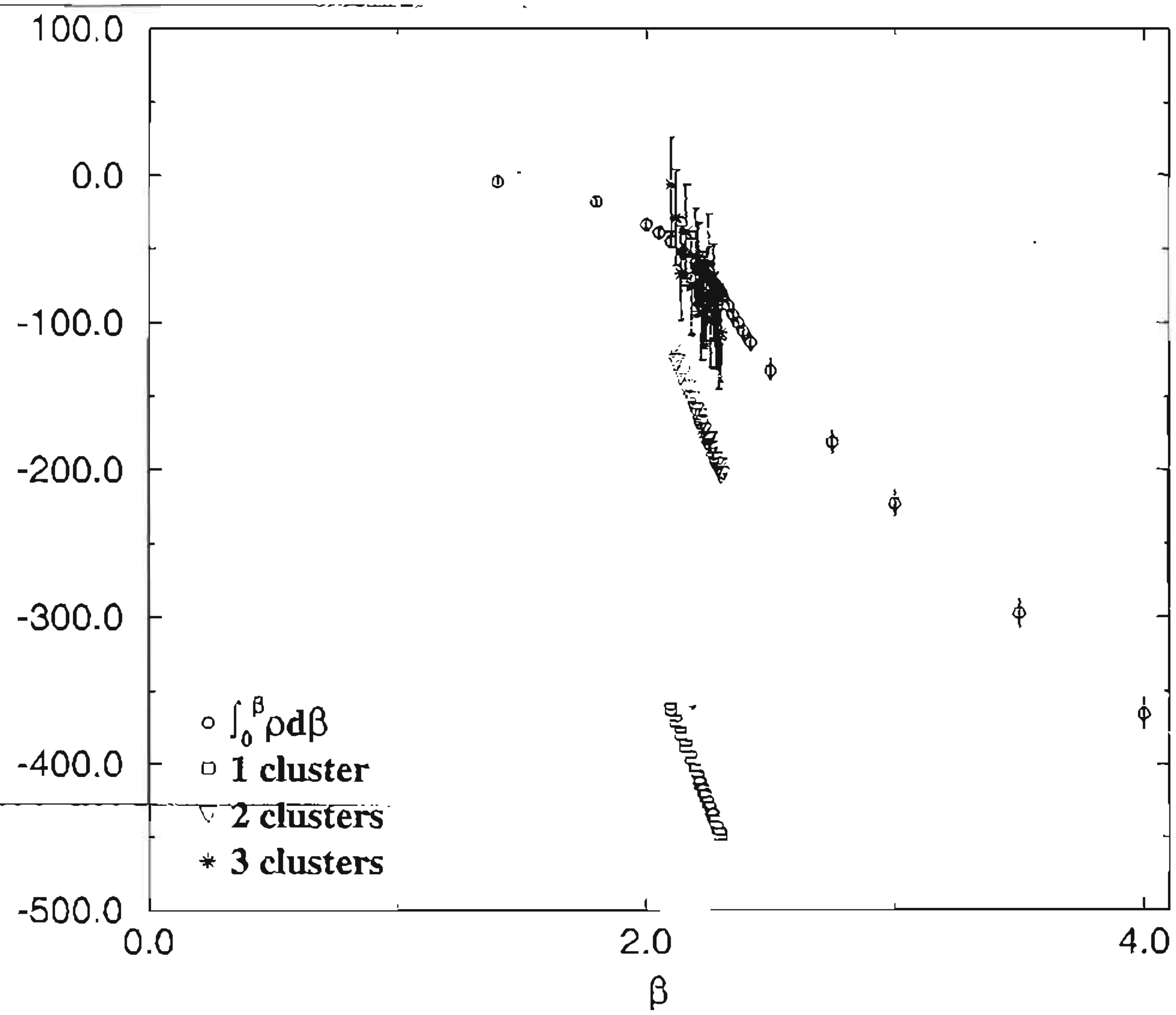
C_1, C_2, C_3 NEEDED. GOING TO HIGHER C_n's ERROR INCREASES C_4 COMPATIBLE WITH 0.

- DO THE CLUSTER EXPANSION FOR MAX ABELIAN GAUGE: COMPATIBLE WITH OTHER GAUGES.

$\langle\mu\rangle$ IN THE MAX. ABELIAN PROJECTION EQUAL WITHIN ERRORS TO $\langle\mu\rangle$ IN OTHER PROJECTIONS.



100.0
50.0
0.0
-50.0
-100.0
-150.0
-200.0
-250.0
-300.0
log<μ>
1.0
2.0
3.0
4.0
β
Plaquette Projection
Polyakov Projection
log<μ>, direct determination in MAB


100.0
0.0
-100.0
-200.0
-300.0
-400.0
-500.0
0.0
2.0
4.0
β
$\int_0^\beta \rho d\beta$
1 cluster
2 clusters
3 clusters

- SU(3) PURE GAUGE [A.D.G., L.Montesi, B.Lucini, G.Paffuti] II

$\langle\mu^a\rangle$ $a=1,2$ $\Phi^a_{diag}=\begin{pmatrix}2&&0\\&-1&\\0&&-1\end{pmatrix}$ $a=1$ $T^a=\begin{pmatrix}1&0&0\\0&-1&0\\0&0&0\end{pmatrix}$ $a=1$

$\Phi^a_{diag}=\begin{pmatrix}1&&\\&1&\\&&-2\end{pmatrix}$ $a=2$ $T^a=\begin{pmatrix}0&0&0\\0&1&0\\0&0&-1\end{pmatrix}$ $a=2$

- ρ FOR DIFFERENT ABELIAN PROJECTIONS
- $a=1$ VS $a=2$
- SIZE DEPENDENCE
- $T<T_c$
- $T>T_c$ $\rho\simeq -2L_s-12$
- $T\simeq T_c$ FINITE SIZE SCALING

$$\rho/L_s^{1/\nu}=f(\tau L_s^{1/\nu})+\frac{a}{L_s^3}+O(L_s^{-6})$$

$\rho_{PEAK}\propto L_s^3$

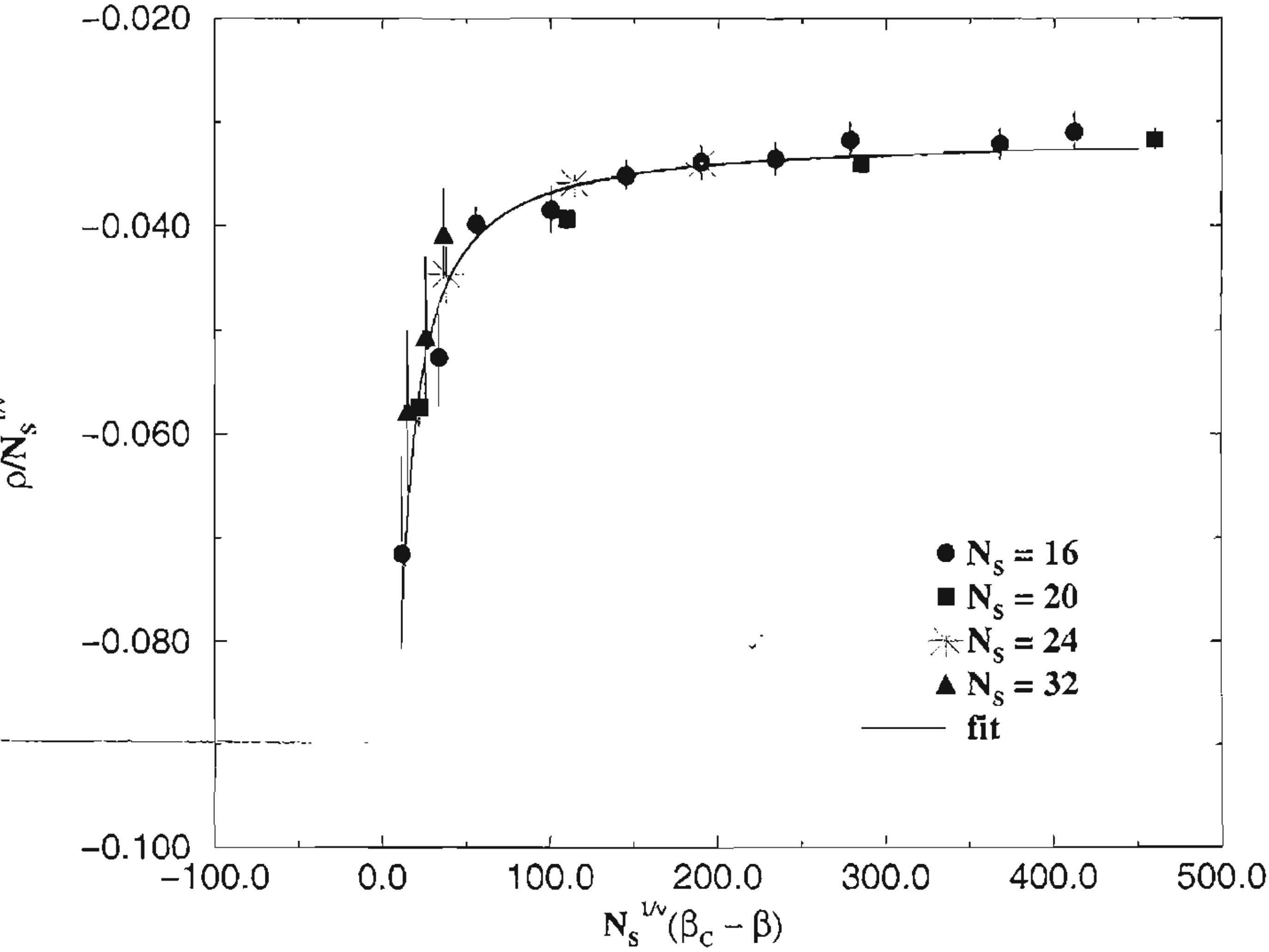
$\nu=.33(7)$ $\beta_c=5.6925$ $N_t=4$

CONSISTENT WITH 1st ORDER ($\nu=1/3$)
AND WITH RESULTS FROM $\langle L\rangle$

- CONCLUSION: IN PURE GAUGE SU(2) SU(3) GAUGE THEORY CONFINED PHASE IS DUAL SUPERCONDUCTING. DECONFINED IS NORMAL AND MAGNETIC CHARGE SUPERSELECTED

A DETAILED STUDY OF SUPERSELECTION
[M.D'Elia, A.D.G., B.Lucini] figs.

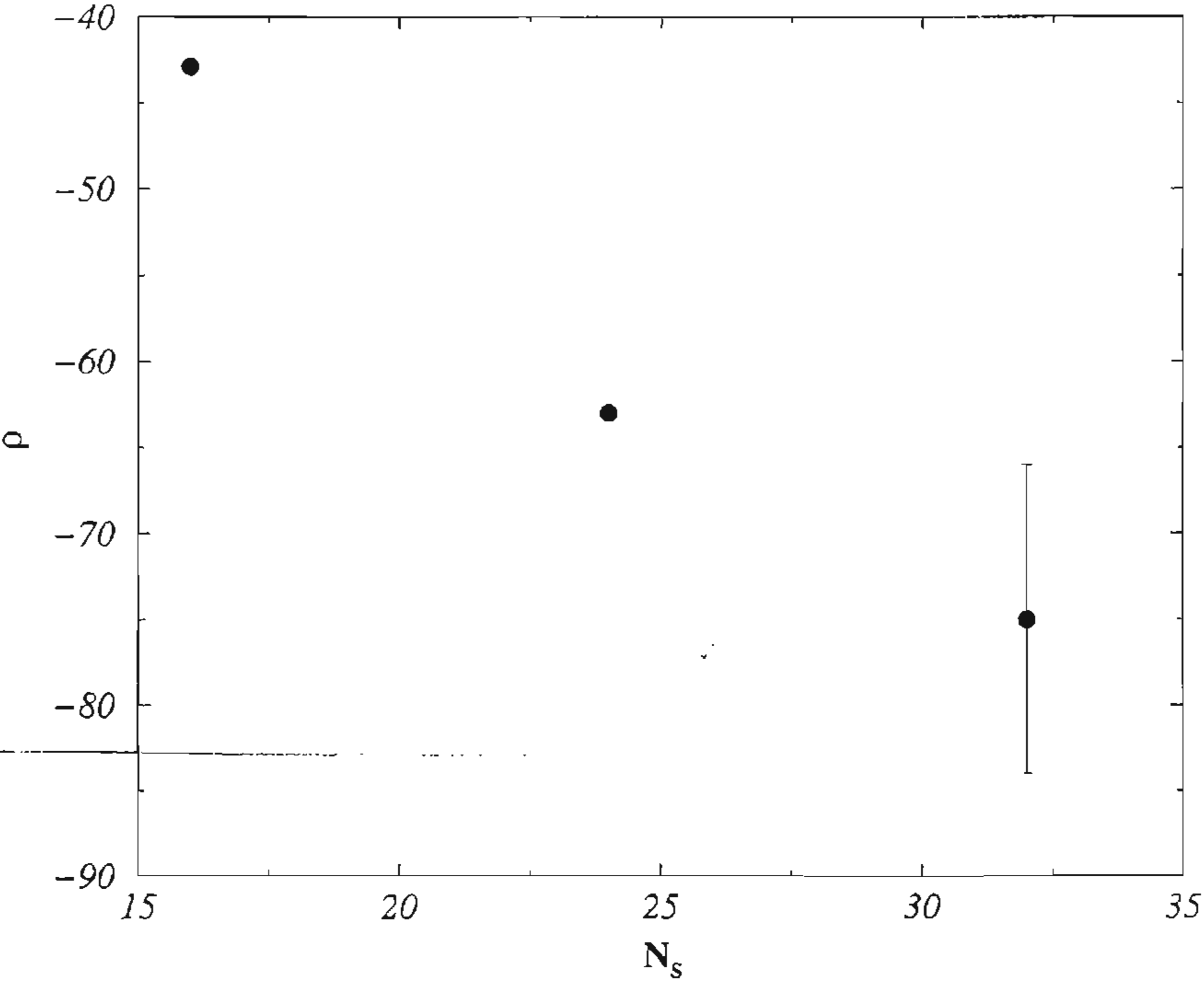
$SU(3)_5$



SCALING $\rho/N_S^{1/\nu} = f(\tau N_S^{1/\nu})$

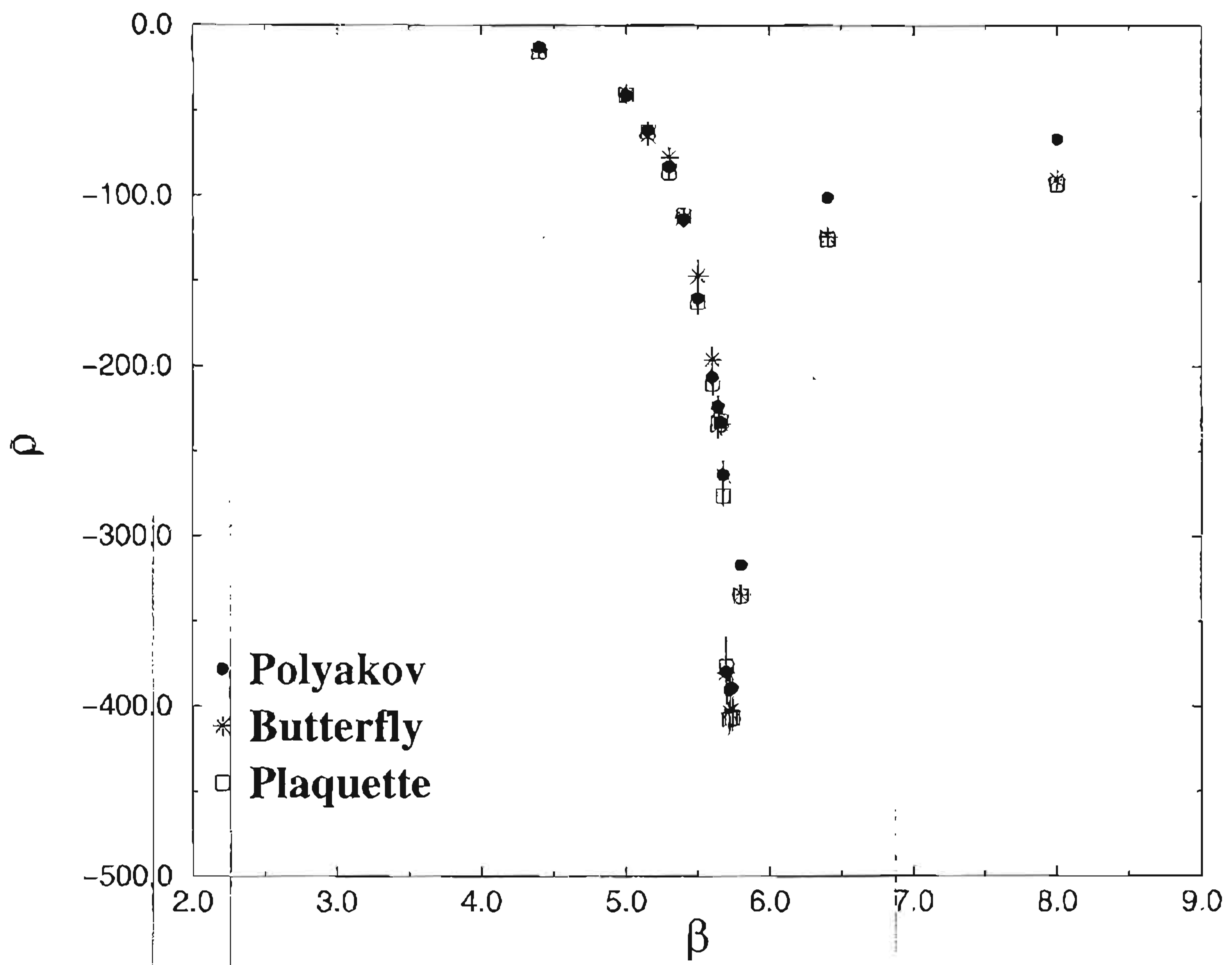
CURVE $\left\{ \begin{array}{l} \beta_c = 5.6925 \\ \nu = .33(1) \end{array} \right.$

$SU(3)_4$



ρ vs N_S $\qquad$ $\beta > \beta_c$

$\rho = -2N_S - 12 \Rightarrow \langle \mu \rangle = 0$



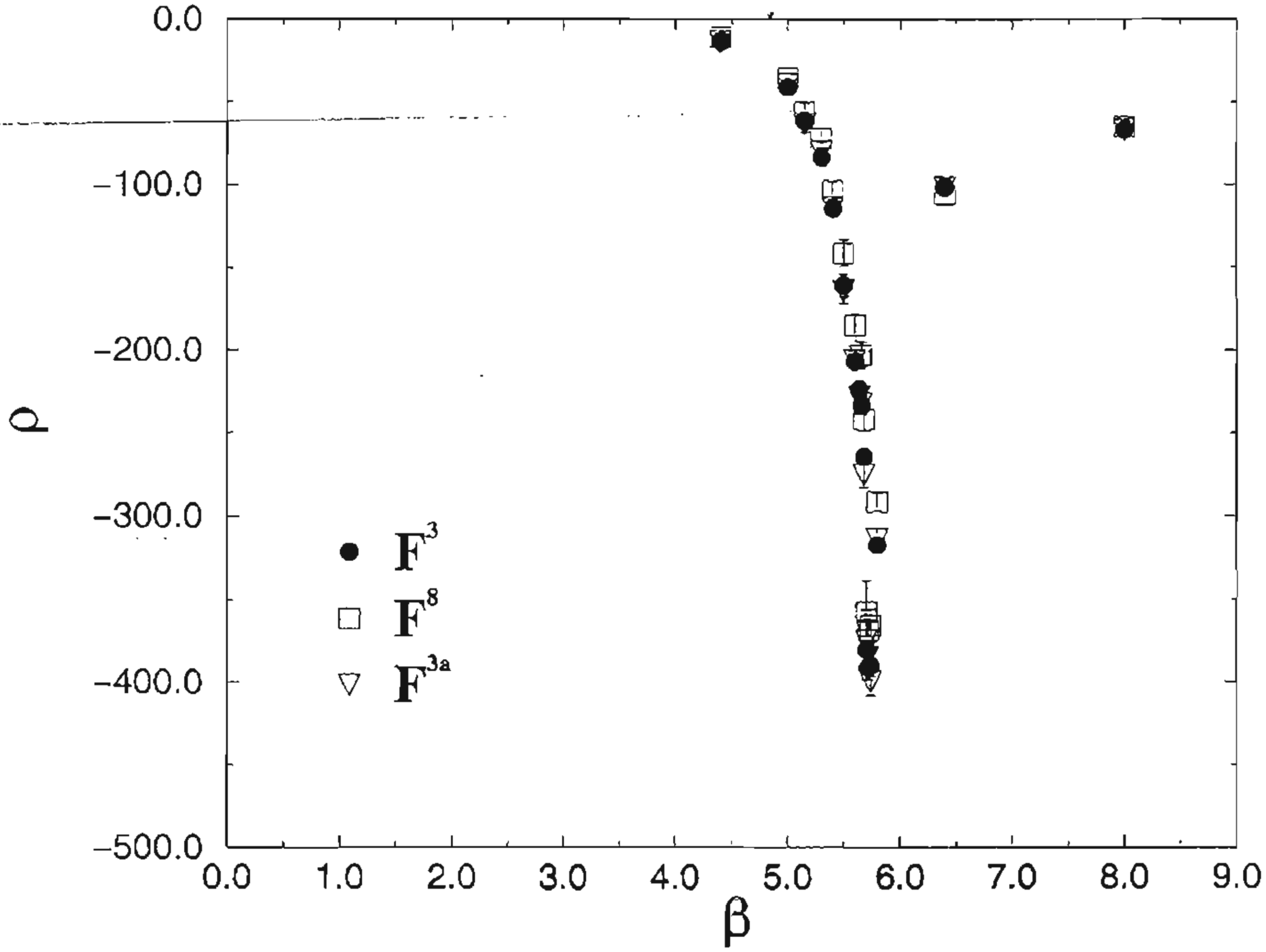
PURE GAUGE SU(3)

ρ VS β FOR DIFFERENT ABELIAN PROJ.'S

$12^3 \times 4$

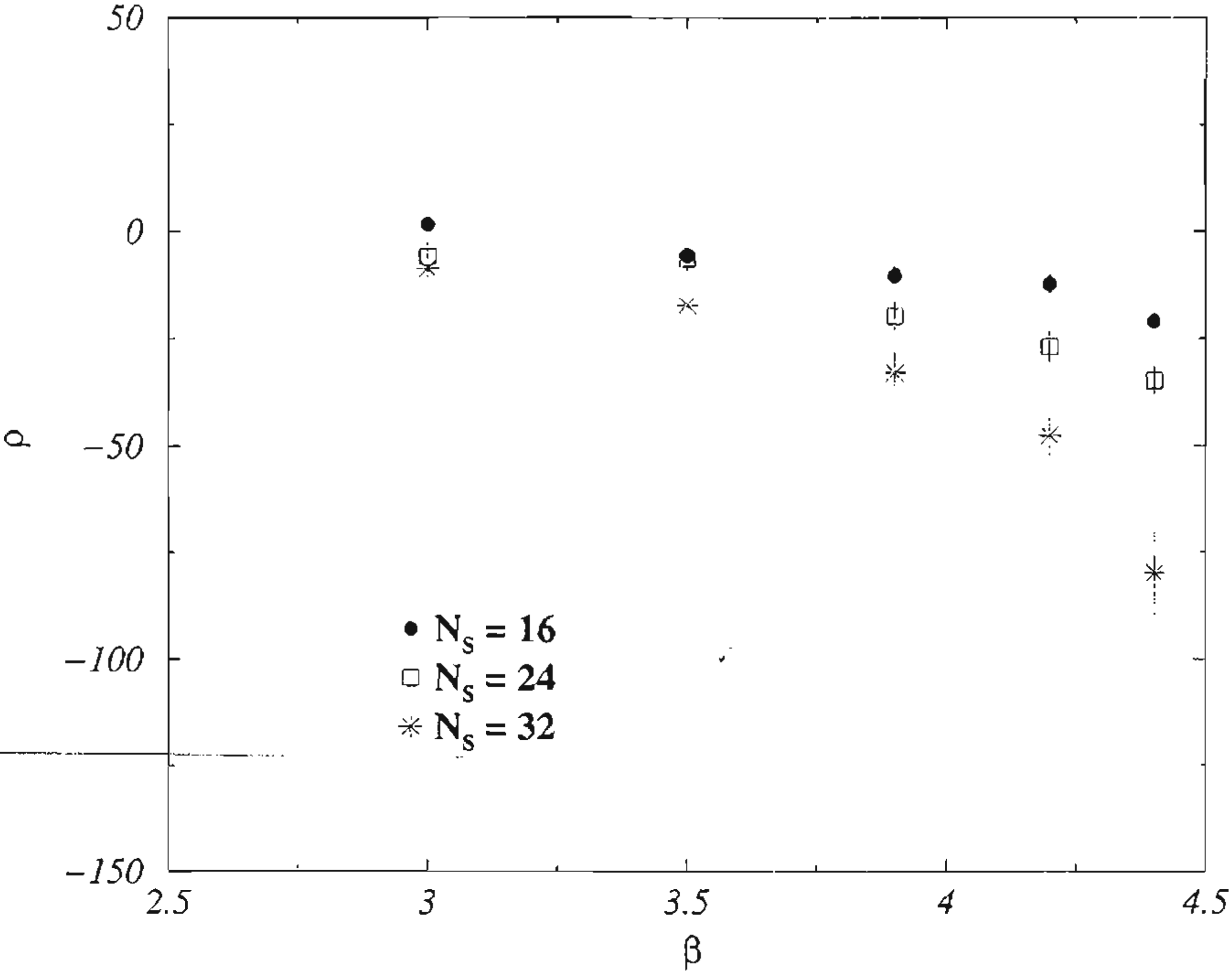
A.D.G, B. LUCINI, L. MONTESI, G. PAFFUTI PHYS. REV D61 034503 (2000)

SU(3) 2   a = 1   a = 2



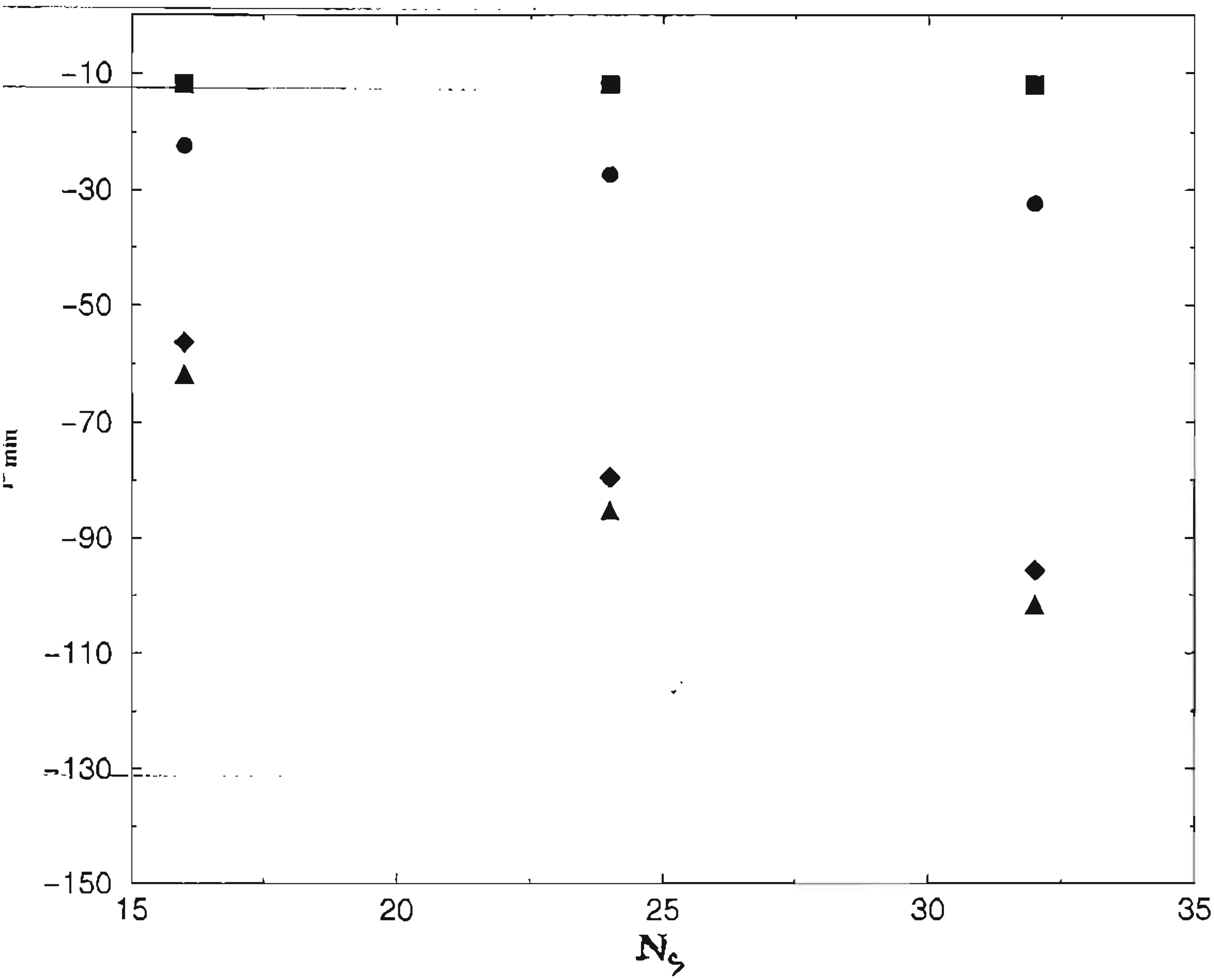
0.0
-100.0
-200.0
-300.0
-400.0
-500.0
ρ
0.0 1.0 2.0 3.0 4.0 5.0 6.0 7.0 8.0 9.0
β
F³
F⁸
F³ᵃ

SU(3)3



SU(3) PURE GAUGE $4 \times N_s^3$

$\beta < \beta_c$



ρ Vs N_S AT LARGE β SU(2)

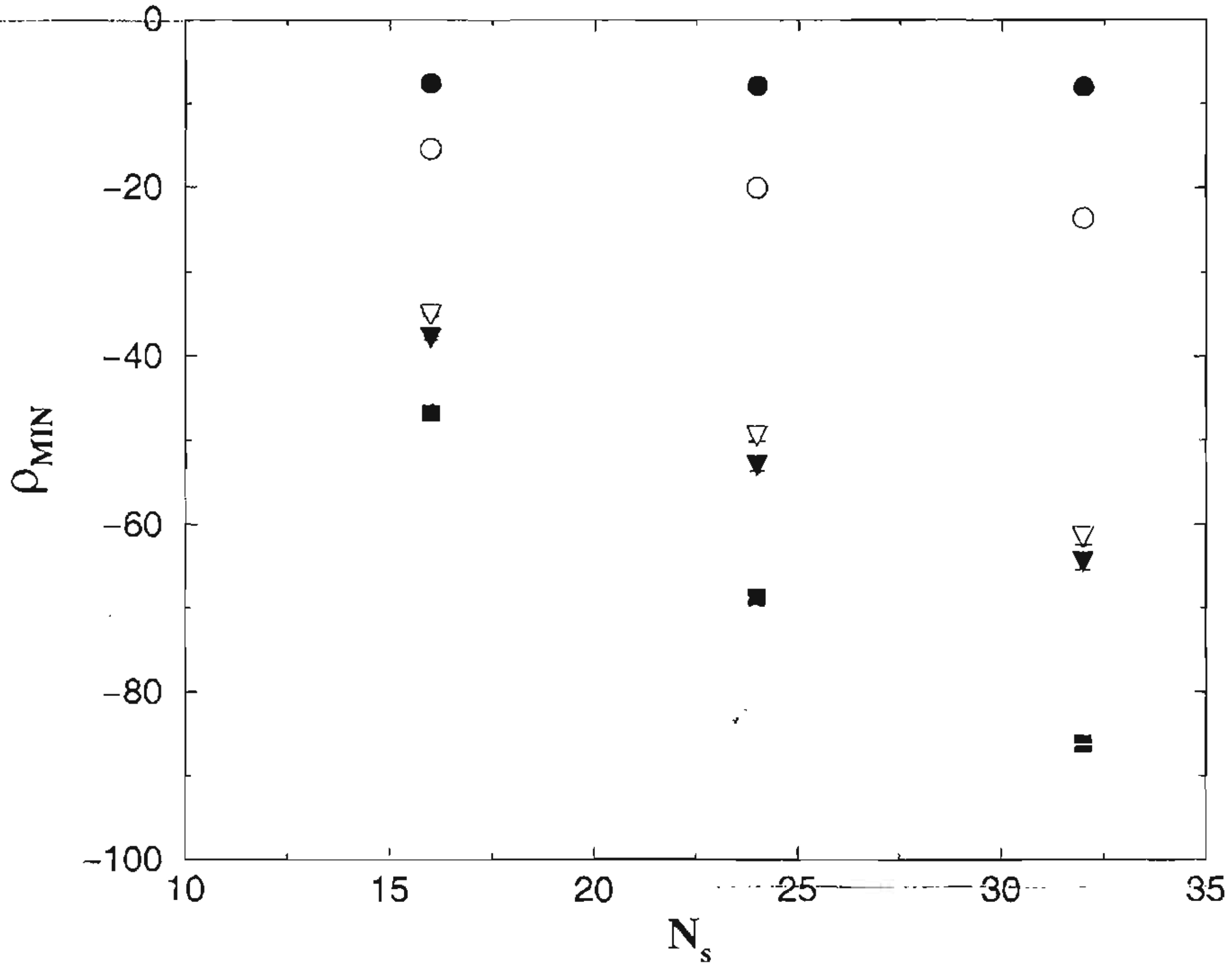
■ DIPOLE 2, -2

● SINGLE MONOPOLE $m=2$

◆ SINGLE MONOPOLE $m=4$

▲ 2 MONOPOLES WITH $m=2$ at $d=2$

M. D'Elia A.D.G., B. LUCINI
PHYS. REV D69, 077504 (2004)



ρ VS N_s AT LARGE β SU(3) G.T.

● DIPOLE 2 , −2

○ SINGLE MONOPOLE m = 2

▽ 2 MONOPOLES +2 +2

▼ MONOPOLE m = 4

■ MONOPOLE Q = 1 m = 2 ; ANTIMONOPOLE m = −2 , Q = 2

- (IV) DEPENDENCE ON THE ABELIAN PROJECTION
[J. CARMONA et al COLOR CONFINEMENT IV PhysRevD 01]

RANDOM ABELIAN PROJECTION: DO NOT FIX THE GAUGE AND MEASURE

$$\left\langle e^{-\beta \int d^3y \, \mathrm{Tr}\{\Phi_{\mathrm{diag}} \vec{E}(\vec{y},t)\} \vec{b}_\perp(\vec{x}-\vec{y})} \right\rangle$$

- AN AVERAGE OVER ABELIAN PROJECTIONS

- SAME RESULTS AS IN SPECIFIC ABELIAN PROJECTIONS
  SU(2) $\beta_c = 2.2986$ $\nu = .63$; SU(3) $\beta_c = 5.6925$ $\nu = 1/3$

- EMPIRICAL PROOF THAT DUAL SUPERCONDUCTIVITY OR ABSENCE OF IT IS AN ABELIAN PROJECTION INDEPENDENT STATEMENT

- THEORETICAL ARGUMENT (WITH EMPIRICAL INPUT)

• $$\mu_U(\vec{x},t) = e^{-\beta \int d^3y \, \mathrm{Tr}\{U(\vec{y},t)\Phi_{\mathrm{diag}} U^\dagger(\vec{y},t) \vec{E}(\vec{y},t)\} \vec{b}_\perp(\vec{x}-\vec{y})}$$

CREATES A SINGULARITY AT $\vec{x}, t$ IN THE GAUGE IDENTIFIED BY $U(\vec{y},t)$, WITH THE TOPOLOGY OF A MONOPOLE.

• $\mu_U(\vec{x},t)$ WILL CREATE THE SAME SINGULARITY IN ANY GAUGE OBTAINED FROM IT BY A TRANSFORMATION WHICH IS CONTINUOUS AROUND $(\vec{x},t)$

• IN ANY ABELIAN PROJECTION THERE IS A FINITE NUMBER OF MONOPOLES / $(\mathrm{fermi})^3$
[SITES WITH EQUAL EIGENVALUES OF Φ] (fig)

$$\langle \mu_U(\vec{x},t)\rangle = \frac{1}{Z}\int [\Pi\, dA_\mu(x)]\, e^{-S}\, \mathrm{Tr}\; e^{-\beta \int d^3y\, \mathrm{Tr}\{\Phi^a_{diag}\, U^+ \vec{E}\, U\}\, \vec{b}_\perp(\vec{x}-\vec{y})}$$

BY A GAUGE TRANSFORM $\quad A_\mu \to U^+ A_\mu U + i\partial_\mu U^+ U$

S IS INVARIANT, AND THE MEASURE IS INVARIANT APART FROM A SET OF ZERO MEASURE ! $U^+\vec{E}U \to \vec{E}$, I.E. $\langle \mu_U\rangle$ IS EQUAL TO $\langle\mu\rangle$ IN THE RANDOM GAUGE.

$|\langle\mu\rangle| \neq 0$
$\langle\mu\rangle = 0$
ARE ABSOLUTE STATEMENTS, INDEPENDEN OF THE CHOICE OF THE ABELIAN PROJECTION

AND SO IS DUAL SUPERCONDUCTIVITY.

- UNQUENCHED THEORY . QCD

IN THE SPIRIT OF $N_c \to \infty$ ONE EXPECTS THAT QUARKS DO NOT MODIFY THE MECHANISM OF CONFINEMENT

THE DEFINITION OF $\langle \mu \rangle$ STAYS UNCHANGED
WE SHALL ANALYZE THE RESULTS AFTER A SURVEY OF THE PHENOMENOLOGY OF THE DECONFINING TRANSITION.

WE SHALL IN PARTICULAR CONSIDER $N_F = 2$ QCD

THE BEHAVIOR IS ANALOGOUS TO THE QUENCED CASE (see trf)

THE TWO dx DIFFER BY

$$2\pi q\ \delta^3(\vec{x}-\vec{y})\ [\theta(x_0)-\theta(x_0-t)]$$
$$+2\pi q\ \delta(x_1)\delta(x_2)\delta(x_0)\ [\delta(x_0-y_0)-$$

WHICH IS A $2\pi q$ VALUED CLOSED LOOP.

- THE DIFFERENCE BETWEEN THE TWO APPROACHES IS IN THE POSITION OF THE DIRAC STRINGS

THE ADVANTAGE OF OUR APPROACH IS TO HAVE AN EXPLICIT OPERATOR $\langle\mu(\vec{x},t)\rangle$

FM PROVE THAT

(i) $\langle\mu\rangle \neq 0$ $T<T_c$ (VILAIN ACTION)
PC $=0\ T>T_c$ WILSON ACTION

(ii) THAT μ IS A DIRAC LIKE OPERATOR AND GAUGE INVARIANT

(iii) $T>T_c$ IS SUPERSELECTED

ANALOGY IN 2D ISING

DEF

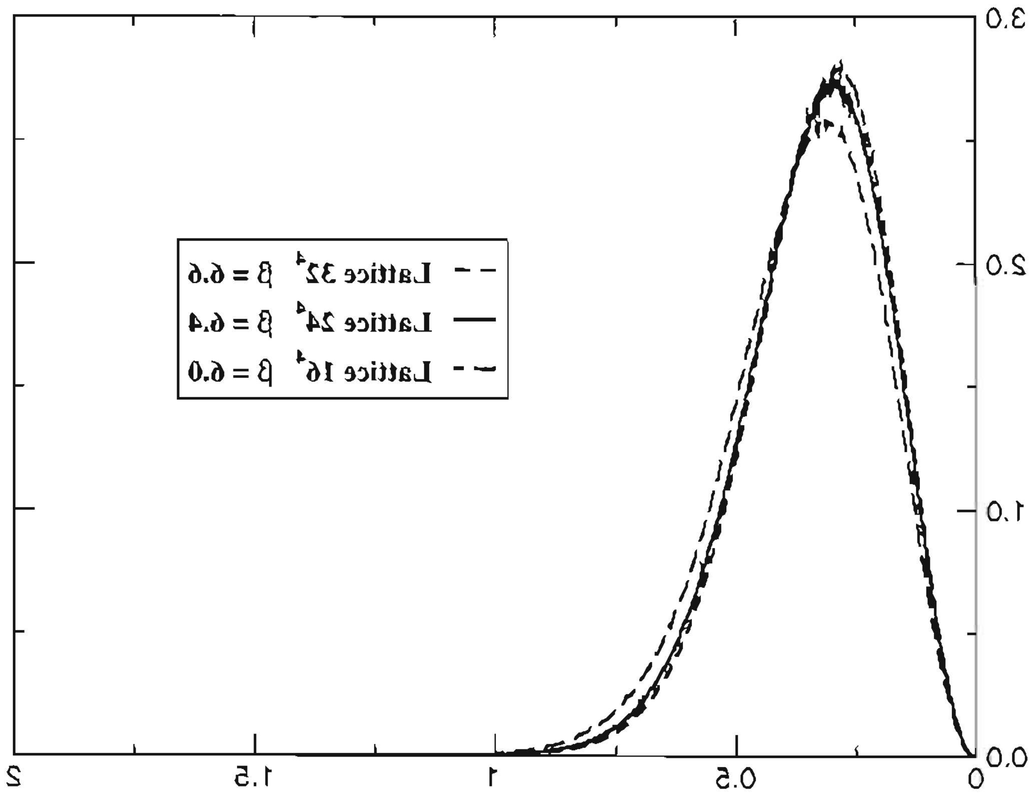
$\langle\bar{\mu}\ \ \mu\rangle$

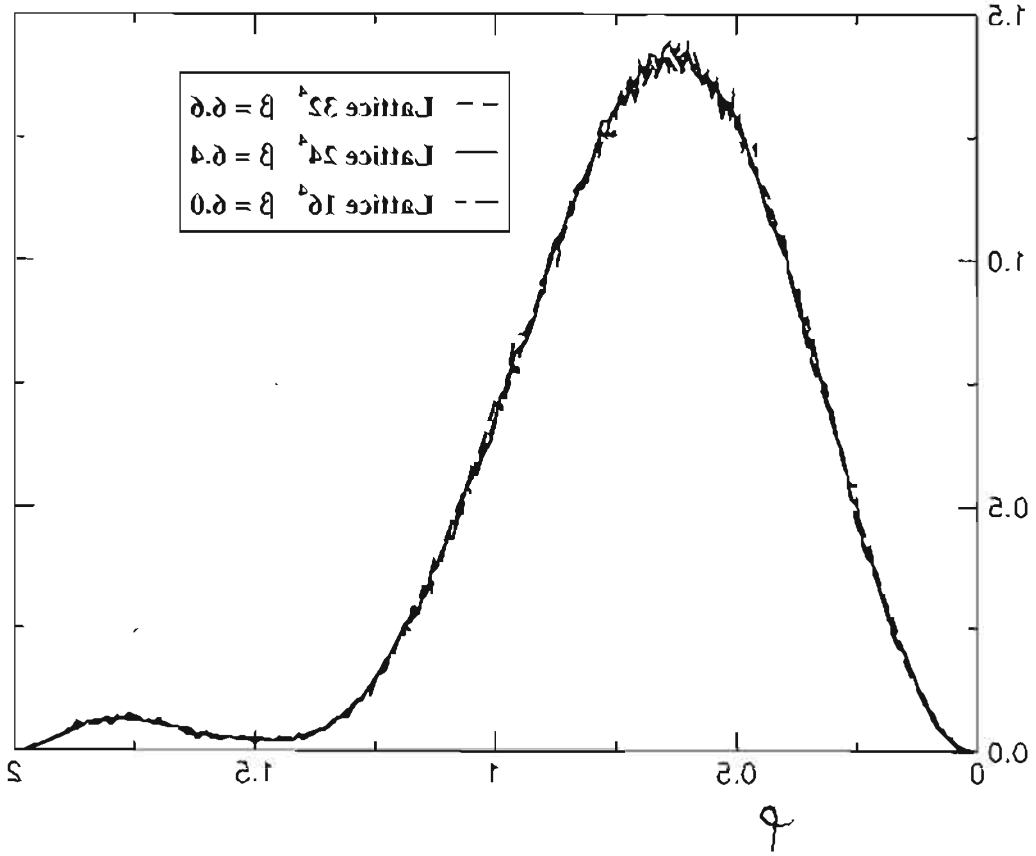
INDEPENDENT ON PATH

SEND PATH TO ∞

μ CREATES KINKS

DENSITY OF MONOPOLES

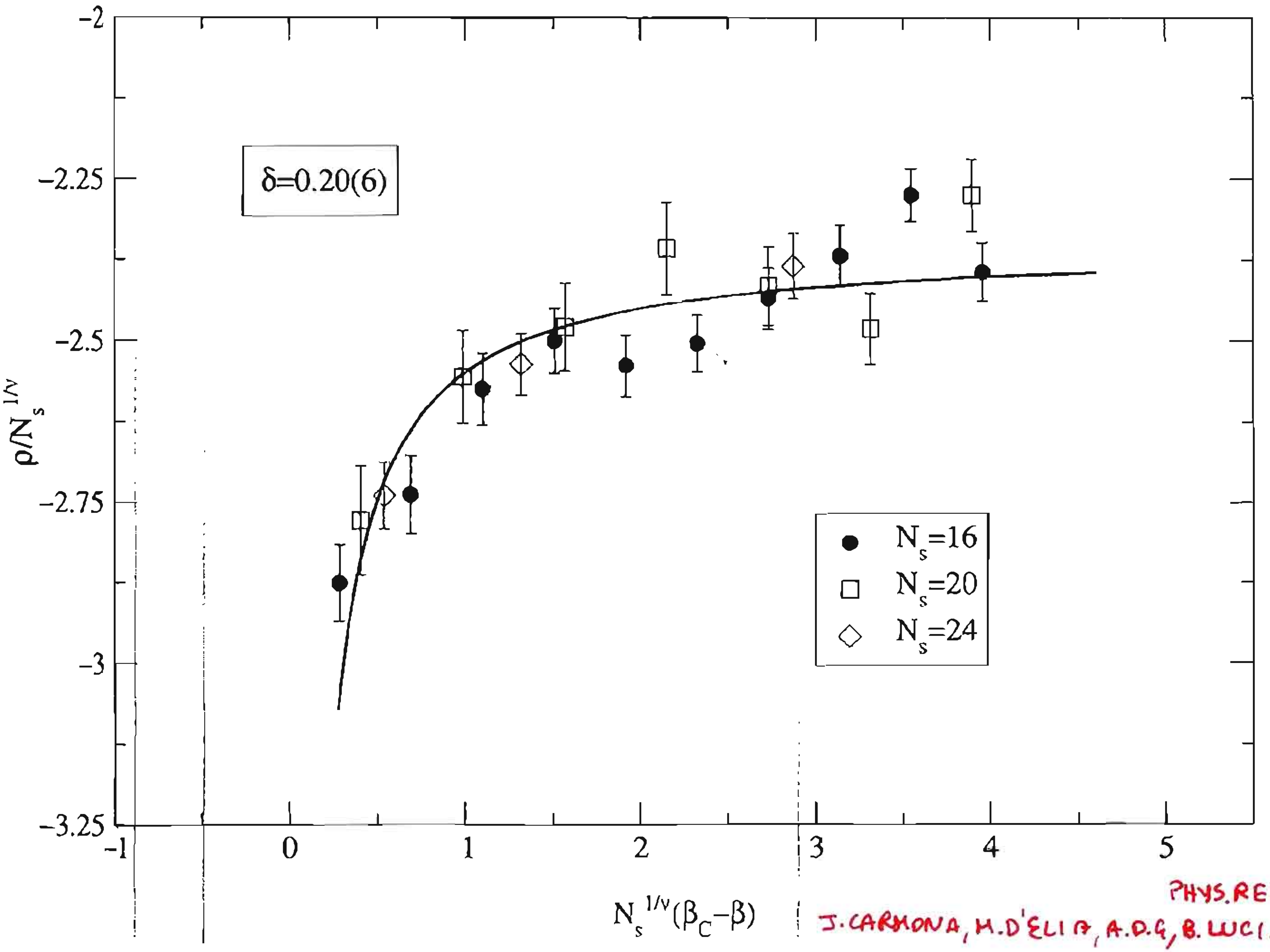




DISTRIBUTION OF THE DIFFERENCES OF THE ENERGY EIGENVALUES AT THE LATTICE SIZE. φ CORRESPONDS TO A MONOPOLE

LATTICES WITH THE SAME PHYSICAL VOLUME

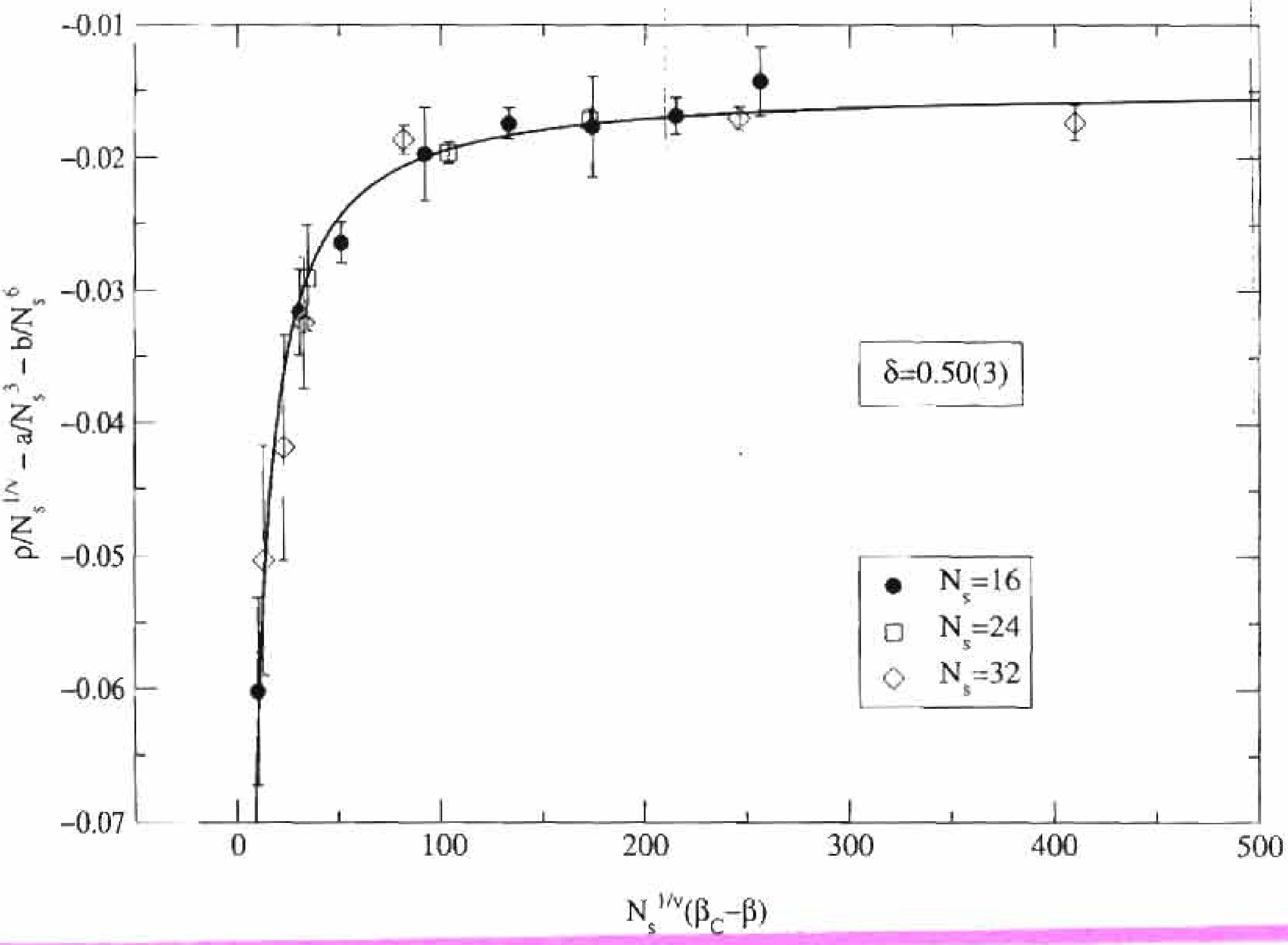
RANDOM GAUGE SU(2)



PHYS.REV. D64 114507-1 (2001)

J. CARMONA, M. D'ELIA, A.D.G, B. LUCINI, G. PAFFUTI

RANDOM GAUGE SU(3)



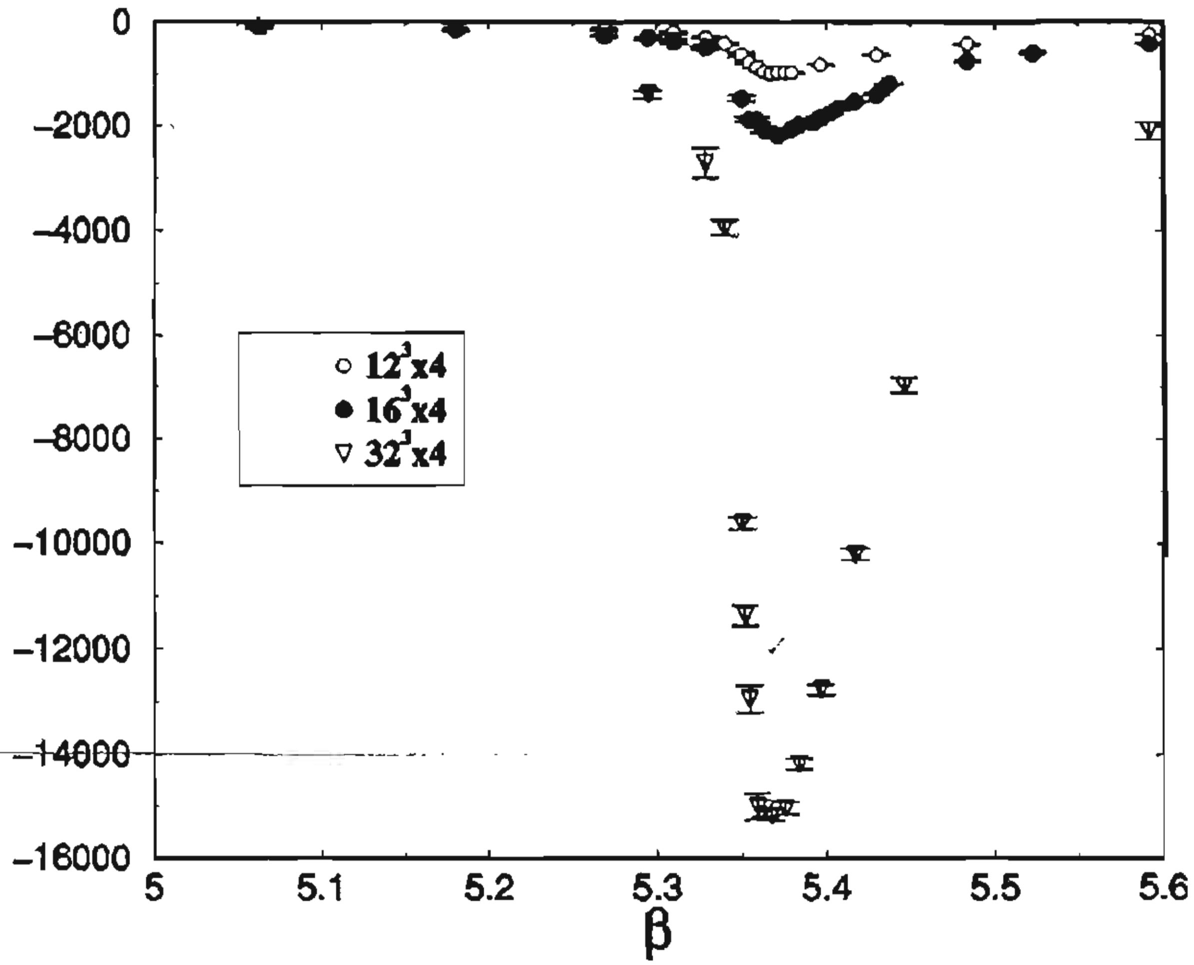


FIG. 2. Behavior of ρ around the phase transition at various lattice sizes.

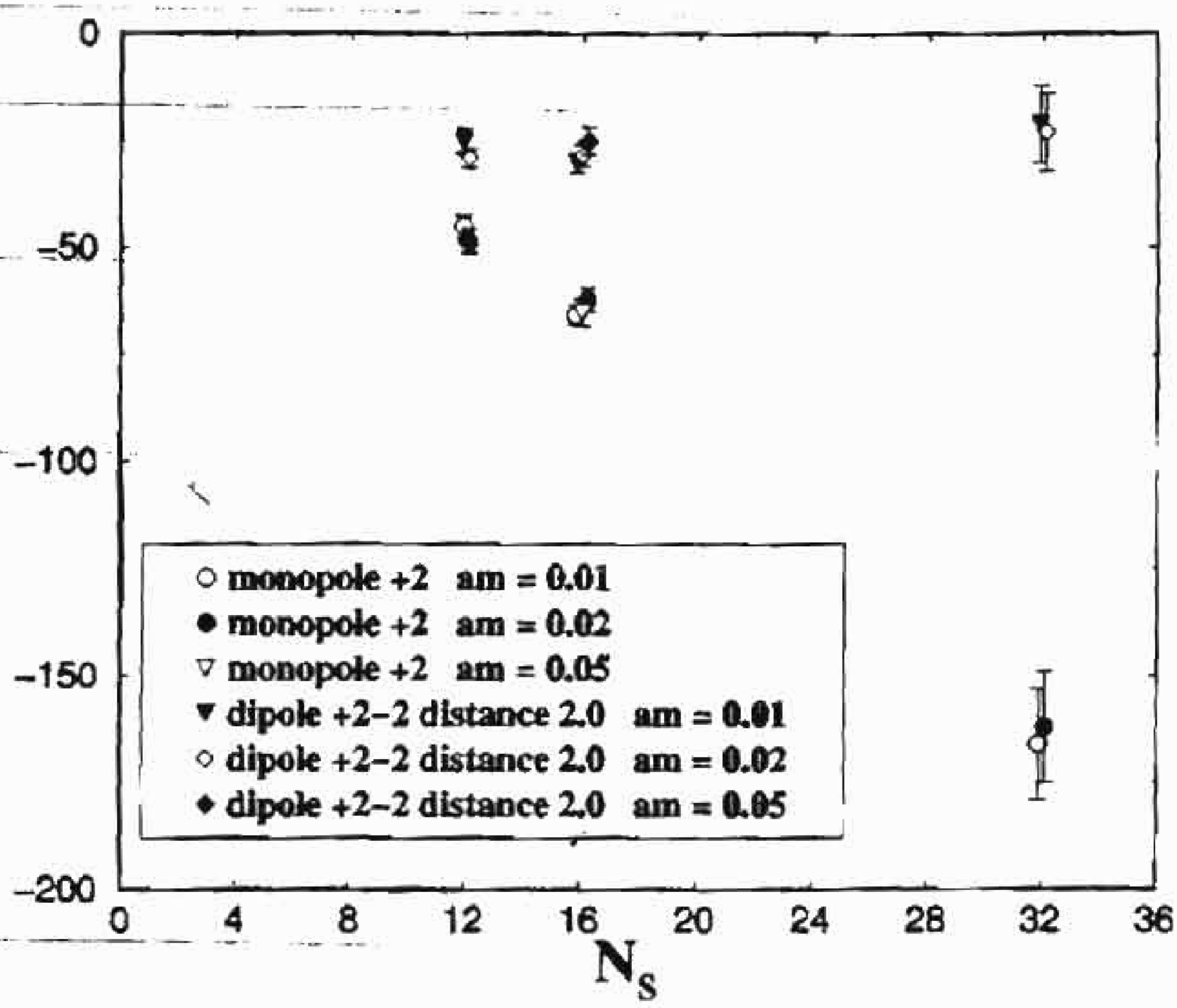


FIG. 3. Weak coupling behavior of ρ at various lattice sizes.

$\beta > \beta_c$ SUPERSELECTION OF MAGNETIC CHARGE

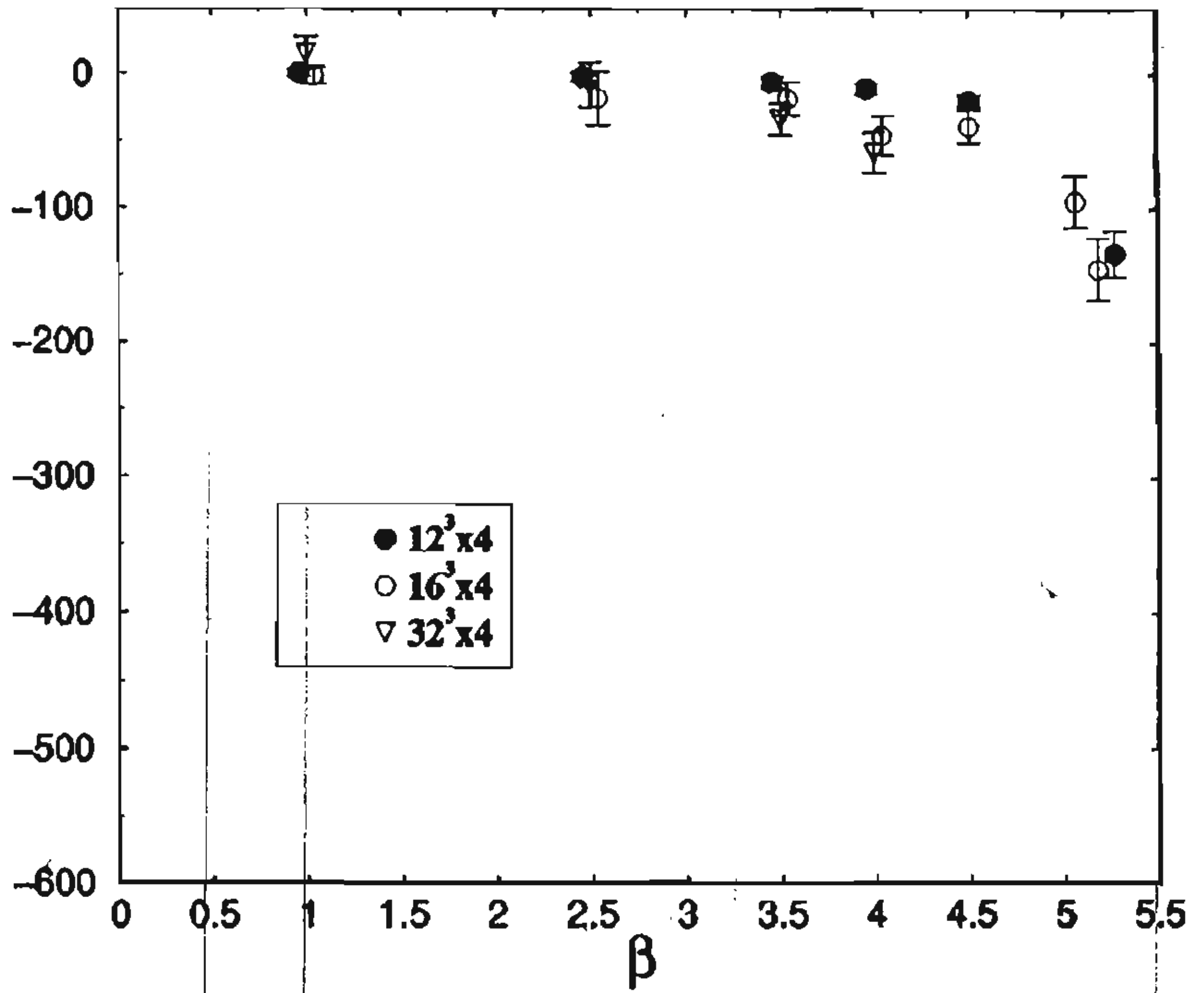


FIG. 4. Strong coupling behavior of ρ at various lattice sizes and $am = 0.1335$.

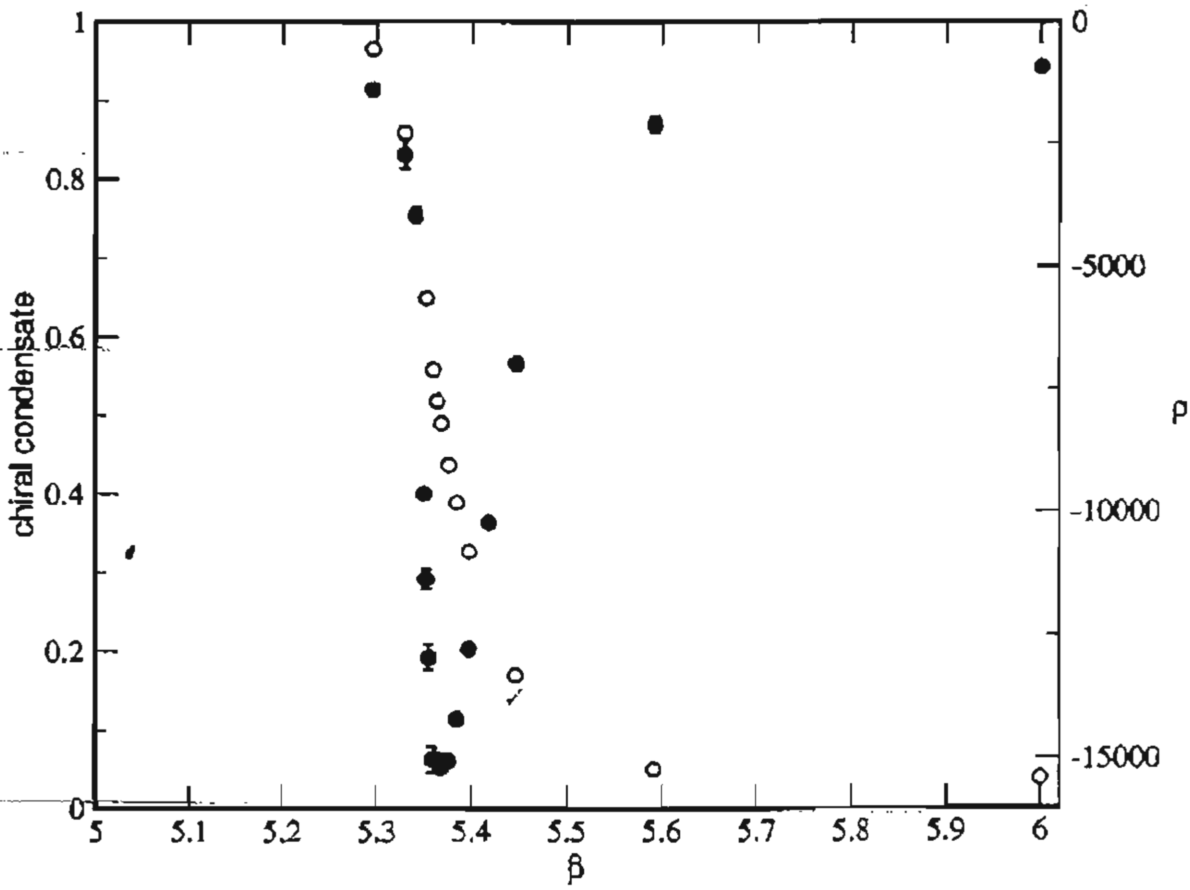


FIG. 1. Chiral condensate (open circles) and ρ (filled circles) on the $32^3\times 4$ lattice.