PARMA XIV SNFT

1st WEEK

$$\mathcal{L}_{SM} = i\bar{\Psi}\gamma^\mu D_\mu \Psi - \frac{1}{4} F_{\mu\nu}F^{\mu\nu}$$

unification problem

F.F.

$$+ D_\mu H^+ D_\mu H - V(H^+H)$$

hierarchy problem

Alessandro Strumia

Fabio Zwirner

Alberto Zaffaroni

$$+ (y_{ij}\, \bar{\Psi}_i H \Psi_j + h.c.)$$

flavour problem

Andrea Romanino

UNIFICATION PROBLEM

* WHY $G_{SM} = SU(3) \times SU(2) \times U(1)$?
* ARE g_1 g_2 g_3 RELATED ?
* WHY
$$
\begin{aligned}
L &= (1, 2, -1/2) \\
E^c &= (1, 1, +1) \\
Q &= (3, 2, +1/6) \\
U^c &= (\bar{3}, 1, -2/3) \\
D^c &= (\bar{3}, 1, +1/3)
\end{aligned}
$$
?

[charge quantization, anomaly cancellation]

GRAND UNIFICATION

Idea: at a scale M_{GUT} e.w. and strong int. described by a gauge theory based on a <u>simple</u> group $G \supset SU(3) \times SU(2) \times U(1)$

$$G \xmapsto[\text{SB}]{} SU(3) \times SU(2) \times U(1)$$

constraint: G representations should contain (Q, U^c, D^c, L, E^c).

e.g. G: $SU(5)$, $SO(10)$, E_6, ...

↳ the smallest, rank 4

① GAUGE COUPLING UNIFICATION

At the classical level: $\underset{U(1)}{g_1} = \underset{SU(2)}{g_2} = \underset{SU(3)}{g_3}$

but $g_Y \neq g_1$

$$\mathrm{tr}(T_5^A T_5^B) = \frac{1}{2}\delta^{AB}$$ for the fundamental $SU(5)$ representation

$$\mathrm{tr}(T_5^Y)^2 = \frac{5}{6}$$

$$\rightarrow \quad T^{U(1)} = \sqrt{\frac{3}{5}}\, T^Y \quad \& \quad g_1 = \sqrt{\frac{5}{3}}\, g_Y$$

$$g_1 T^{U(1)} \equiv g_Y T^Y$$

$$\left.\begin{aligned} &\sqrt{\tfrac{5}{3}}\, g_Y = g_2 = g_3 \\ &\sin^2\vartheta_W \equiv \frac{g_Y^2}{g_Y^2 + g_2^2} = \frac{3}{8} \end{aligned}\right\} \text{not-so-good}$$

Much better when running effects are included:

$$\frac{1}{\alpha_i(Q)} = \frac{1}{\alpha_i(m_Z)} + \frac{b_i}{2\pi} \log \frac{Q}{m_Z} \qquad \alpha_i \equiv \frac{g_i^2}{4\pi}$$

$$\begin{pmatrix} b_1 \\ b_2 \\ b_3 \end{pmatrix}_{MSSM} = \begin{pmatrix} 33/5 \\ 1 \\ -3 \end{pmatrix} + \text{"}\delta n_H\text{"} \quad \text{cfr.} \begin{pmatrix} 41/10 \\ -19/6 \\ -7 \end{pmatrix}_{SM}$$

includes 2 higgs doublets + SUSY partners

extra Higgs doublets

Inputs: $[M_{GUT},\ \alpha_U \equiv \alpha(M_{GUT})]$ traded for:

$$\alpha_{em}^{-1}(m_Z)\Big|_{\overline{MS}} = 127.934$$

$$\sin^2\vartheta(m_Z)\Big|_{\overline{MS}} = 0.231$$

Outputs (MSSM)

$$\alpha_3(m_Z)\Big|_{LO} = \alpha_{em}(m_Z)\,\frac{56 - 2\delta n_H}{(120+6\delta n_H)\sin^2\vartheta(m_Z) - (24+3\delta n_H)}$$

	$\delta n_H = 0$	$\delta n_H = 2$
$\alpha_3(m_Z)\Big\vert_{LO}$	$\underbrace{0.118}_{\text{successful!}}$	1.13 ← not a misprint!

For $\delta n_H = \emptyset$

$$\log\left(\frac{M_{GUT}}{m_Z}\right) = \pi\,\frac{3 - 8\sin^2\vartheta(m_Z)}{14\,\alpha_{em}(m_Z)}\Big|_{\overline{MS}}$$

$$M_{GUT} \approx 2\cdot 10^{16}\,\text{GeV}$$ → not-so-fa[r] from $M_{Pl} = 2.4\,10$[…] Ge[V]

$$\alpha_U = \frac{28\,\alpha_{em}(m_Z)}{36\sin^2\vartheta_W(m_Z) - 3}\Big|_{\overline{MS}}$$

$$\alpha_U \approx \frac{1}{25}$$

② CLASSIFICATION OF PARTICLES

SU(5) SO(10)

$10 \equiv (Q, U^c, E^c)$

$\bar{5} \equiv (L, D^c)$

$16 \equiv [\underbrace{Q, U^c, E^c, L, D^c}_{\text{1 family}}, \nu^c]$

really striking

Exercise: $\mathrm{tr}(T^Y_5)^2 = 2\left(-\frac{1}{2}\right)^2 + 3\left(\frac{1}{3}\right)^2 = \frac{5}{6}$ o.k.

ELECTRIC CHARGE QUANTIZATION:

$U(1)_{em} \rightarrow Q$ unconstrained

$SU(5) \supset U(1)_{em} \rightarrow \mathrm{tr}\, Q = 0$ for all irr. representation

e.g. $\bar{5}$: $Q(\nu) = 0$, $Q(e) = -1$ $\rightarrow 3Q(d^c) = 1$

ANOMALIES:

Cancellation of gauge anomalies in the SM: non trivial.

$SO(4n+2) \rightarrow \mathrm{tr}\left(T^A_R \{T^B_R, T^C_R\}\right) = 0$ for any R

→ anomaly free group

$16 \equiv 10 + \bar{5} + 1$ ↗ does not contribute to SU(5) anomalies

③ B/L VIOLATION

$R \supset (\text{QUARKS}, \text{LEPTONS})$

irreducible representation of G

→ lepton/quark distinction is no longer fundamental

Notation: $T^A = \begin{cases} T^a & \text{SM generators (12)} \\ T^{\hat{a}} & \text{remaining generators} \end{cases}$

in $SU(5)$: $\dim G \equiv 24 \rightarrow \#(T^{\hat{a}}) = 24 - 12 = 12$

easy to visualize, for the fundamental repr:

$$T^A \equiv \begin{array}{c} \left(\begin{array}{c:c} T^a \ \text{Gluons} & T^{\hat{a}} \\ \hdashline T^{\hat{a}} & T^a \end{array} \right) \begin{array}{l} \}\, 3 \\ \}\, 2 \end{array} \\ \begin{array}{cc} \underbrace{\qquad\qquad}_{3} & \underbrace{\qquad}_{2} \end{array} \end{array} \qquad (T^A)^+ = T^A$$

W, Z, γ

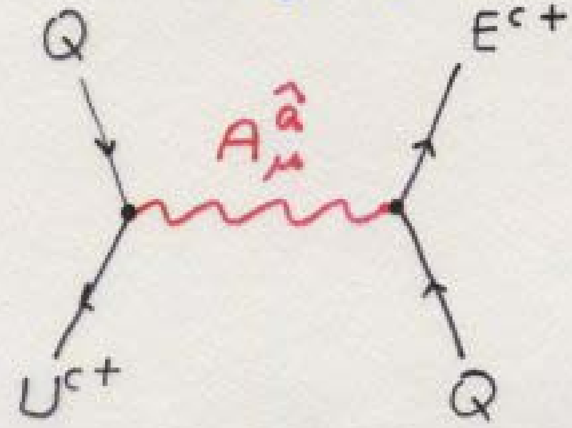
$T^{\hat{a}} \sim (3, 2, -5/6)$
$\quad (\bar{3}, 2, +5/6)$

$\dashrightarrow A_\mu^{\hat{a}}$ have masses $\simeq M_{GUT} = 2 \cdot 10^{16}$ GeV from the breaking

$SU(5) \longrightarrow SU(3) \times SU(2) \times U(1)$

$A_\mu^{\hat{a}}$ mediate B/L violating transitions:
[not the only source of B/L violation]

$$\mathcal{L}_{gauge} \supset g\,\overline{\Psi}_Q \bar{\sigma}^\mu A_\mu^{\hat{a}}\, \Psi_{U^c} + g\,\overline{\Psi}_Q \bar{\sigma}^\mu A_\mu^{\hat{a}}\, \Psi_{E^c} + g\,\overline{\Psi}_L \bar{\sigma}^\mu A_\mu^{\hat{a}}\, \Psi_{D^c} + h.c.$$



dim 6

four-fermion operators:

$\rightarrow QQU^{c+}E^{c+}$

$\rightarrow QL\, U^{c+}D^{c+}$

$$\tau(p \rightarrow e^+\pi^0) = 10^{35}\left[\frac{M_X}{10^{16}\,\mathrm{GeV}}\right]^4 \cdot \left[\frac{0.015\,\mathrm{GeV}^3}{a}\right]^2 \cdot \left[\frac{1/25}{\alpha_U}\right]^2 \mathrm{yr}$$

$\langle 0|uud|p\rangle \simeq a = 0.015(1)\,\mathrm{GeV}^3$

SK limit:

$\uparrow$ undetectable

$\tau(p \rightarrow e^+\pi^0) > 5.4 \times 10^{33}$ yr 90% CL

[future SK: 10^{34} yr]

- B violation at $E \sim M_{GUT} \rightarrow$ baryogenesis in the early universe

④ FERMION MASS RELATIONS

$R \supset (\text{QUARKS}, \text{LEPTONS})$

→ mass relations expected

In SUSY-GUTs, we need 2 Higgs doublets.

SU(5):

$$H_u \sim 5 \equiv (H_u^D, H_u^T) \qquad D \equiv SU(2)_L \text{ doublet}$$

$$H_d \sim \bar{5} \equiv (H_d^D, H_d^T) \qquad T \equiv \text{color triplet}$$

minimal SU(5) Yukawa couplings:

$$w = 10_i\, y_{u_{ij}} 10_j\, H_u + 10_i\, y_{d_{ij}} \bar{5}_j\, H_d$$

$y_{u,d}$ are 3x3 matrices in generation space

$y_u = y_u^T$ not restrictive

$$10\, y_u\, 10\, H_u = Q\, y_u\, U^c H_u^D + Q\, y_u\, Q\, H_u^T + U^c\, y_u\, E^c H_u^T$$

$$10\, y_d\, \bar{5}\, H_d = Q\, y_d\, D^c H_d^D + E^c\, y_d\, L\, H_d^D + Q\, y_d\, L\, H_d^T + U^c\, y_d\, D^c H_d^T$$

■ = couplings to D, as in SM

■ = couplings to T, new source of $\cancel{B}/\cancel{L}$.

$\rightarrow \quad y_e = y_d^T$ $\quad$ $\underline{\text{at } Q = M_{GUT}}$

$\rightarrow \underbrace{m_b = m_\tau}_{\text{o.k. if } \tan\beta \text{ in appropriate range}} \qquad \underbrace{m_s = m_\mu \qquad m_d = m_e}_{\text{out by a factor } O(1)}$

$m_s \approx \frac{m_\mu}{3} \qquad m_d \approx 3 m_e$

not really bad ...

PARAMETER COUNTING:

$y_u \leftrightarrow 12$	redefinitions:
	$U(3)$ on $10 \rightarrow 9$
$y_d \leftrightarrow 18$	$U(3)$ on $\bar{5} \rightarrow 9$
30	18

$\rightarrow (30-18) = 12$ observables =

6 eigenvalues + 3 mixing angles

+ 3 phases $\begin{cases} 1 \text{ as in CKM} \\ 2 \text{ new phase, not constrained by present data.} \end{cases}$

They can be useful for baryogenesis

SUMMARY:

1. gauge coupling unification at LO → <u>a success</u>
2. classification properties → powerful [SO(10)!]
3. B/L violation → p-decay baryogenesis
4. fermion masses & flavor → not bad ($m_b = m_\tau$)

crucial test: p-decay!

<u>but</u>

I DT SPLITTING PROBLEM

$$H_{u,d} \equiv \left[H^D_{u,d} \,,\; H^T_{u,d} \right]$$

mass ≈ e.w. scale

$SU(2)_L \times U(1)_Y \to U(1)_{em}$

mass ≈ M_{GUT}

p-decay amplitudes $\sim \left(\frac{1}{m_T}\right)$

How to achieve this splitting?

$\simeq 10^{16}\,\text{GeV}$ expected

$$w = m_\Sigma \, tr\, \Sigma^3 + tr\, \Sigma^2 + m\, H_d H_u$$

$$+ H_d \Sigma H_u + \cdots$$

$y_{u,d}$ terms

$\Sigma \sim 24$ (adj) of $SU(5)$

Its VEV breaks $SU(5) \longrightarrow SU(3) \times SU(2) \times U(1)$

$$\langle \Sigma \rangle = \begin{pmatrix} 2\sigma & 0 & 0 & 0 & 0 \\ 0 & 2\sigma & 0 & 0 & 0 \\ 0 & 0 & 2\sigma & 0 & 0 \\ 0 & 0 & 0 & -3\sigma & 0 \\ 0 & 0 & 0 & 0 & -3\sigma \end{pmatrix} \qquad \sigma \simeq 10^{16}\,\text{GeV}$$

After $SU(5)$ breaking:

$$w = m H_d H_u + H_d \langle \Sigma \rangle H_u + \cdots$$

$$= (m + 2\sigma)\, H_d^T H_u^T +$$

$$\underbrace{(m - 3\sigma)}_{\mu \text{ in MSSM}}\, H_d^D H_u^D + \cdots$$

$\mu \simeq$ e.w. scale

$\underbrace{(m + 2\sigma)}_{m_T} \simeq 10^{16}\,\text{GeV} \rightarrow$ m and σ should be adjusted with a precision of

$$10^{14} \equiv \left(\frac{M_{GUT}}{\text{e.w. scale}} \right)$$

DT splitting problem: central problem in all GUT_s [it is the GUT version of the hierarchy problem]

technically solved by SUSY <u>but</u> the splitting can be <u>upset</u> by:

(i) radiative corrections when SUSY is broken [e.g. theories with additional singlets coupled to both D and T]

(ii) Non renormalizable operators from physics beyond M_{GUT}.

GUT as an effective theory valid up to M_{Pl}

$$\frac{H_d \Sigma \Sigma H_u}{M_{Pl}} = \frac{H_d \langle\Sigma\rangle^2 H_u}{M_{Pl}} + \cdots$$

allowed by gauge symmetry

$$\approx \frac{(10^{16})^2}{10^{18}} = 10^{14}\,\text{GeV}!$$

- Avoidable if Σ not required in $\cancel{SU}(5)$
e.g. $\cancel{SU}(5)$ through COMPACTIFICATION of an extra dimension.

III P-DECAY

From gauge boson exchange: probably undetectable

From coloured higgsino exchange: $\tilde{H}^T_{u,d}$

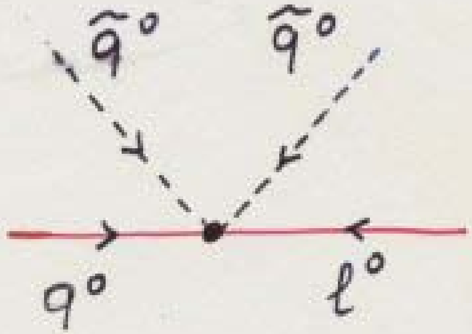
$$w = Q\, y_u\, Q\, H_u^T + U^c\, y_u\, E^c\, H_u^T$$
$$+ Q\, y_d\, L\, H_d^T + U^c\, y_d\, D^c\, H_d^T$$
$$+ m_T\, H_d^T H_u^T + \cdots$$

At $E \ll m_T$:

$$w = \frac{1}{2m_T} Q\, y_u\, Q\, Q\, y_d\, L \qquad |\Delta B| = 1 \text{ terms}$$
$$+ \frac{1}{m_T} U^c\, y_u\, E^c\, U^c\, y_d\, D^c + \cdots$$

local, dim 5 operators controlled by m_T

and by quark masses:



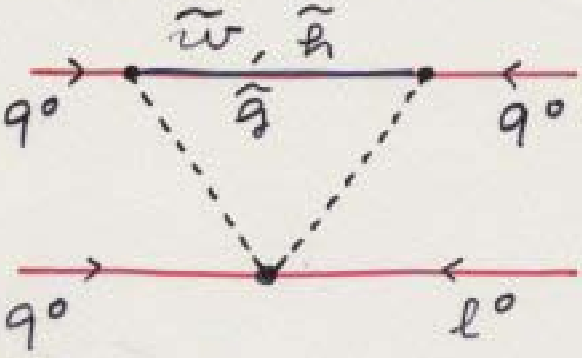
$$y_u = \frac{\sqrt{2}\, m_u}{v \sin\beta} \qquad y_d = \frac{\sqrt{2}\, m_d}{v \cos\beta}$$

Notice: $y_u y_d \propto \dfrac{1}{\sin 2\beta}$ minimized by $\tan\beta \simeq 1$

FROM  TO  :

→ RENORMALIZATION EFFECTS FROM m_T DOWN TO THE e.w. SCALE

→ "DRESSING"



→ FROM q°, ℓ° TO q, ℓ

→ RENORMALIZATION EFFECTS FROM e.w. SCALE DOWN TO ~ 1 GeV

- DOMINANT CONTRIBUTION (at small $\tan\beta$):

$$\approx (\text{loop factor}) * \frac{g^2 M_2}{m_T\, m_{SUSY}^2} *$$

$$* \left[V_{CKM}^T\, y_u^{diag}\, K^+\, V_{CKM}\right]_{ij} \left[V_{CKM}^*\, y_d^{diag}\right]_{k\ell} *$$

$$* (\nu_\ell\, d_i)(d_j\, u_k)$$

controlled by :

→ m_T, SUSY spectrum

→ QUARK MASSES m_u, m_d, $\tan\beta$, V_{CKM}

$$U_u^T\, y_u\, U_u\, K = y_u^{diag}$$ → 2 additional phases of min. SU(5)

dominant amplitude :

$(\nu_\mu\, u\, d\, s) \;\propto\; m_c\, m_s\, \lambda^2$

$(\nu_\tau\, u\, d\, s) \;\propto\; m_c\, m_b\, \lambda^4$

$\lambda \equiv$ Cabibbo angle

preferred channel :

$$p \to \bar{\nu}\, K^+$$

cf. $p \to e^+ \pi^0$ of $A^{\hat{a}}_\mu$-exchange

present limit :

$$\tau(p \to \bar{\nu} K^+) > 2 \times 10^{33}\ yr$$

90% CL [SK]

minimal SU(5) prediction :

$[m_{SUSY} \simeq M_2 \simeq 1\, TeV,\ \ K = \mathbb{1}]$

$$\tau(p \to \bar{\nu} K^+) \simeq 10^{32}\, |\sin^2 2\beta| \left(\frac{m_T}{10^{17} GeV}\right)^2 yr$$

Is minimal SU(5) ruled out?

- larger m_T? → no, otherwise $\alpha_3(m_Z)$ too large
- larger m_{SUSY}? → $m_{SUSY} \lesssim O(\mathrm{TeV})$ by naturalness
- $\tan\beta = 1$ → still possible?
- $K \neq \mathbb{1}$ → no distructive interference
- hadronic matrix element? → $\langle 0 | u u d | p \rangle \simeq a$
  $a = 0.003\,\mathrm{GeV}^3$ adopted
  cf. $a = 0.015(1)\,\mathrm{GeV}^3$

↓

minimal SU(5) is <u>dead</u>.

SO(10) in similar troubles...

non minimal GUT survive but often baroque constructions!

Are they simplest ways out?

[extra dimensions? more on this later on...]