Low-dimensional defects in lattice YM theories

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Confinement, continuum vs. lattice

Confinement is well understood in $U(1)$ cases.

Both in continuum and lattice terms

Non-Abelian case studied numerically on the lattice

Confining configurations: "strings populated by particles"

Defined, however, in specifically lattice terms

Continuum: strings in higher dimensions

Is synthesis possible?

B) Quantum vacuum fields

we will discuss mostly vacuum fields

$\{A_\mu^a(x)\}$ (generated by computer)

Two ways to derive, say, Coulomb potential

Classically,

$$\Delta \varphi = \text{const}\, \rho(r)$$

$$\varphi = -\frac{\text{const}}{r}$$

→ "Influence of one particle on the other in empty vacuum"

Quantum case:

$$\langle A_\mu(x), A_\nu(0) \rangle_{\text{vacuum}}$$

averaging over configurations which are zero-point vacuum fluctuations (route through Wilson loop)

(Questions)

Field theory and lattice, general remarks

Lattice YM $\equiv$ Euclidean YM

However some unusual features:

a) UV cut off is known explicitly and study of UV power divergences is possible

For example, vacuum-energy density:

$$\langle \alpha_s (G^a_{\mu\nu})^2 \rangle = \frac{\text{const}}{a^4} \left[1 + a_1 \alpha_s + \ldots \right]$$

First term counts number of degrees of freedom. $\{ \epsilon \sim \int \omega(k) d^3k$

All constants are known

Longest perturbative series, ever found explicitly.

c) Measurements on false vacuum

A common "poisoned" question:
"how you would observe monopoles, at least in principle?"

The answer seems to be:
"In no way.
But: from absence of tachyons in this room, it does not follow, that the SM is wrong."

In other words, monopoles (vortices) are condensed and correspond to tachyonic modes. Excitations $\equiv$ glueballs.

What is more amusing, and in positive, is that one can make measuments on tachyonic modes.

For example:

$$\langle |\varphi_M|^2 \rangle \sim \text{const}\ \mathcal{L}^{mon}_{tot}$$

where $\mathcal{L}^{mon}_{tot}$ is the total length of the monopole trajectories.

c') "Overheated" vacuum of QCD

Very preliminary, more details later.

Usually, one assumes that if "old" vacuum is no longer vacuum at all, then the new vacuum has nothing in commom with the old one.

If it were so, we could not make measurements on old vacuum, living in the new state.

Second order phase transition:

change is very smooth,

critical exponents after the phase transition, are related to the exponents before the transition.

In reality (lattice measurements) probability of a link to belong to a monopole trajectory is of order

$$\theta_{link} \sim (a \cdot \Lambda_{QCD})^3$$

$$\sim \exp\left(-\,const/g^2(1/a^2)\right)$$

Non-pert. physics, direct

d) Measurements of size

Particle data tables contain limits on sizes of elementary particles

The notion of size is not trivial, in view of radiation, which happens at all distances

Measuring size assumes knowledge, theoretical knowledge, of cross section of a standard process (summation of radiation)

In the Euclidean case, means lattice, one also needs a theoretical input

$$M^{Dirac}_{mon} = \frac{1}{8\pi}\int_a^\infty \vec{H}^2 d^3r = \frac{const}{a}$$

As far as we see that the non-Abelian action, associated with an "object", is ultraviolet divergent we claim seeing "elementary object".

[At scales available for lattice studies at given time]

In principle, observation of point-like objects would be no less important for (Euclidean) theory, than observation of quarks.

The main discovery/paradox/challenge, to my mind, is that in YM theories gluons are **not** the only elementary objects (so far)

The lectures are in fact centered around this point, and this point alone

However, there is no straight line (theory) and we will try various avenues, with very limited success, if any.

Coexistence of magnetic and electric charges: beyond field th.

Motivation: in case of YM,
after trading one gluon for monopoles,
charged gluons still exist

Two types of monopoles (see also A. DiGiacomo lectures)

point-like, or Dirac monopole,
singular fields

$$\partial_\mu {}^*F_{\mu\nu} \equiv j_\nu^{mon}$$

$$\partial_\nu j_\nu^{mon} = 0$$

$$^*F_{\mu\nu} = \frac{1}{2}\epsilon_{\mu\nu\alpha\beta}F_{\alpha\beta}$$

extended, Polyakov - 't Hooft monopoles

$$F_{\mu\nu} = G^a_{\mu\nu}n^a - \frac{1}{g}\epsilon^{abc}\overset{\text{no action}}{D_\mu n^b D_\nu n^c}$$

where $n^a = \frac{H^a}{|H^a|}$, H^a is Higgs

Monopole trajectory is defined as

$$H^a = 0$$

three conditions in 4d $\Rightarrow$ 1d object (trajectory)

In pure YM, there is no Higgs

One could think in terms of a composite Higgs field: ('t Hooft)

In case of SU(2) YM such a comp. Higgs difficult to introduce.

Examples of failures:

$$H^a \equiv G^a_{12} \quad \text{violation of Lorentz inv.}$$

$$H^a \equiv \epsilon^{abc} G^b_{\mu\nu} G^c_{\mu\nu} = 0$$

This chain can be continued: no Higgs composite in terms of gauge-field products. (any number of them)

The only resort left:

$$(j^{mon}_\mu)^a = D_\nu \tilde{G}^a_{\mu\nu}$$

However, this is not a trajectory.

<u>What is it?</u>

Where we are now

There is talking around that dual formulations of YM theory are string theories, not field theories

Dual formulation is identical rewriting of the original theory.

Nobody has done this. Moreover, theoretically limit $N_c \to \infty$ commonly discussed.

Moreover, without UV regularization theory is not a theory yet but an educated guess.

How we can utilize ~~precise~~, exact complete lattice formulation and possibility to make simulations to advance to a new field of string theories?

Of course, computer would not write for us the action of the corresponding string theory.

But there are possibilities:

A) Topological excitations of the original formulations can well become fundamental variables of a dual formulations. Di Giacomo's lectures

Moreover, it is "natural" for duality-related excitations to condense in the vacuum state of the original formulation.

<u>Lesson</u>: look for strings in the vacuum state of YM theories.

Old news: strings are well-known to be present in the vacuum.

Greensite's
Di Giacomo's
lectures

Is it the end of the story?

No.

Remember: μ-meson is called 'meson'
because it was ~~too~~ mistaken for
π-meson, predicted by Yukawa.

Similar, there are strings and strings

Theoretically, the suspect is
fundamental (infinitely thin) string
living in extra dimensions and treated
classically.

Thus, we must understand, what is
signature of fundamental string?
Just create the language in terms
of physical observables.
Following textbooks on
Quantum Geometry I am teaching
that

fundamental = fine tuning between
infinite action and entropy.

At this moment,

I am trying to develop intuition,

why (or why not) strings are

natural duality-related excitations.

I am trying to explain that

our old friend Dirac string,

in the quantum vacuum of ~~ch~~ charged

particles could be zero ("minus one")

approximation for "fundamental" strings

(thin)

Fighting Dirac string.

Dirac monopole:

$$A_\varphi = \frac{1}{g}\,\frac{(1-\cos\theta)}{2r\sin\theta}\,, \quad A_r = A_\theta = 0$$

Potential has singularity, $\sin\theta = 0$, along the Dirac string.

To get rid of the string and save monopole as a particle one postulates:

a) Dirac quantization condition, to avoid Aharonov-Bohm effect

b) Dirac veto, charged particles are not allowed to touch the string

c) ignores the issue of energy of the string. Naively:

$$E \sim \int H^2 d^3r \sim \frac{\ell\,(\text{Flux})^2}{a^2}$$

Zwanziger formalism

One photon should serve two charges, electric and magnetic

Introduce, first, two photons, A_μ and B_μ, and, then, impose the condition:

$$F_{\mu\nu}(A) = {}^*F_{\mu\nu}(B)$$

the number of d.o.f. is correct then

$$\mathcal{L}_{ZW} = \frac{1}{2}(m[\partial\wedge A])^2 + \frac{1}{2}(m\cdot[\partial\wedge B])^2$$

$$+ \frac{i}{2}(m[\partial\wedge A])(m\cdot{}^*[\partial\wedge B])$$

$$- \frac{i}{2}(m\cdot[\partial\wedge B])(m\cdot{}^*[\partial\wedge A]) +$$

$$+ i g_e j^e\cdot A + i g_m j^m\cdot B$$

where $[\partial\wedge A]_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$ ....

m is a space-live vector

Propagators, in Feynman gauge,:

$$\langle A_\mu, A_\nu \rangle = \langle B_\mu, B_\nu \rangle = \frac{\delta_{\mu\nu}}{k^2}$$

$$\langle A_\mu B_\nu \rangle = \frac{i}{k^2 (km)} {}^*[m \wedge k]_{\mu\nu}$$

which reproduce classical effect, Coulombic interaction of charges (same) and of magnetic monopole with a moving electric charge.

So far, everything is fine.

Next steps:

a) Spontaneous symmetry breaking

b) radiative corrections

In both cases, there are problems

Evaluation of radiative corrections is straightforward, beginning with J. Schwinger (1966)

$$\langle B_\mu, B_\nu \rangle(k) = \frac{\delta_{\mu\nu}}{k^2}(1 - L) + \underbrace{\frac{1}{(km)^2}(\delta_{\mu\nu} - m_\mu m_\nu) L}_{\text{Dirac string self-energy}}$$

$$L = \frac{\alpha_{el}}{6} \ln \frac{\Lambda_{uv}^2}{k^2}$$

The problem is that

$$\langle A_\mu, A_\nu \rangle(k) = \frac{\delta_{\mu\nu}}{k^2}(1 - L)$$

and electric and magnetic couplings run the same, violating the quantization condition

The reason: virtual particles (electrons) strike the Dirac string. Indeed,

$$e A_\mu^{ext} \bar{\Psi}_e \gamma_\mu \Psi_e$$

involves full A_μ^{mon}, string included

'98

The guess is easy to check

$$\vec{H}_{mon} \equiv \vec{H}_{rad} + \vec{H}_{string}$$

Interaction of two monopoles, classically:

$$V_{M,\bar{M}}(r) = \frac{1}{4\pi} \int \vec{H}^{(1)}_{rad} \cdot \vec{H}^{(2)}_{rad} \, d^3r$$

i.e. one keeps only radial part of the field

The radiative corrections bring in $\vec{H}_{string}$

$$\frac{1}{2} \int d^3r \left[\vec{H}^{(1)}_{string} \cdot H^{(2)}_{rad} + \vec{H}^{(1)}_{rad} \cdot \vec{H}^{(2)}_{string} \right]$$

$$= -2 \int \vec{H}^{(1)}_{rad} \cdot \vec{H}^{(2)}_{rad} \, d^3r$$

Note that everything is finite, and this is a source of confusion.

The coefficient does not depend on the shape of the string and explains the wrong running of the magnetic charge

To maintain consistent field th. one has to remove the effect "by hand" which is very difficult to implement technically and goes beyond field th. in fact

Also, if mass is added for 'one' photon

$$\delta \mathcal{L} = \frac{1}{2} m_V^2 A_\mu^2$$

(let it be due to spontaneous symm. breaking)

the "other" photon develops singularities:

$$\langle B_\mu B_\nu \rangle = \frac{X_{\mu\nu}}{k^2 + m_V^2}$$

$$X_{\mu\nu} = \delta_{\mu\nu} - \frac{k_\mu n_\nu + k_\nu n_\mu}{(kn)} + \frac{k_\mu k_\nu}{(kn)^2} + \frac{m_V^2}{\underline{(kn)^2}} (\delta_{\mu\nu} - n_\mu n_\nu)$$

which is a trace of the Dirac string.

The reason is clear intuitively:

if charged particles condense, they are everywhere and touch the Dirac string.

A way out: nullify the condensate at the line where the Dirac string rests.

This is classical

Abrikosov - Nielsen - Olesen solution

The way out ; again:

one should not put monopoles into existing vacuum ($\langle \varphi_0 \rangle \neq 0$) but allow them to change the vacuum (nonlocally) itself.

Changing vacuum costs energy, which classically is decisive factor (QM entropy counts)

Along the red line

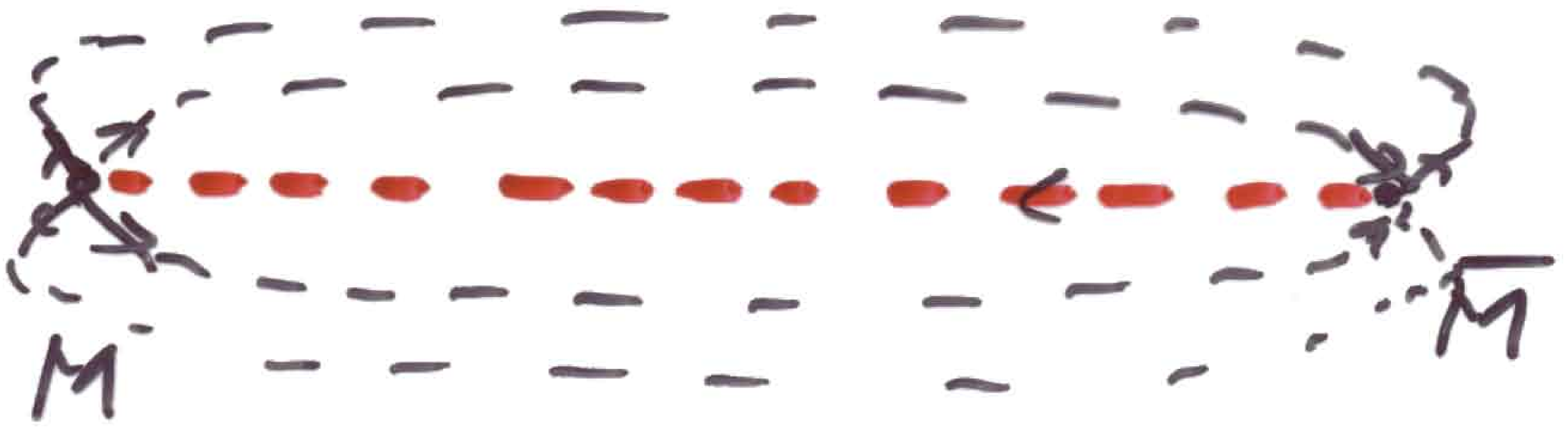
$$\langle \varphi \rangle \equiv 0$$

to allow the Dirac string not to be ~~char~~ touched by charged particles

Sheets of magnetic flux.

Known classical solution, with elementary (Lagrangian) Higgs

$\langle \varphi \rangle \neq 0$



$\underline{A^2 |\varphi|^2}$

Radial magnetic field of a monopole streams to the other one

$\langle |\varphi|^2 \rangle \neq \langle \varphi \rangle^2$

Speculations on YM:

*) No Higgs → thickness goes to zero

**) No $\langle \varphi \rangle$ classical, since color conserved

Replaced by vacuum value $\langle A^2 \rangle$ which is UV divergent. Therefore

$$\text{Action} \sim \frac{\text{Area}}{a^2} \qquad \{ (A^3_\mu)^2 A^+_\mu A^-_\mu$$

***) Former Dirac string becomes dynamical

The action should be balanced by another quantum effect, entropy.

Conclusions (to parts I, II)

a) There are beautiful thing which can be done on the lattice, unusual facets of field th.

In particular, power-like UV divergences become meaningful.

Physics of powers of $(a \cdot \Lambda_{QCD})$, not logs of it

b) if we wish to consider both monopoles and charged particles the actual object might well be surfaces, swept by "former" Dirac strings, which become dynamical because of violation of the Dirac veto