PARMA, September 05

PLAN OF THE LECTURES

I • INTRODUCTION

- BRANES
- COMPACTIFICATIONS
- EFFECTIVE THEORIES

II • WARPING AND HOLOGRAPHY

- Randall-Sundrum MODELS
- HOLOGRAPHIC INTERPRETATION

III • COMPACTIFICATION WITH FLUXES

- TYPE IIB COMPACTIFICATION
- MODULI STABILIZATION
- GKP and KKLT

IV • THE PROBLEM OF THE VACUUM

- LANDSCAPE
- STATISTIC OF VACUA

- In this series of lectures "STRING THEORY" means

  effective theory with extra-dimensions containing gravity which can be derived by a consistent string theory.

- STRING THEORY CONTAINS GRAVITY

$T = \frac{1}{\alpha'}$

$\uparrow$

$\sim$ Planck

– MASSLESS FIELDS :

$$S_{\mu\nu} = (g_{\mu\nu}, B_{\mu\nu}, \phi)$$

– MASSIVE FIELDS :

$$m^2 \sim \frac{n}{\alpha'} \qquad n \in \mathbb{N}$$

- STRING THEORY PREDICTS EXTRA-DIMENSIONS

  CONSISTENT STRING THEORIES TYPICALLY LIVE IN 10 (or 11) DIMENSIONS

- STRING THEORY NATURALLY INCORPORATES SUPERSYMMETRY

String theory has two fundamental parameters:

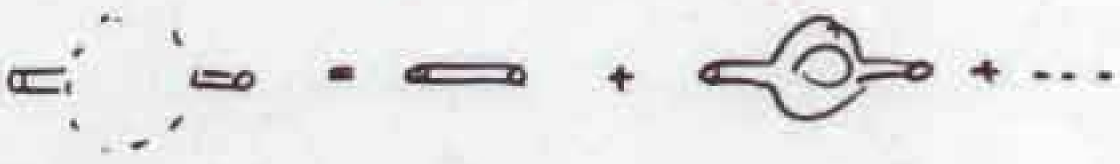
$$(\alpha', g_s)$$

• α' determines the string tension:

α' length square: $\left\{ \begin{array}{l} \ell_s = \sqrt{\alpha'} \\ \\ M_s = \dfrac{1}{\sqrt{\alpha'}} \end{array} \right.$

and determines the masses of excited states $m^2 \sim \dfrac{n}{\alpha'}$

• g_s is the loop expansion parameter



g_s

we are only interested in effective actions for massless modes ($g_{\mu\nu}$, etc...)

$$S_{\text{STRING}} = S_{\text{MASSLESS}} + S^{(\alpha')}_{\substack{\text{HIGHER}\\ \text{DERIVATIVES}\\ \text{CORRECTIONS}}} + S^{(g_s)}_{\text{LOOP}}$$

$$(R^2, R^3, \ldots)$$

TYPICALLY (SUPER)-GRAVITY

$$S = M_s^8 \int \sqrt{g}\left(R\, e^{-2\phi} + \ldots \right) d^{10}x$$

$g_s = \langle e^{\phi} \rangle$ is a VEV for a scalar field (DILATON)

• NOTICE :

$$M_P^8 \int \sqrt{g}\, R \Rightarrow \boxed{M_P^8 \sim \frac{M_s^8}{g_s^2}}$$

typically $M_S \sim M_P \sim$ Planck scale

TYPE II SUPERGRAVITY

In most string theories we have at least the massless fields

$$(g_{\mu\nu}, B_{\mu\nu}, \phi)$$

interacting, at low energies, with the action

$$M_{pl}^{D-2}\int d^Dx\, e^{-2\phi}\sqrt{g}\left(R + (\partial\phi)^2 + H_{\mu\nu\rho}^2\right)$$

$$H_{\mu\nu\rho} = \partial_{[\mu} B_{\nu\rho]}$$
$$H_{(3)} = dB_{(2)}$$

Supersymmetry requires a partner for $g_{\mu\nu}$

$$g_{\mu\nu} \longrightarrow \psi_\mu^\alpha \quad \text{spin } 3/2$$

and partners λ^α with spin 1/2 for other fields.

Example: minimal sugra $D=4$

$$M^2\int d^4x\, \sqrt{g}\left(R + \psi_\mu \Gamma^{\mu\nu\rho} D_\nu \psi_\rho\right)$$

- multiplet $(g_{\mu\nu}, \psi_\mu^\alpha)$
- transformations:

$$\delta g_{\mu\nu} \rightarrow \delta e_\mu^\nu \sim \bar{\varepsilon}\Gamma^\nu\psi_\mu$$
$$\delta\psi_\mu^\alpha \sim D_\mu \varepsilon^\alpha$$

- Supersymmetry $N=1$: 1 Weyl spinor parameter

For greater D and greater susy, many more bosonic fields are required:

roughly: number of degrees of freedom do not match anymore

$$g_{\mu\nu} \sim D^2$$
$$\psi_\mu^\alpha \sim 2^{D/2}$$

In D=10, the maximal supersymmetric theory has N=2 – two 10d Weyl spinorial parameters

Fermionic sector: $(\psi_\mu^\alpha, \lambda^\beta)$

$$\begin{array}{ccc} & (\psi_\mu^\alpha, \lambda^\beta) & \\ \swarrow & & \searrow \\ \text{type IIA} & & \text{type IIB} \\ \text{NON CHIRAL} & & \text{CHIRAL} \\ (\epsilon_1, \bar{\epsilon}_2) & & (\epsilon_1, \epsilon_2) \end{array}$$

Bosonic sector:

$$\begin{array}{ccc} & (g_{\mu\nu}, B_{\mu\nu}, \phi) & \\ \text{IIA} \swarrow & & \searrow \text{IIB} \\ \text{ODD FORMS} & & \text{EVEN FORMS} \\ (A_\mu, A_{\mu\nu\rho}) & & (\tilde{\phi}, \tilde{B}_{\mu\nu}, A^+_{\mu\nu\rho\sigma}) \end{array}$$

these are called Ramond-Ramond forms

The fields interact with

$$\int d^{10}x\, e^{-2\phi}\sqrt{g}\left(R+(\partial\phi)^2+H_{\mu\nu\rho}^2\right)+\sqrt{g}\sum_d F_{\mu_1\ldots\mu_d}^2+\ldots$$

+ fermionic interactions

- $A_{\mu_1\ldots\mu_d}$ are generalized electromagnetic potentials with field strength – curvature

$$F_{\mu_1\ldots\mu_{d+1}}=\partial_{\{\mu_1}A_{\mu_2\ldots\mu_{d+1}\}}$$

or, in the language of forms:

$$F_{(d+1)}=d\,A_{(d)}$$

- Electric - magnetic duality:

$\tilde{F}_{\mu\nu}=\varepsilon_{\mu\nu\tau\rho}F_{\tau\rho}$ generalizes to $\tilde{F}_{\mu_1\ldots\mu_{D-d-1}}=\varepsilon_{\mu_1\ldots\mu_D}F_{\mu_{D-d}\ldots\mu_D}$

$\tilde{F}=*F$

$\tilde{F}_{(D-d-1)}=*F_{(d+1)}$

magnetic — electric

magnetic photon A_μ^D

$\tilde{F}_{\mu\nu}=\partial_{\{\mu}A^D_{\nu\}}$

- the magnetic field $\tilde{A}_{\mu_1\ldots\mu_{D-d}}$ is non local with respect to $A_{\mu_1\ldots\mu_d}$

$\boxed{\text{These fields are abelian}}$

no known theory for non-abelian (YM) p-forms

Including duals,

type IIB contains all even potentials

$(\tilde{\phi},\ \tilde{B}_{\mu\nu},\ A^{+}_{\mu\nu\tau\rho},\ B_{\mu_1\ldots\mu_6},\ A_{\mu_1\ldots\mu_8})$

$$dB_{(6)} = * dB_{(2)}$$
$$d\tilde{\phi} = * dA_{(8)}$$

SELF-DUALITY: $dA_{(4)} = * dA_{(4)}$

and type IIA contains all odd potentials

$(A_\mu,\ A_{\mu\nu\rho},\ A_{\mu\nu\rho\tau\sigma},\ A_{\mu_1\ldots\mu_7})$

$$dA_{(1)} = * dA_{(7)}$$
$$dA_{(3)} = * dA_{(5)}$$

$$\boxed{\text{Type II} = \text{IIA} + \text{IIB contains all possible } p\text{-form potentials } A_{(p)}}$$

A theorem in string theory says that no elementary string state is charged under the RR-forms $A_{(p)}$

- fermion λ^α is neutral
- all excited string states with masses$^2 \sim \frac{1}{\alpha'}$ are neutral

But non perturbative states are : D-branes

An object charged under $A_{(p)}$ is a p-dimensional spatially extended object : a p-brane :

$\boxed{p=0}$

(e, A_μ)

$$e\int A_\mu v^\mu dt = e\int A_\mu \frac{dx^\mu}{dt}\,dt = e\int A_\mu dx^\mu = \int_{\substack{\text{world}\\ \text{volume}}} A_{(1)}$$

$\boxed{p\geq 1}$

Σ

$$\mu_p \int_\Sigma A_{(p+1)}$$

$\Sigma =$ (p+1)-worldvolume $\left\{\begin{array}{l} p \text{ space dim.} \\ 1 \text{ time}\end{array}\right.$

As an electron has a mass, a brane has a tension

$$\tau_p \int_\Sigma \sqrt{g}$$

this replaces $m\int ds$ of a 0-brane = particle

and thus modifies spacetime; the interaction of gravity in the presence of a source

$$\int \sqrt{g}\, R\, d^{10}x \; + \; \tau_p \int_\Sigma \sqrt{g}$$

deforms the metric to:

$$ds^2 = H^{-1/2}(r)\, dx_\mu^2 + H^{1/2}(r)\, dy_i^2$$

$$H(r) \sim 1 + \frac{T}{r^{D-p-3}} \qquad D=10$$

(Laplace eq. in D-p-1 variables y_i)

- y_i are the transverse variables; $r=\sqrt{\sum y_i^2}$
- x_μ are (p+1) coordinates along the brane

We need two theorems from string theory :

1. There exist Dp-branes in type II string theory. Dp is charged under $A_{(p+1)}$.
Type II B contains Dp-branes with p <u>odd</u>;
Type II A contains Dp-branes with p <u>even</u>.

- Dp preserves half of the supersymmetries (BPS object)

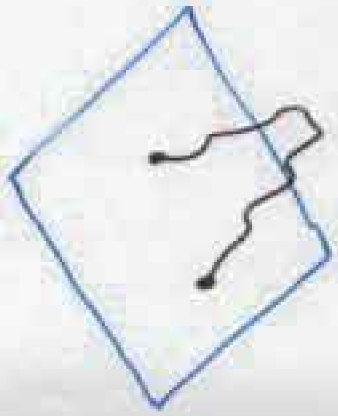
- There is a relation between charge and tension :

$$T_p = \frac{1}{g_s}\mu_p \qquad g_s = e^{-\phi}$$

(non-perturbative, solitonic like objects, like monopoles in QFT)

$$\boxed{\text{For every } p\text{, there is a Dp-brane charged under } A_{(p+1)} \text{ in type II string theory}}$$

2. Quantizations of Dp-branes is done using open strings. This gives vector fields and scalars living on the worldvolume of Dp-branes (collective coordinates)



(A_μ, ϕ_i)

$i = 1, \dots, D-p-1$

ϕ_i parametrize fluctuation of the Dp position in the $D-p-1$ transverse directions

$$\mathbb{R}^D = (x_0, x_1, \dots, x_p) + (y_{p+1}, \dots, y_D)$$

Branes move by minimizing the volume spanned by their motion

$\xi \to \mathbb{R}^D$

$$\tau_p \int_\Sigma \sqrt{g^{(\Sigma)}_{\text{INDUCED}}}\, d\xi$$

$$\left(g^{(\Sigma)}_{\text{INDUCED}}\right)_{\alpha\beta} = G_{IJ}\, \partial_\alpha X^I \partial_\beta X^J$$

TRIVIAL EMBEDDING : $\xi_\alpha = x_\alpha$

$\xi_\alpha \to \mathbb{R}^D$: $x_\alpha \to X^I(x_\alpha) = (x_0, \dots x_p, Y_{p+1}(x_\alpha) \dots)$

$$G_{IJ}\, \partial_\alpha X^I \partial_\beta X^J = \left(\begin{array}{c|c} \cancel{g_{\alpha\beta}} & \\ \hline g_{\alpha\beta} + & G_{IJ}\, \partial_\alpha Y^I \partial_\beta Y^J \end{array} \right)$$

The action becomes

$$\tau_p \int \sqrt{g_{IND}} = \tau_p \int \sqrt{g} + \frac{1}{2}\sqrt{g}\, G_{IJ}\, g^{\alpha\beta} (\partial_\alpha Y^I)(\partial_\beta Y^J) + \dots$$

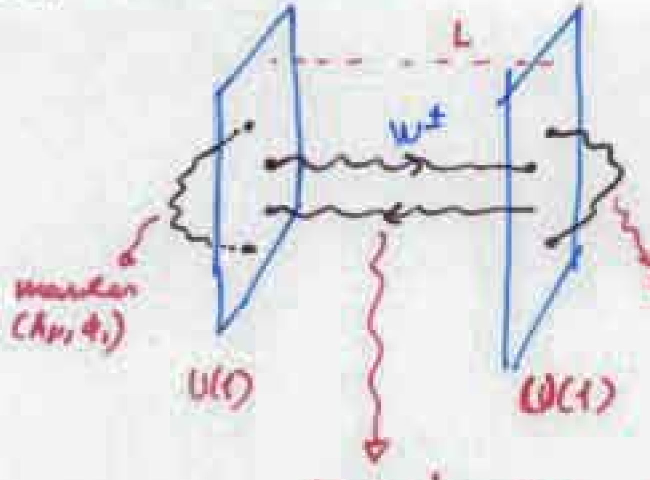
HIGHER DERIVATIVES

MASSLESS SCALAR FIELDS Y^I CORRESPONDING TO FLUCTUATIONS OF THE BRANE

NOTICE :

- WE WANT POSITIVE KINETIC TERM FOR Y^I (NO GHOSTS!) SO τ_p IS BETTER TO BE POSITIVE !

new fields may become light when two D-branes approach each other:



L = distance

$$m \sim \frac{L}{\alpha'} \ll \frac{1}{\sqrt{\alpha'}} \quad \text{if } L \ll \frac{1}{\sqrt{\alpha'}}$$

The theory on coincident Dp-branes is well described by a non-abelian YM theory: in this case U(2) with adjoint scalars

Separation of branes is an Higgs phenomenon:

$$U(2) \longrightarrow U(1) \times U(1)$$

$$\left(A_\mu, \phi = \begin{pmatrix} L & 0 \\ 0 & -L \end{pmatrix}\right) \xrightarrow[\text{Higgs}]{}$$ two photons + $W^{\pm}$ massive vector fields with masses $\sim L$ (identified with the open strings between the branes)

More generally,
on N coincident Dp-branes we have a U(N) gauge theory

U(N) SYM theory

we can obtain real gauge groups ($SO(n)$, $Sp(n)$) with a trick:

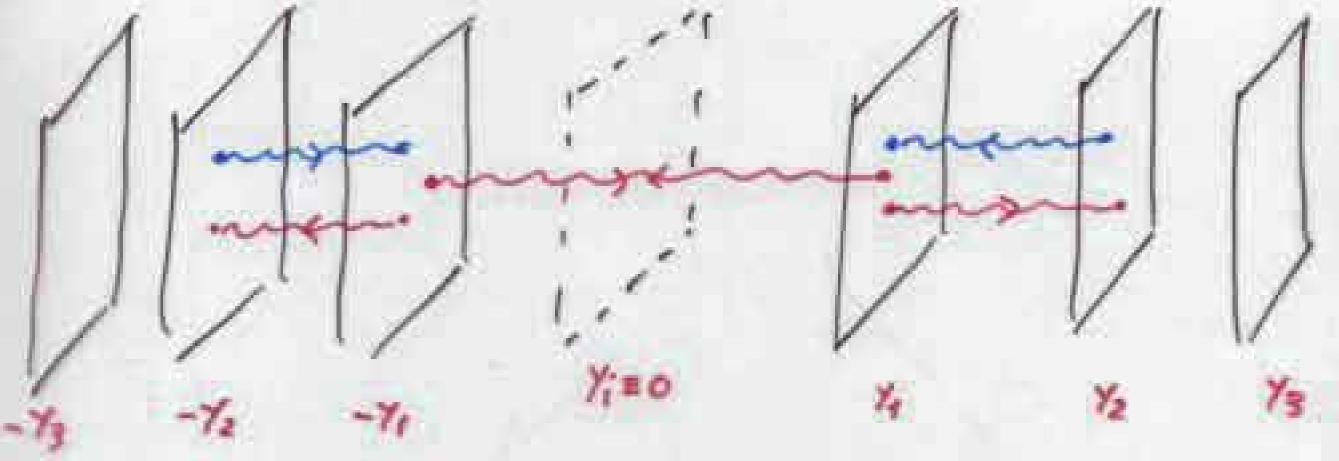
Mod out spacetime transverse to a Op-brane

$$\mathbb{Z}_2 \times \Omega :$$

ORIENTATION REVERSAL ON STRINGS

$\mathbb{Z}_2$: $y_i \to -y_i$

Ω:



EXERCISE: $U(2m)$ gauge fields become $SO(2m)$ after this projection.

The new object at $y_1 = 0$ is an ORIENTIFOLD PLANE:

- it carries negative charge $\quad \mu_{Op} = -2^{p-5}\mu$
- it carries negative tension $\quad T_{Op} = \mu_{Op}/g_s$

There are no fields on Op-plane.
(fluctuations would have scalars with negative kin. term (ghosts) since $T < 0$)

BPS CONDITION:

1/2 BPS: PRESERVES HALF OF THE SUSY:

$$\begin{cases} Q^{(1)}|\text{brane}\rangle = 0 \\ Q^{(2)}|\text{brane}\rangle \neq 0 \end{cases} \Longrightarrow \quad \tau = \frac{1}{g_s}\mu$$

$$Q = \{Q^{(1)}, Q^{(2)}\}$$

$\boxed{\begin{matrix} g_s = 1 \\ \alpha' = 1 \end{matrix}}$

	τ	μ	SUSY PRESERVED
D3	1	1	$Q^{(1)}$
O3	-1/2	-1/2	$Q^{(1)}$
$\overline{\text{D3}}$	1	-1	$Q^{(2)}$

D3 + O3 $\rightarrow$ preserves half of the susy

D3 + $\overline{\text{D3}}$ $\rightarrow$ breaks all susy

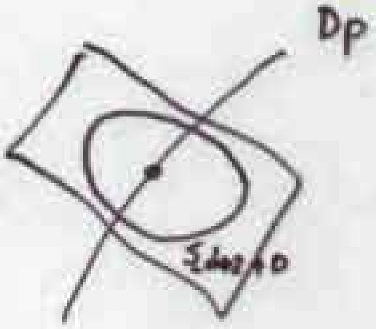
$F_{(p+2)}$ are "quantized".

Recall that in 4d the existence of charged states implies, quantum mechanically, the

DIRAC QUANTIZATION CONDITION

$$\underset{\substack{\uparrow \\ \text{electron} \\ F_{\mu\nu}}}{q_e} \cdot \underset{\substack{\uparrow \\ \text{monopole} \\ \tilde{F}_{\mu\nu}}}{q_m} = 2\pi n \qquad n \in \mathbb{Z}$$

The analogous condition for p-forms is:



for electric-magnetic pairs

$$\underset{\substack{\uparrow \\ \text{Dd-brane}}}{F_{d+2}} = * \underset{\substack{\uparrow \\ \text{(D-d-4)-brane}}}{F_{D-d-2}}$$

define electric and magnetic charges

$$q_e = \int_{S_{D-d-2}} F_{D-d-2} \qquad q_m = \int_{S_{d+2}} F_{d+2}$$

then $\boxed{q_e \, q_m = 2\pi n}$

• IN PARTICULAR : charges (fluxes) are quantized.

NOTE: the same is true, for topological reasons, for integrals over non-contractible cycles:

$$\int_X F_{D-d-2} = \text{quantized in integer units}$$

KK REDUCTION

HOW TO REACH 4D ? WE MUST COMPACTIFY

$$R^{10} = R^{1,3} \times M$$

COMPACT SIX DIM MANIFOLD

OLD KALUZA-KLEIN IDEA :

EXAMPLE : $M = T^6$

$$\phi(x,y) = \sum \phi_{\vec{n}}(x)\, e^{i\vec{n}\vec{y}/R} \qquad y_i \sim y_i + 2\pi m_i R_i$$

4 DIMENSIONAL MODES

EQ MOTION 10D

$$\Box\phi = 0 \longrightarrow$$

EQ MOTION 4D

$$\Box\phi_{\vec{n}} + \frac{|n|^2}{R^2}\phi_n = 0$$

$$m \sim \frac{n}{R}$$

There is a massless mode $\phi_0(x)$.

HOW SMALL IS R ? EXTRA DIMENSIONS MUST BE INVISIBLE. NATURAL CHOICE IN STRING THEORY

$$R \sim \ell_s$$

$$\ell_s = \frac{1}{M_s} = \sqrt{\alpha'}$$

STRING LENGTH

- with a $(4+n)$-dimensional theory

$$\boxed{M_{PL}^{n+2} \int d^{4+n}x \sqrt{g}\, R}$$

- $$\frac{1}{G_{NEWTON}} \sim M_{PL}^{n+2}$$

with n compact extra dimensions, by dimensional reduction

$$\underbrace{M_{PL}^{n+2} \cdot V}_{\downarrow} \int d^4x \sqrt{g}\, R$$

$$\boxed{\left(M^{PL}_{(4)}\right)^2 = \left(M^{PL}_{(4+n)}\right)^{2+n} \cdot \text{Volume}}$$

$$V \sim R^n$$

- $M_{(4+n)}$ is the FUNDAMENTAL SCALE OF GRAVITY

FOR TYPE II: $n = 6$

$$M_s^8 \int e^{-2\phi} \sqrt{g}\, R \Rightarrow \left(M_{PL}^{(10)}\right)^8 = M_s^8 e^{-2\phi} = \frac{M_s^8}{g_s^2}$$

- $M_{(4)}$ IS KNOWN: $\sim 10^{19}$ GEV

- $M_{(4+n)}$ can be lowered by increasing the volume (R)

SCENARIOS FOR EXTRA DIMENSIONS :

- MICRO : $R \sim 10^{19}$ GeV
- MINI : $R \sim 10^{16}$ GeV
- MIDI : $R \sim$ TeV
- MAXI : $R \geqslant 1$ mm
- NON COMPACT (see next lecture)

— FROM MICRO TO MAXI THE 4 SCENARIOS ARE COMPATIBLE WITH AND, TO A CERTAIN EXTENT, REQUIRES STRING THEORY

— ALSO COMPATIBLE WITH EXPERIMENTS? IN SOME CASE EVIDENCE FOR MIDI E MAXI CAN BE AROUND THE CORNER ?.
(SEE STRUMIA'S LECTURES)

ORTHODOX PARADIGM

- MICRO :

  All sizes $R \sim 10^{19}$ GeV, the quantum scale for gravity $M_5^{(10)} \sim M_{Pl}^{(10)} \sim 10^{19}$ GeV

OR

- MINI :

  $M_{(10)}$ = grand unification scale 10^{16} GeV by increasing R .

  (Forbidden in the 80's based on heterotic string ($g_s \gg 1$), resurrected by duality: HORAVA - WITTEN)

UNORTHODOX PARADIGM (ADD)

LARGE EXTRA DIMENSIONS

- MIDI :

  $1/R \sim$ TeV : Example $n=6$, $M_{(10)} \sim 8000$ TeV

  SIGNATURE : KK MODES WITH MASSES $\sim$ TEV

  $m^2 \sim n^2/R^2$ $1/R \sim$ TEV

- MINI :

  $M_{(4+n)} \sim$ TeV : QUANTUM GRAVITY SCALE NEAR !

  $R \sim 1$ mm $(n=2)$ $\longrightarrow$ $R \sim 1$ fermi $(n=6)$

  - Radiation in extra dim
  - string states as resonances
  - black hole at LHC
  - modification Newton law $R \lesssim 1$ mm

KK too light ? SM has no KK up to TeV : it is OK if SM lives on branes .

- KK THEORY RESURRECTED IN THE 80's :

ISOMETRIES OF M $\Longrightarrow$ MASSLESS GAUGE FIELDS IN 4 D

$$g^a_{\mu i}(x,y) = A^a_\mu(x)\, K^a_i(y) \Rightarrow \text{massless } A^a_\mu$$

EXAMPLE : $M = S^5$
ISOMETRY $SO(6)$ $\Rightarrow$ $SO(6)$ gauge fields

WITTEN KILLED KK PROGRAM IN 83 :
no way of getting CHIRAL fields

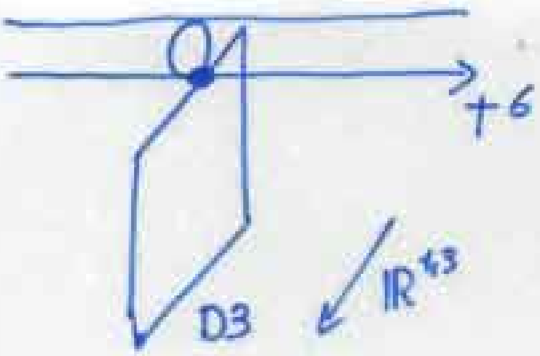
- HETEROTIC STRING REVOLUTION IN 85 :

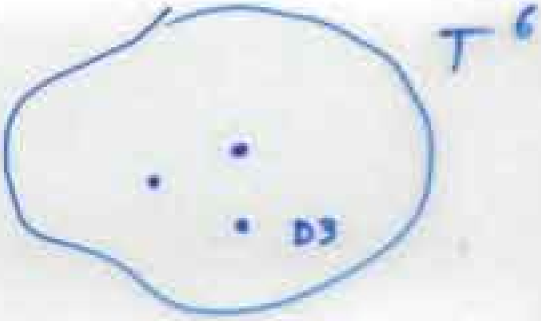
$N=1$ 10D theory with $E_8 \times E_8$ gauge group.

- NOWADAY IS MUCH EASIER TO USE D-branes (on type I = open strings)

$$\boxed{\text{MATTER FIELDS TYPICALLY LIVES ON LOCALIZED D-branes}}$$

EXAMPLE : IIB on T^6 with D3 branes





- we can localize gauge fields on D3, have multiple groups, chiral fields, etc ...

• we can have also D(3+p) - branes with p - direction compactified on T^6

- Bi-fundamental matter fields :

massless if Dp - Dp' intersects.

- Sometimes chiral :

Matter fields depend on collections of branes : plenty of papers with ALMOST Standard Model on MSSM construction.

CONSISTENCY CONDITIONS : TADPOLE CANCELLATION

or better: CHARGE CONSERVATION

BY GAUSS LAW : total charge in a compact space must be zero.

Proof : the eqs of motion for sources of a $A_{(p+1)}$ form :

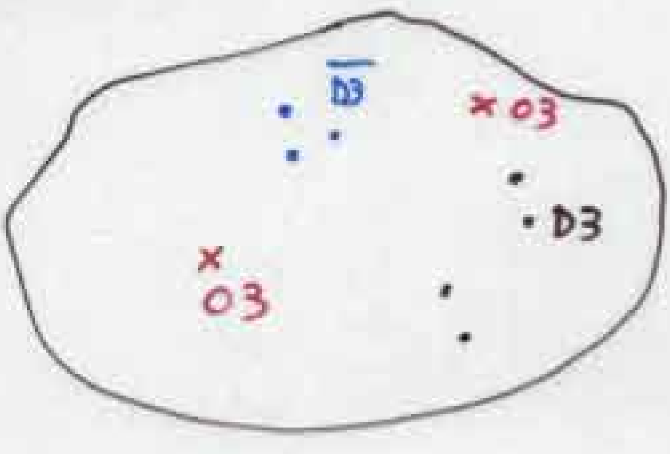
$$d * F_{(p+2)} = \sum \mu_i \, \delta(x_i^\circ)$$

by integration :

$$0 = \int_M d * F_{(p+2)} = \sum \mu_i \quad \blacksquare$$

↑ M compact

• If we include D3 branes on M we must also include O3 planes or other negative charge objects, like anti-D3 branes :



$$N_{D3} + N_{\overline{D3}} = \frac{N_{O3}}{2}$$

$\mu_{Op} = -2^{5-p}\mu_{D3}$

When M preserves supersymmetry?

Consider the background:

$$\begin{cases} g_{\mu\nu} = (g_{(4)} \otimes g_M) \\ B = F_{(p)} = 0 \\ \psi = \lambda = 0 \end{cases}$$

Susy variations reduce to

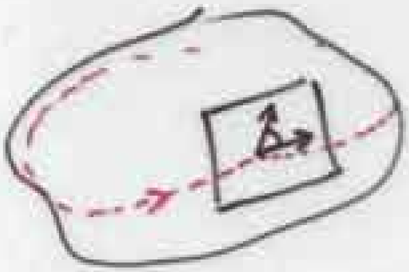
$$\delta \text{ bosons} = \text{fermion} \equiv 0$$
$$\delta \psi_\mu \sim D_\mu \varepsilon$$

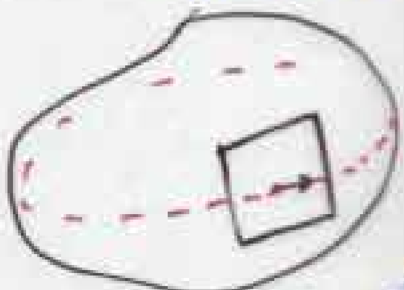
Susy is preserved if $D_\mu \varepsilon = 0$: THERE EXISTS A COVARIANTLY CONSTANT SPINOR ON M

Geometric interpretation:

HOLONOMY GROUP

ON SPINORS





TM : fiber $\mathbb{R}^6$ with SO(6) action

fiber $\mathbb{C}^4$

$$\text{Holonomy} = \left\{ \begin{array}{c} \text{group of all elements} \\ \text{of } SO(6) \text{ obtained} \\ \text{by parallel transport} \\ \text{of vectors} \end{array} \right\}$$

$$v \xrightarrow[\text{PARALLEL TRANSPORT}]{} v' = gv \quad g \in \text{Hol}$$

Now $SO(6) \simeq SU(4)$

$\underline{6}$: vector

$\underline{4}$: spinor

Since there is a covariantly constant ϵ

$$\mathbb{C}^4 = \mathbb{C} \oplus \mathbb{C}^3$$

$$4 \to 1 + 3$$

$$SO(6) \sim SU(4) \to SU(3) \equiv Hol(M)$$

- Manifolds with holonomy $SU(3)$ are very special: they are named

CALABI - YAU Manifold

CY admit Ricci-flat metrics, so that our supersymmetric background has the form

$$(\eta_{\mu\nu},\ g^{(6)}_{\mu\nu})$$

FLAT METRIC : ZERO COSMOLOGICAL CONSTANT IN 4D

- We can have even more susy:

  - Two covariantly constant spinors, ϵ_1 and ϵ_2:

$$Hol(M) = SU(2)$$

one proves: $\quad M = T^2 \times K_3$

  - Four covariantly constant spinors

$$Hol(M) = \{0\}$$

$$M = T^6$$

On M with $Hol(M) = SU(3)$ we have $N=2$ in 4 dimensions:

IIB 10 d $\qquad\qquad\qquad$ $d=4$

$\varepsilon_1, \varepsilon_2 \qquad \Longrightarrow \qquad \tilde{\varepsilon}_1, \tilde{\varepsilon}_2 \quad N=2$

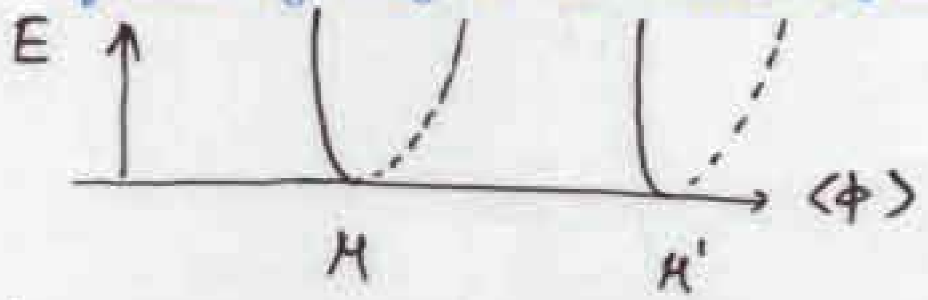
(of opposite chirality)

How can we get $N=1$?

- change 10d string (heterotic, type I)
- introduce D3 brane: we get gauge fields and we break half of the susy
- introduce fluxes (see next lecture)

If we preserve susy we have typically many flat directions:

If we deform $M \to M'$ (both supersymmetric) both compactifications have zero energy, then they are both vacua of our theory and they must be connected by some scalar fields getting a vev with a flat potential



Deformations parameters in our supersymmetric solutions are massless fields in 4d with flat potential (MODULI)

Example: T^6 : take only $g_{\mu\nu}$ for simplicity.

$g_{IJ}(x,y) \longrightarrow$

$g_{\mu\nu}(x)$	graviton
$g_{\mu i}(x)$	vector fields $U(1)^6$ isometries
$g_{ij}(x)$	massless scalars

Since $ds^2 = g_{\mu\nu}\, dx_\mu\, dx_\nu + g_{ij}(x)\, dy^i dy^j$

$G_{ij} = \langle g_{ij}(x) \rangle \neq 0$ means that the torus T^6 has radii corresponding to the metric G_{ij}

Varying the $\boxed{\text{size and shape}}$ of T^6 : VEVs for scalar fields

EXAMPLE 2 : CALABI-YAU

Reduction much more difficult: massless modes are classified by cohomology groups of M, and still represents size and shape of M

EFFECTIVE ACTION ?

It should be computed case by case.
Supersymmetry helps.
For example, for $N=1$ compactifications we know that the form of SUGRA is:

KINETIC TERMS determined by a real function $K(\Phi,\bar{\Phi})$

$$\int d\theta^2 d\bar{\theta}^2 \, K(\Phi,\bar{\Phi}) \rightsquigarrow \partial_i\partial_{\bar{J}} K \, \partial\Phi^i \partial\bar{\Phi}^{\bar{J}}$$

KAHLER POTENTIAL

$$G_{i\bar{J}} = \partial_i \partial_{\bar{J}} K$$

POTENTIAL TERMS determined by an holomorfic function $W(\Phi)$

$$V = e^{K}\left(\sum_{i\bar{J}} G^{i\bar{J}} D_i W D_{\bar{J}} \bar{W} - 3|W|^2 \right)$$

$W(\Phi)$ SUPERPOTENTIAL

II

By warping we mean:

$$ds^2 = f^2(y)\, dx_\mu^2 + ds_M^2 \qquad y \in M$$

in a compactification with branes

observers — measures energy E' — measures energy E

$$E' = \frac{f(y)}{f(y')} E$$

all energy rescaled depending on position in interior space

- Based on warping there is another unortodox possibility for the existence of extra-dimensions: RANDALL-SUNDRUM MODELS:

  - space with extra <u>non-compact</u> dimensions

  - "localized" gravity: i.e. gravity is four-dimensional

Consider 1 extra dimension:

Requiring Poincaré invariance in 4d, the most general spacetime is:

$$ds^2 = (dx)^2 + a^2(x)\, dx_\mu dx^\mu$$

$$(x_\mu, x) \to (-,+,+,+,+)$$

By dimensional reduction, the 4d Planck mass (or Newton constant) is:

$$M^3\int d^5x \sqrt{-g}\, R \longrightarrow M^3 \int_{-\infty}^{+\infty} dx \int d^4x_\mu \sqrt{-g_{(4)}}\, R_{(4)}\, a^2(x)$$

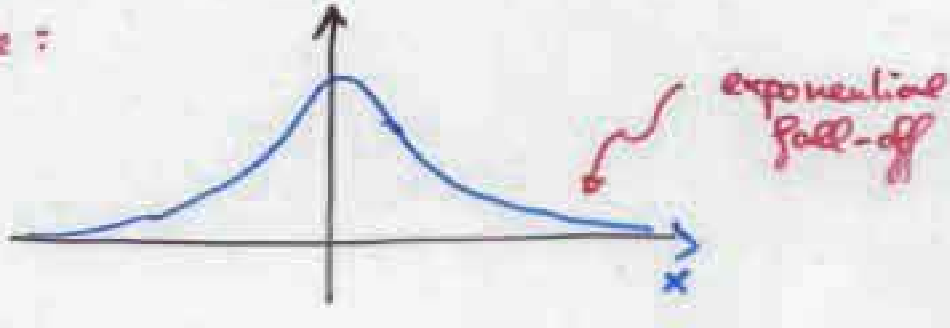
$$\downarrow$$

$$\underbrace{\left(M^3\int dx\, a^2(x)\right)}_{\text{4d Planck scale } M_{Pl}^2} \int d^4x_\mu \sqrt{-g_{(4)}}\, R_{(4)}$$

We do not need x necessarily compact, but

$$\int dx\, a^2(x) < +\infty$$

Example:



An interesting space with exponential warp factor is

AdS$_5$: $$ds^2 = dx^2 + e^{-2kx}(dx_\mu dx^\mu)$$

$$x \in (-\infty, +\infty)$$

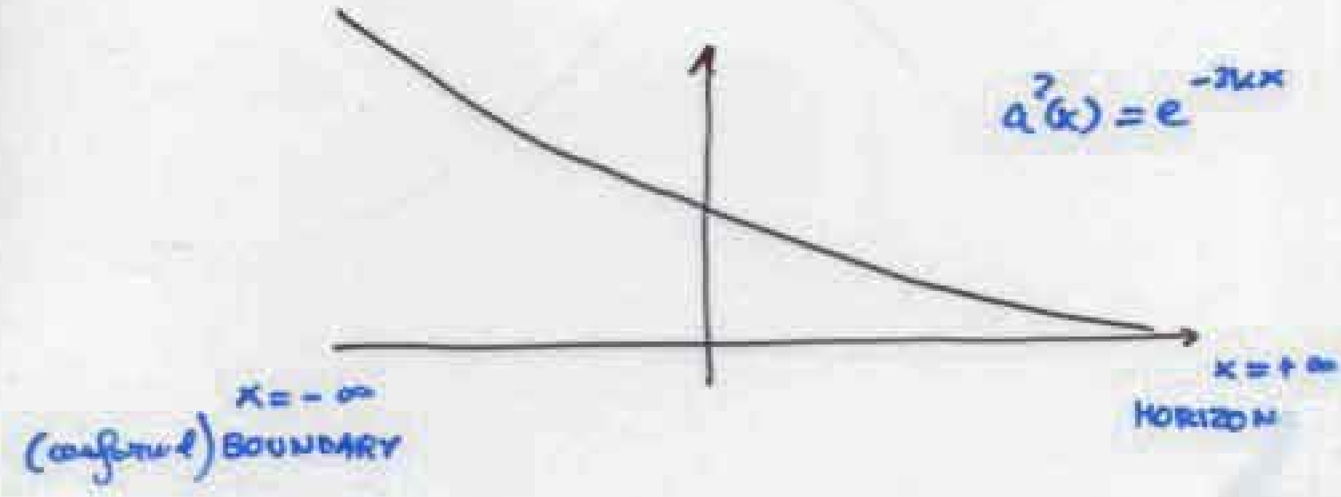
- Curvature is constant ($\sim k$)

$\downarrow$

vacuum for the 5d theory with cosmological constant

$$\int d^5x \sqrt{-g}\,(2M^3 R - \Lambda)$$

$$(\Lambda = -24 M^3 k^2)$$

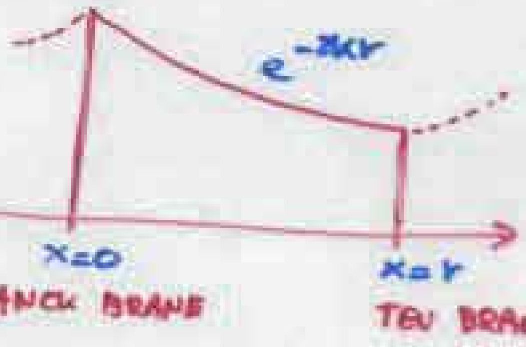


Now, the integrated warp factor diverges

$$\int_{-\infty}^{+\infty} dx\; a^2(x) = \int_{-\infty}^{+\infty} dx\; e^{-2kx} = +\infty$$

the boundary contribution

However the 4d M_{Pl} is finite if we consider only a slice of AdS_5:



RSI MODEL

Spacetime cannot end: we need boundaries ≡ branes

$$\int d^5x\sqrt{-g}\left(2M^3R-\Lambda\right)-\tau_P\int d^4x\sqrt{-g}\,\delta(x=0)-\tau_T\int d^4x\sqrt{-g}\,\delta(x=r)$$

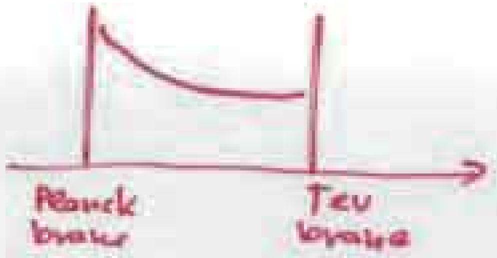
$(\Lambda=-24M^3k^2)$

Consider a background $R^{1,3}\times \underset{\overset{|||}{S^1/Z_2}}{I}$

Einstein eqs:
$$\begin{cases}6\ddot{\phi}=-\frac{\tau_P}{M^3}\,\delta(x=0)-\frac{\tau_T}{M^3}\,\delta(x=r)\\ 6(\dot{\phi})^2=-\Lambda/M^3\end{cases}$$

- in the bulk: $\ddot{\phi}=0\Rightarrow\phi$ linear
  $\phi=2kx \qquad 24k^2=-\Lambda/M^3$
- effect of branes:
  matching the delta functions at $x=0$, $x=r$ $\qquad \tau_P=-\tau_T=24M^3k$

NOTE: one brane has negative tension. No problem if it does not fluctuate. Negative tension objects (orientifolds) COMMON AND REQUIRED IN STRING THEORY.



- we can have fields on both branes

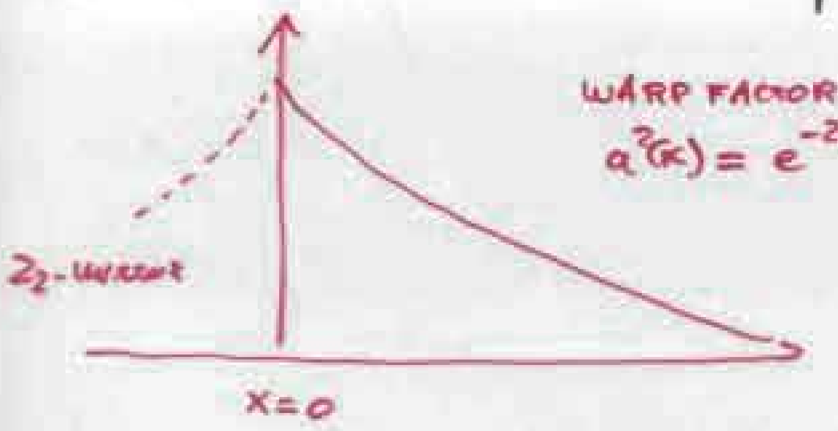
- possible solution of hierarchy problem:

  an observer on the TeV brane sees scales controlled by a factor of e^{-2kr} with respect to an observer on the Planck brane

- RS I contains SM on a brane and KK graviton modes

  two scales: TeV and Planck. gravity is strongly interacting and there are KK already at the TeV

We can also have a non-compact internal dimension:



RS II MODEL

- KK MODES ARE NOW A CONTINUUM

- VERY SHORT DISTANCES : 5 DIM

$$F \sim \frac{m_1 m_2}{r^3}$$

- LARGE DISTANCES : 4 DIM

$$\frac{1}{G_N} \sim 2M^3 \int_0^{\infty} dx\, a^2(x) = 2M^3 \int_0^{\infty} e^{-2kx} dx < +\infty$$

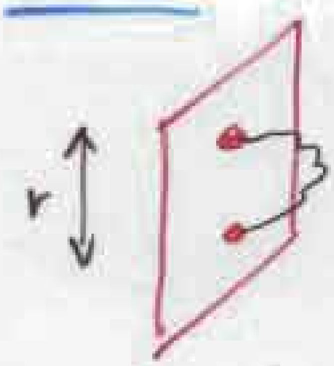
WITH SMALL CORRECTION TO NEWTON LAW DUE TO KK GRAVITON:

$(g_{\mu\nu}, g_{\mu\nu}^{kk})$

$$F = \frac{m_1 m_2}{r^2} + O\left(\frac{1}{r^4}\right)$$

- this mild correction is non trivial because KK modes in non-compact space are continuously distributed in mass, suggesting STRONG CORRECTION TO GRAVITY

NOTE: how do we compute corrections?



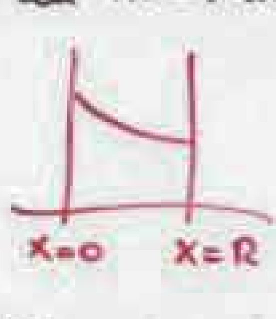
5d graviton = $g_{\mu\nu}^{(4)}$ + massive KK modes

we compute the Green function

$$G(x_\mu, x\,;\, x'_\mu, x') \sim \sum\!\!\!\!\!\!\int_n \frac{\psi_n(x_\mu, x)\,\psi_n(x'_\mu, x')}{E_n}$$

eigenvalues and eigenfunctions of the KK modes

Since spectrum is continuum, we regulate (with an IR brane at position R)

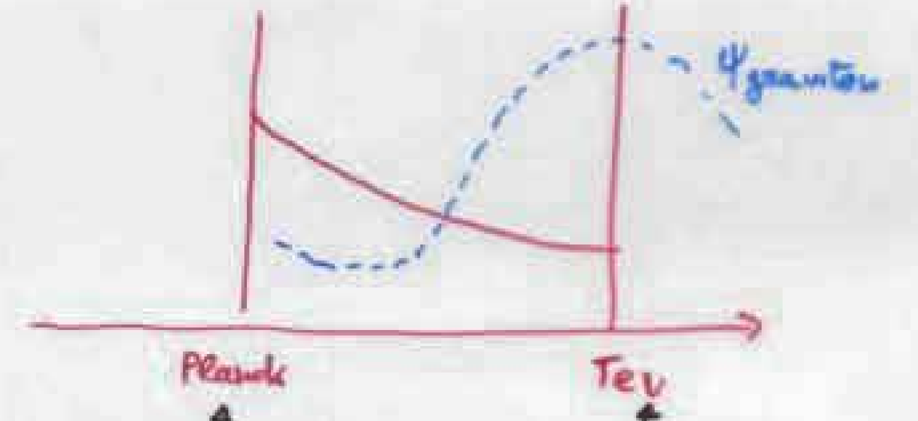


ψ_n discrete solutions of $\text{Einstein}(\psi_n) = E_n \psi_n$

and then remove the regulator

$$\psi_n \sim \text{Bessel functions} \underset{R\to\infty}{\Longrightarrow} F^2 = \frac{1}{|x|^2} + O\left(\frac{1}{|x|^4}\right)$$

HOLOGRAPHIC INTERPRETATION OF RS[1/4]



observer on Planck brane see M_{pe} as scale of gravity

due to redshift the scale where gravity strong interacts is lowered to TeV:
strongly interacting graviton and KK modes

What is the "real" scale of gravity?

- the 5d interpretation of this somehow confusing fact is that the graviton wavefunction is peaked on the TeV brane.

- there is an alternative explanation of this fact:

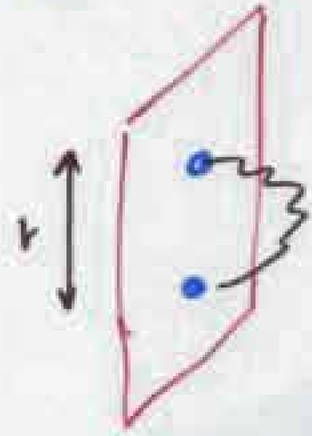
RS models are equivalent to $\left\{\begin{array}{l}\text{Purely 4d models} \\ \text{where ordinary} \\ \text{gravity interacts} \\ \text{with and hidden} \\ \text{sector which is} \\ \text{(almost conformal)}\end{array}\right.$

$$\text{5d } \overset{(5)}{g_{\mu\nu}} \equiv \text{4d } \underbrace{g^{(4)}_{\mu\nu}}_{\text{massless}} \oplus \text{CFT}$$

$$\begin{array}{c} ||| \\ \left\{ \underbrace{g^{(4)}_{\mu\nu}}_{\text{massless}},\ \underbrace{g^{(4)\,\text{KK}}_{\mu\nu}}_{\text{massive}} \right\} \end{array}$$

Consider first $\boxed{\text{RS II}}$: corrections to gravity

• in 5d GR we compute the gravitational force between objects on the brane:



compute a Green function for the 5d Einstein eqs

$$F \sim \frac{1}{r^2} + O\left(\frac{1}{r^4}\right)$$

• In the dual picture with 4d gravity coupled to a CFT we just consider the effects of the interaction $g_{\mu\nu}T^{\mu\nu}$

$g_{\mu\nu}$ graviton propagator $\qquad g_{\mu\nu}\langle T_{\mu\nu}T_{\rho\sigma}\rangle g_{\rho\sigma}$

$$\frac{1}{p^2} \quad + \quad \frac{1}{p^2}\left(p^4 \log p\right)\frac{1}{p^2}$$

since $\langle T_{\mu\nu}(x)T_{\rho\sigma}(0)\rangle \sim \frac{1}{|x|^8}$

$$\langle T_{\mu\nu}(p)T_{\rho\sigma}(-p)\rangle \sim p^4 \log p$$

The correction is

$$\frac{1}{p^4}\, p^4 \log p \sim \log p \underset{\text{Fourier transform}}{\Longrightarrow} \qquad \frac{1}{|x|^4}$$

From "complete" propagator $\langle g_{\mu\nu}\, g_{\rho\sigma} \rangle$ we extract the force and gravitational potential:

Thus explained the correction

$$F \sim \frac{1}{r^2} + \frac{c}{r^4}$$

- we also learn that c is related to $\langle T_{\mu\nu} T_{\rho\sigma} \rangle = c/|x|^8$, called "central charge" of the CFT

- One can do a similar analysis for vector fields: suppose one have a bulk gauge field

$$\int \sqrt{g}\,(2M^3 R - \Lambda)\, dx^5 + \int \sqrt{g}\, \frac{1}{g_M^2} F_{\mu\nu}^2\, dx^5$$

this gives rise to

$$A_\mu \to \Big(\underset{\text{massless zero mode}}{A_\mu^{(0)}}\,,\ A_\mu^{(n)\,KK} \Big)$$

In the dual interpretation

$$(g_{\mu\nu}^{4d}, g_{\mu\nu}^{KK}, A_\mu^{4d}, A_\mu^{KK}) \Longrightarrow g_{\mu\nu}^{4d} + A_\mu^{4d} \text{ coupled to CFT}$$

all fields in the CFT couple to $g_{\mu\nu}$; in this case we also have charged matter

(Call J_μ the current associated with A_μ)

- Since J_μ is made with CFT fields:

$$\langle J(p) J(-p) \rangle \sim p^2 \log p^2$$

so that the gauge propagator, after including ~~(CFT)~~ corrections

$$G(p) \sim \frac{1}{p^2 \ln p}$$

- evolution of gauge couplings in RS is logarithmic (this can be computed using KK)

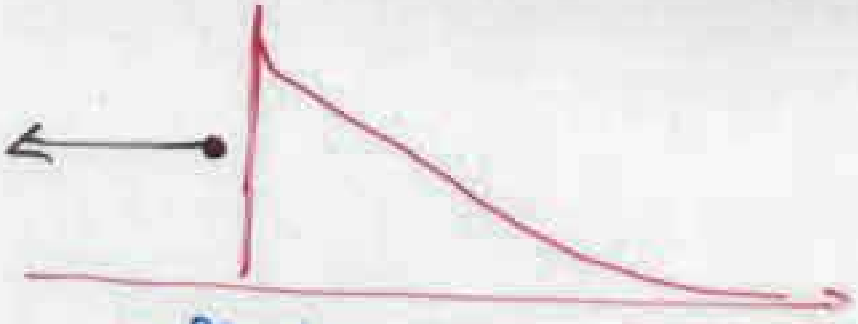
- <u>the message is</u>:

$$\underbrace{(g_{\mu\nu}, A_\mu)\ \text{KK modes}}_{\text{5d part of the game}} \cong \text{EFFECTS OF AN HIDDEN CFT IN 4d}$$

substantially equivalent to "composite" operators $T_{\mu\nu}$, J_μ

The holographic interpretation of RS is equivalent to the so-called <u>AdS - CFT correspondence</u>

Take the Planck brane to infinity:



This is equivalent to

decompactify $\cong$ take 4d $M_{Pl} \to \infty$

$\parallel\!\!\parallel$

decouple 4d gravity

In the correspondence:

$$\left(g_{\mu\nu}^{4d},\ g_{\mu\nu}^{KK}\right) \to g_{\mu\nu}^{4d} + \text{CFT}$$

+ other fields

$\Downarrow$

- $g_{\mu\nu}^{4d}$ decouples ($M_{Pl} \to \infty$)
- $g_{\mu\nu}^{KK}$ reconstructs a truly 5d graviton propagating in non-compact space - KK description is now misleading

$$g_{\mu\nu}^{5d} \longrightarrow \text{CFT}$$

gravity theory in 5d

4d conformal field theory

- Fields in an AdS_5 background can be thought as "composite" of some 4d CFT

$$g_{\mu\nu} \longrightarrow T_{\mu\nu}$$
$$A_\mu \longrightarrow J_\mu$$
$$\phi \longrightarrow \mathcal{O}$$

(some scalar operator with the same quantum number of ϕ)

This is known as AdS/CFT CORRESPONDENCE in string theory – many checks of this correctness.

REMARK :

I. SIMMETRIES

$SO(4,2) \equiv$ ISOMETRIES OF AdS_5 $\equiv$ CONFORMAL GROUP IN 4d

II. GAUGE FIELDS IN AdS

GAUGE FIELDS IN THE BULK $\equiv$ GLOBAL SYMMETRIES IN THE CFT

$g_{\mu\nu} \longrightarrow$ Poincaré $(T_{\mu\nu})$

$A_\mu \longrightarrow$ global U(1) current (J_μ)

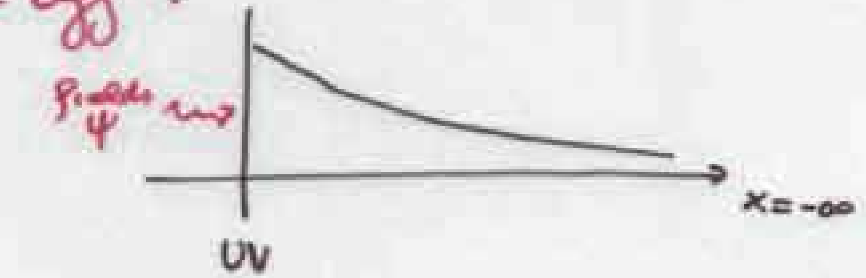
(with minor subtleties) the AdS/CFT prescription is:

$$Z(\phi^{(0)}) = \left\langle e^{i\int \phi^{(0)}(x)\mathcal{O}(x)d^4x} \right\rangle_{CFT} = e^{S_{AdS_5}(\bar{\phi}(x,y))}$$

• This boundary-bulk relation is called HOLOGRAPHY. It explains how a 5d theory can be possibly equivalent to a 4d one!

• This also applies to RS II:

Just consider the RS II as a "regularized" AdS5 where the Planck brane is an UV cut-off:



AdS/CFT then gives a proof of the holographic interpretation of RS II since it can be shown that:

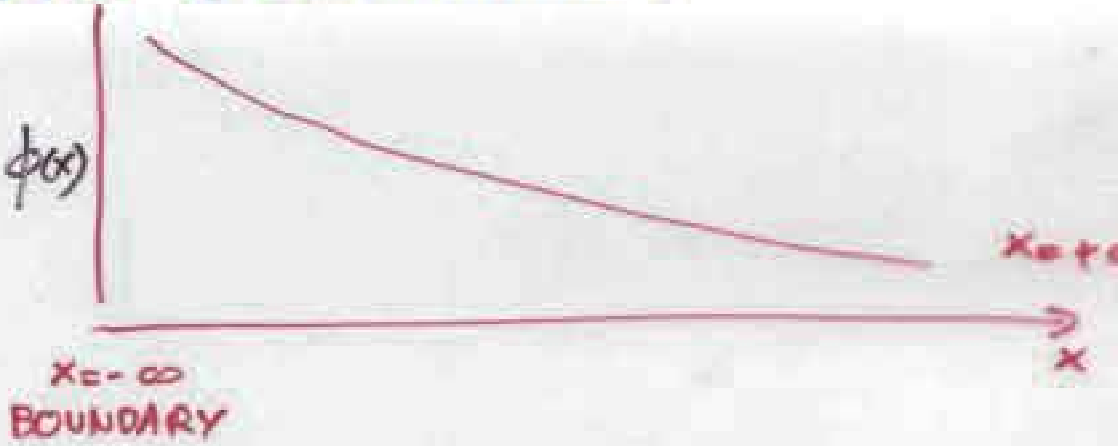
$$S_{brane}(g^{(0)}_{\mu\nu}, \psi) + S_{bulk}(\bar{g}_{\mu\nu}) \Big| \equiv S_{UV}(g^0, \psi) + S_{CFT} + \int\sqrt{g}R$$

eventual coupling to UV fields

eqs motion with boundary condition $g_{\mu\nu}|_{UV} = g^0_{\mu\nu}$

Einstein term is generated in computation

AdS/CFT is QUANTITATIVE :



given a field $\phi(x_\mu, x)$ in AdS_5 and the correspondent operator O there is a natural coupling

$$\phi(x)\, O(x)$$ source-operator

Examples $\begin{cases} g_{\mu\nu}(x)\, T_{\mu\nu}(x) \\ A_\mu(x)\, J_\mu(x) \end{cases}$

However $\phi(x_\mu, x)$ is 5d, and $O(x)$ is 4d. We couple them at the boundary

In QFT consider the generating functions for correlators of $O(x)$:

$$Z(\phi_0) = \left\langle e^{i\int d^4x\, \phi_0(x) O(x)} \right\rangle_{CFT}$$

• Consider $\phi_0(x)$ as the boundary value of a 5d field $\phi(x_\mu, x)$:

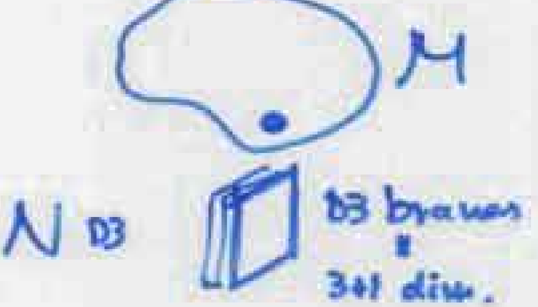
determine $\Phi(x_\mu, x)$ as the solution of 5d equation of motions (Einstein + ...) with boundary condition $\phi^{(0)}(x_\mu)$

String realization for RS ?

- WARP FACTORS ARISE IN THE PRESENCE OF
  - I – BRANE SOURCES
  - II – FLUXES

- I/II ARE COMPLEMENTARY IN THE SENSE OF THE AdS/CFT CORRESPONDENCE

I. Recall that Dp-branes deform spacetime due to their tension.



$$ds^2 = H^{-1/2}(r)\, dx_\mu^2 + H^{1/2}(r)\, d\Omega_M^2$$

$$H(r) = 1 + \frac{N}{r^4}$$

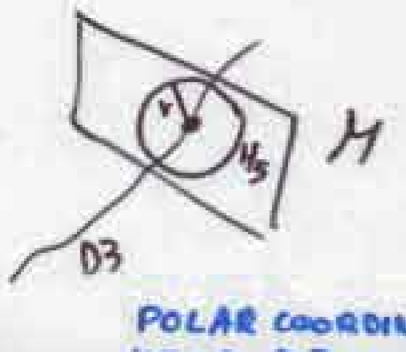
if $N \gg 1$ $\qquad H(r) \sim N/r^4$

$$\Downarrow$$

$$ds^2 = r^2 dx_\mu^2 + \frac{1}{r^2}\left(dr^2 + r^2 d\Omega_M\right)$$

$$\Downarrow \; r = e^{-kx}$$

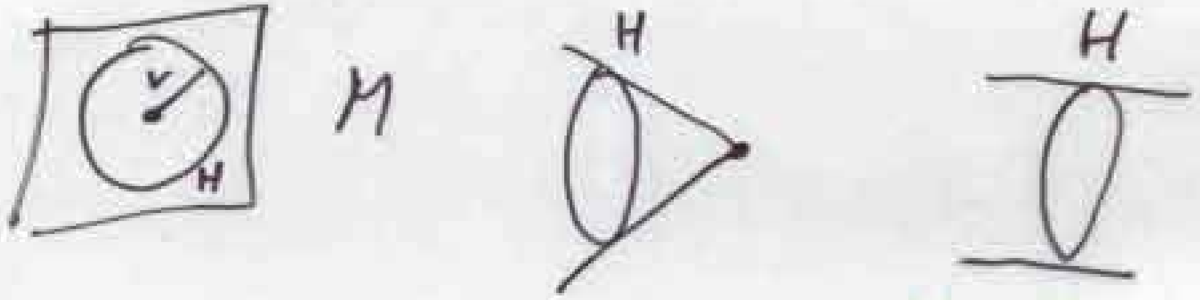
$$\boxed{ds^2 = \left(e^{-2kx} dx_\mu^2 + dx^2\right) + d\Omega_M}$$



POLAR COORDINATES NEAR D3

AdS_5 x compact

II. fluxes. It is a complementary view:



one can consider the compact 5d space H and put some $F_{(5)}$ flux

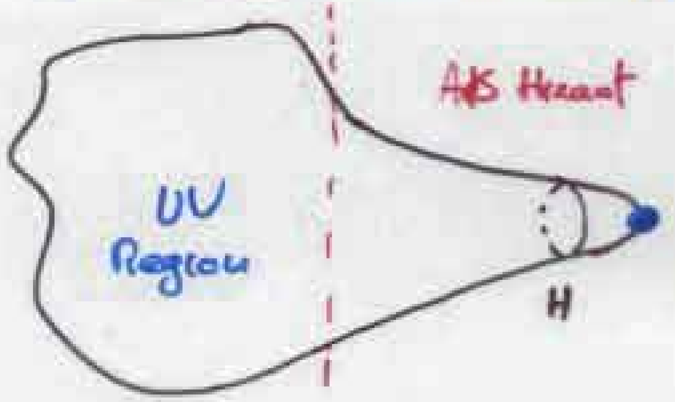
D3 branes charged

$$ds^2 = ds^2_{(5)} + ds^2_{\substack{H \\ +\text{flux}}}$$

$$\int_H F_{(5)} = N$$

backreaction of $F_{(5)}$ on the metric typically produces a solution of the Einstein eqs of motion that is AdS_5 in 5d.

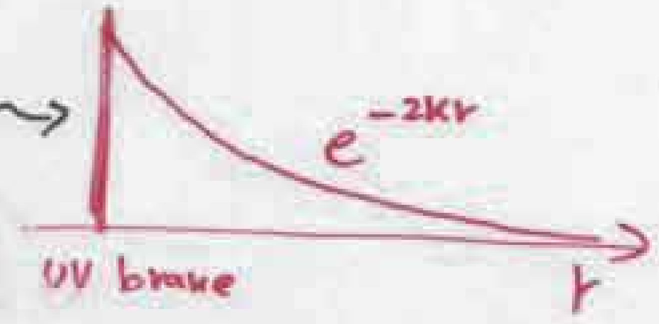
- Stringy realization of RSII



PUT LARGE NUMBER OF BRANES N OR LARGE FLUX OF F_5

fields ψ and couplings parametrizing the UV physics

$$S(\psi) = \int_{\substack{\text{momenta} \\ k \gg \Lambda_{UV}}} d\Phi \, e^{S_{\text{string}}(\Phi)}$$



- FLUXES/BRANES ARE ALMOST EQUIVALENT, EXCHANGED BY THE AdS/CFT correspondence:

    - The CFT holographically dual to the RSII model just constructed is the YM theory living on the D3 sources.

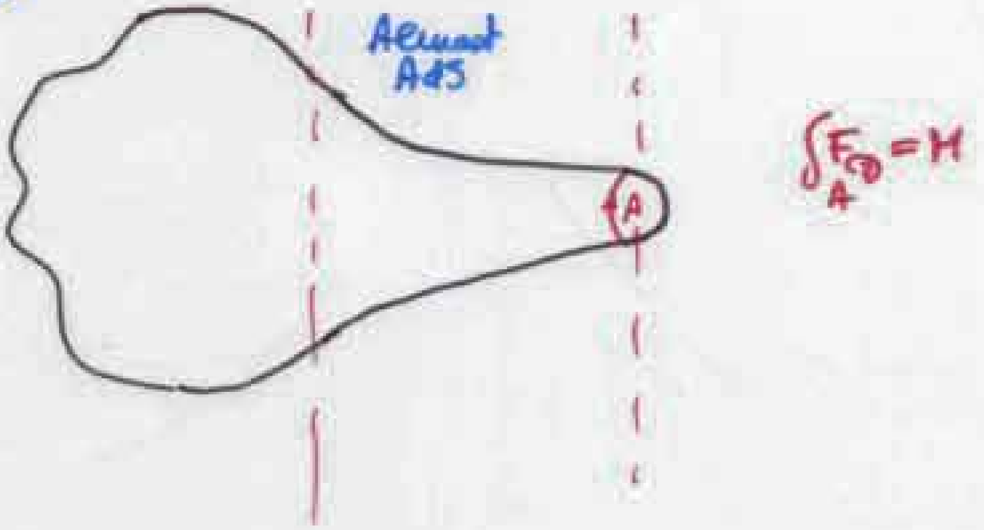
String realization of RSI

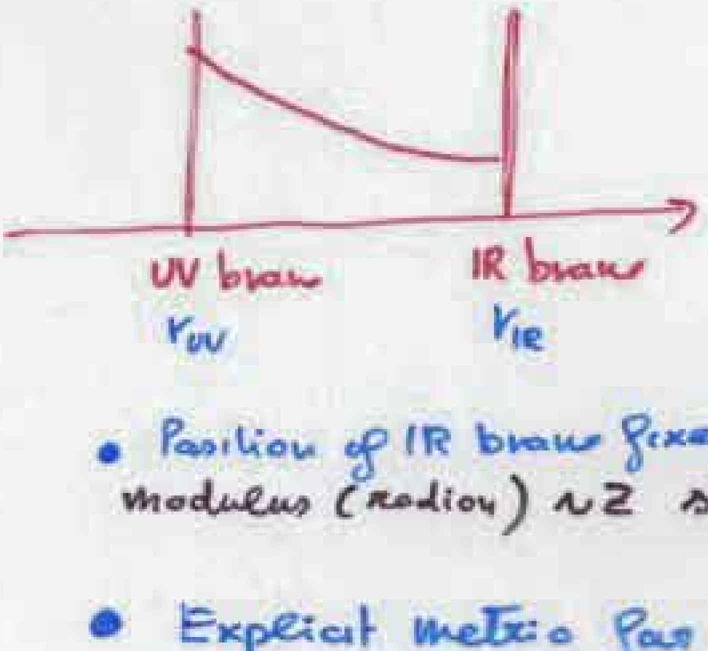
SEE FIRST LECTURE II

We need to cut-off the IR: the dual theory is now an ALMOST conformal theory:

QFT
- conformal from Planck to TeV scale
- conformal invariance broken at the TeV scale

Stringy realization as GKP:





- Position of IR brane fixed: corresponding modulus (radion) $\sim Z$ stabilized
- Explicit metric for $r < r_{UV}$ known:

$$ds^2 = H^{-1/2}(r)\, dx_\mu^2 + H^{1/2}(r)\, ds_{CY}^2$$

- From Planck to Tev:

$$H(r) \sim \frac{1}{r^4 \log r}$$

(logarithmically corrected AdS_5)

- Below Tev:

spacetime $\sim \mathbb{R}^{1,6} \times S^3$

- the dual gauge theory is also known:

$$SU(N+M) \times SU(N) \qquad \mathcal{N}=1 \text{ SYM}$$

with bi-fundamentals chiral fields

- Almost conformal at high energies ($M \ll N$)
- Confining at low energies

$$z \sim e^{-k/Mg_s}$$

scale of the confinement gauge theory

- Extremely interesting physics (cascading gauge theory)

III COMPACTIFICATION WITH FLUXES

Main features:

- NON TRIVIAL WARPING
- STABILIZATION OF MODULI
- MANY STABILIZED VACUA

With "warping" we mean:

$$ds^2 = f^2(y)\, dx_\mu^2 + ds^2_M \qquad y \in M$$

In a compactification with branes

measure energy E'

measure energy E

$$E' = \frac{f(y')}{f(y)} E$$

all energy scales depends on position in internal space

It was longly believed that COMPACTIFICATIONS with warp factors were forbidden:
as usual, brane/fluxes have changed the point of view.

EXAMPLE: TYPE IIB COMPACTIFICATION

$$S_{IIB} \sim \int d^{10}x \sqrt{g}\left(R - \frac{\partial\tau\,\partial\bar{\tau}}{2(\mathrm{Im}\,\tau)^2} - \frac{G_{(3)}\bar{G}_{(3)}}{12\,\mathrm{Im}\,\tau} - \frac{\tilde{F}_5^2}{4\cdot 5!}\right)$$

$$+ \int \frac{C_4 \wedge G_{(3)} \wedge \bar{G}_{(3)}}{\mathrm{Im}\,\tau}$$

where fields have been reorganized

$$\left.\begin{array}{l} (g_{\mu\nu}, B_{\mu\nu}, \phi) \\ (\tilde{\phi}, \tilde{B}_{\mu\nu}, A_{\mu\nu\tau\rho}) \end{array}\right\} \Rightarrow \quad \begin{array}{l} \tau = \tilde{\phi} + i e^{-\phi} \\ G_{(3)} = F_{(3)} - \tau H_{(3)} \end{array}$$

$$\left(F_{(3)} = d\tilde{B},\ H_{(3)} = dB,\ \tilde{F}_{(5)} = *\tilde{F}_{(5)} \right)$$

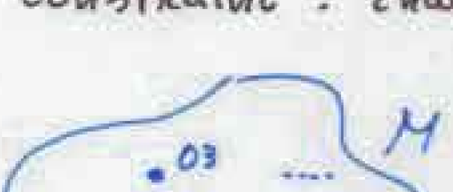
a subtlety: $\quad \tilde{F}_{(5)} = \underset{\substack{\parallel \\ dC_{(4)}}}{F_{(5)}} - \frac{1}{2}\tilde{B}_{(2)} \wedge H_{(3)} + \frac{1}{2} B_{(2)} \wedge F_{(3)}$

MODIFIED BIANCHI IDENTITIES

Look for solutions:

$$\begin{cases} ds^2 = e^{2A(y)} dx_\mu^2 + e^{-2A(y)} \tilde{g}_{ij}(y)\, dy_i\, dy_j \\ \tilde{F}_{(5)} = d\alpha(y)\, dx^0 \wedge dx^1 \wedge \dots dx^3 + *(\qquad) \end{cases}$$

constraint: charge conservation for $\tilde{F}_{(5)}$



$$d*\tilde{F}_{(5)} = d\tilde{F}_{(5)} = dF_{(5)} + \int H_{(3)} \wedge F_{(3)}$$

$$\Downarrow$$

$$\int H_{(3)} \wedge F_{(3)} + Q_3 = 0$$

$\uparrow$

~ number D3 − number O3 − number $\overline{D3}$ + ...

The trace of Einstein eqs give

$$\nabla^2 e^{2A} = e^{2A} \frac{G\bar{G}}{\mathrm{Im}\,\tau} + e^{-6A}\left[(\partial\alpha)^2 + (\partial e^{4A})^2\right] + e^{2A}\left(T^m_m - T^\mu_\mu\right)$$

integrating over COMPACT M we get:

$$0 = |G|^2 + (\partial\alpha)^2 + |\partial e^{4A}|^2 + \text{sources}$$

so in absence of localized sources,

$$\left.\begin{array}{l} G = 0 \\ \alpha = \text{constant} \end{array}\right\} \text{no fluxes}$$

$$e^A = \text{constant} \rightarrow \text{no warping}$$

If the contribution of sources is negative we can get a solution.

Consider a Dp brane wrapped on a (p-3) cycle in M:

$$S_{Dp} \sim \tau_p \int \sqrt{g} + \mu_p \int C_{p+1}$$

General Relativity exercise: compute T_{MN}:

$$T_{\mu\nu} = -\tau_p e^{2A} \eta_{\mu\nu} \delta_\Sigma \; ; \quad T_{ij} = -\tau_p \underbrace{\delta^\Sigma_{ij}}_{\text{supported on the cycle}}$$

so that:

$$\left(T^i_i - T^\mu_\mu\right) = (7-p)\tau_p \delta_\Sigma$$

since we like $p \leq 7$

$$\boxed{\text{We need negative tension object to evade the no-go theorem}}$$

Fortunately, we have plenty of orientifolds ...

Moreover $d\tilde{F}_{(5)} = H_3 \wedge F_3 + \rho_3(\text{sources})$ gives the eq.

$$\nabla^2 \alpha = e^{2A} \frac{G * \bar{G}}{\text{Im}\,\tau} + e^{-6A} \partial_\mu \alpha\, \partial^\mu e^{2A} + e^{2A} \rho_3$$

which combined with trace of Einstein eqs:

$$\nabla^2 (e^{4A} - \alpha) = \frac{e^{2A}}{\text{Im}\,\tau} \left| i G_{(3)} - * G_{(3)} \right|^2 + e^{-6A} \left| \partial (e^{4A} - \alpha) \right|^2$$

$$+ e^{2A} \left[\frac{1}{4} (T^m_m - T^\mu_\mu) - \rho_3 \right]$$

Now the last term is POSITIVE for a great number of branes

$$\frac{1}{4} (T^m_m - T^\mu_\mu) - \rho_3 \geq 0$$

- it is SATURATED for D3, O(3) $(T \sim \mu)$
- it is positive for $\overline{D3}$ $(T \sim -\mu)$

- In a compactification with only D3 and O3 (satisfying charge cancellation) we then get

  - $\alpha = e^{4A}$ warping and $F_{(5)}$ shape related

  - $* G_{(3)} = i G_{(3)}$ ISD = imaginary self dual 3-flux

(BPS solutions)

Noone was able to solve completely the equation of motion nor to write a complete four dimensional effective action, but we can say few general things.

GEOMETRY OF THE SOLUTIONS WITH FLUXES

FLUXES BREAK SUSY TO $N=1$ OR $N=0$.
Indeed we also put branes:

$$D3, O3 \longrightarrow N=1$$
$$\overline{D3} \longrightarrow N=0$$

The supersymmetric case is closely related to the geometry of the internal manifold, as in the case without fluxes.

EXAMPLE: GKP with D3 and O3. BPS CASE

The analysis of susy variations shows that:

$$\left.\begin{array}{l} \bullet\ \alpha = e^{4A} \\ \bullet\ *G_{(3)} = i\, G_{(3)} \end{array}\right\} \text{eqs. motion}$$

$$\left.\begin{array}{l} \bullet\ g_M \text{ IS CALABI-YAU} \\ \bullet\ \tau \text{ IS CONSTANT} \end{array}\right\} \longleftarrow \text{susy}$$

so that the solution is still related to determination of possible CY where to compactify

Generic solution with fluxes are more complicated. In order to discuss them let us see few things about CY

CY GEOMETRY :

- CY ARE ALWAYS COMPLEX MANIFOLD

locally, if we choose, a base of vierbein

$$ds^2 = e_1^2 + e_2^2 + e_3^2 + e_4^2 + e_5^2 + e_6^2$$
$$= z_1\bar{z}_1 + z_2\bar{z}_2 + z_3\bar{z}_3$$

$$z_1 = e_1 + ie_2$$
$$z_2 = e_3 + ie_4$$
$$z_3 = e_5 + ie_6$$

- CY ARE COMPLETELY DETERMINED BY THE EXISTENCE OF TWO CLOSED FORM

KAELHER $\quad J = z_1 \wedge \bar{z}_1 + z_2 \wedge \bar{z}_2 + z_3 \wedge \bar{z}_3$

COMPLEX 3-FORM $\quad \Omega = z_1 \wedge z_2 \wedge z_3$

$$dJ = d\Omega = 0$$

- SU(3) holonomy translates in the statement :

J, Ω ARE SU(3)- INVARIANT

In compactification with fluxes:

$$\delta\psi_\mu = D_\mu \varepsilon + A_\mu \varepsilon$$

contribution from fluxes

$$A_\mu \sim F_{\mu\mu_1\cdots\mu_{p+1}} \Gamma^{\mu_1\cdots\mu_{p+1}}$$

This means that ε is still covariantly closed, but with respect to a different connection:

$SU(3)$ holonomy $\longrightarrow$ $SU(3)$ structure

$SU(2)$ holonomy $\longrightarrow$ $SU(2)$ structure

Typical M which preserves $N=1$ susy with fluxes have G-structures.

EXAMPLE: $SU(3)$ structure

There still exists
$$\begin{cases} J = z_1\wedge\bar{z}_1 + z_2\wedge\bar{z}_2 + z_3\wedge\bar{z}_3 \\ \Omega = z_1\wedge z_2\wedge z_3 \end{cases}$$

but now they have TORSION:

$$\begin{cases} dJ = \mathrm{Im}(W_1\bar{\Omega}) + W_4\wedge J + W_3 \\ d\Omega = W_1\, J\wedge J + W_2\wedge J + W_5\wedge\Omega \end{cases}$$

EFFECTIVE ACTION

In the approximation where we consider fluxes as first order effects, the volume is large and other technical requirements are satisfied, we can say something about the 4d action.

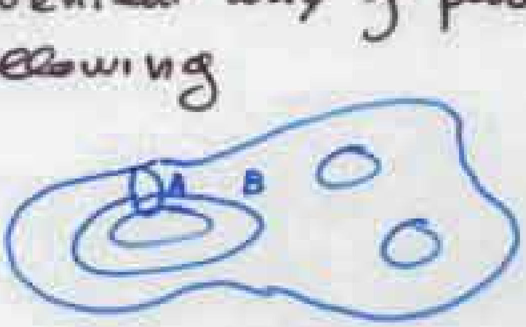
We consider the BPS GKP case, for simplicity.

MODULI : they should be related to SHAPE AND SIZE of the CY M.
Some more geometry shows that the scalar moduli are related to the cohomology group of M :

$$H^{1,1}(M) \qquad \text{KAHLER MODULI}$$
$$H^{2,1}(M) \qquad \text{COMPLEX STRUCTURE MODULI}$$

- COMPLEX STRUCTURE MODULI : they are related to the possible complex coordinates with can put on M

A convenient way of parametrizing them is the following



A_i, B_j basis of 3-cycles with intersection $A_i B_j = \delta_{ij}$

$$z_i = \int_{A_i} \Omega$$

parametrize the complex structure moduli

An important result (SPECIAL GEOMETRY) ubiquitous in $N=2$ gauge and supergravity theories states that

THERE EXISTS A FUNCTION $\mathcal{F}(z_i)$ (PREPOTENTIAL) SUCH THAT:

$$\begin{cases} z_i = \int_{A_i} \Omega \\ \dfrac{\partial \mathcal{F}}{\partial z_i} = \int_{B_i} \Omega \end{cases}$$

$\mathcal{F}$ completely determines the $N=2$ effective action for vector fields in $N=2$ compactifications on Calabi-Yau

- KAHLER MODULI: they deal with possible inequivalent form J we can put on $\mathcal{M}$. Typically the volume of $\mathcal{M}$ is a kahler moduli:

$$J = z_1 \wedge \bar{z}_1 + z_2 \wedge \bar{z}_2 + z_3 \wedge \bar{z}_3 \Leftrightarrow ds^2 = z_1\bar{z}_1 + z_2\bar{z}_2 + z_3\bar{z}_3$$

variation in ds affect J

in $N=1$ compactifications, the volume must lie in a chiral multiplet:

$$\rho \sim (\text{volume, other scalar } b)$$

$$A_{(4)} = b_{\mu\nu} J_{ij}$$

$$\varepsilon_{\mu\nu\rho\sigma} \partial_\nu b_{\rho\sigma} \equiv \partial_\mu b$$

We can compute the kinetic terms for the volume and dilaton by restricting to a reduction on

$$ds^2 = g_{\mu\nu}\,dx^\mu dx^\nu + e^{2u(x)}\, g_{mn}\, dy_m dy_n$$

OVERALL VOLUME

one gets:

$$S \cong \int \sqrt{g}\left(R - 2\,\frac{\partial\tau\,\partial\bar\tau}{|\tau-\bar\tau|^2} - 6\,\frac{\partial\rho\,\partial\bar\rho}{|\rho-\bar\rho|^2}\right)$$

$$\rho = b + i e^{4u}$$

From this:

$$K = -\ln\left[-i(\tau-\bar\tau)\right] - 3\ln\left[-i(\rho-\bar\rho)\right]$$

More complicated is the computation of the kinetic terms for complex moduli z_i. One obtains:

$$K = -\ln\left(-i\int_M \Omega\wedge\bar\Omega\right)$$

WEYL-PETERSON METRIC

THE POTENTIAL COMES INSTEAD FROM

$$S_{10D} = \int dx_4 \int_M dy\, \sqrt{g}\, \frac{|G|^2}{\operatorname{Im}\tau} + \underbrace{\int dx\, \sqrt{g}(z_i)}_{\text{LOCALIZED SOURCES}}$$

Defining $\quad G^{\pm} = \frac{1}{2}(G \pm i * G)$

$$S = \int \frac{G^+ G^+}{\mathrm{Im}\,\tau} - \frac{1}{\mathrm{Im}\,\tau} \underbrace{\int G_3 \wedge \bar{G}_3 + \text{tensions}}_{\text{THEY CANCEL}}$$

$$G = H - \tau F \qquad \int G \wedge \bar{G} \cong \int H \wedge F \cong \text{charge sources}$$

The remaining term after some calculations can be expressed as

$$V = e^{K}\left(\sum_{ij} K^{i\bar{j}} D_i W \, D_{\bar{j}} \bar{W} - 3|W|^2 \right)$$

$$D_i W = \partial_i W - \partial_i K \cdot W$$

with

$$W = \int_M G_{(3)} \wedge \Omega$$

VAFA-WITTEN-GUKOV SUPERPOTENTIAL

MODULI STABILIZATION

- $W = \int G_{(3)} \wedge \Omega$

depends on $\begin{cases} \text{complex moduli:} & z_i = \int_{A_i} \Omega \\ \text{dilaton} & G_{(3)} = H - \tau F \end{cases}$

but not on Kähler moduli

(no volume factor in W)

$$\boxed{\text{So, generically, all complex structure and dilaton moduli will be stabilized by a non-trivial superpotential}}$$

- NOTICE THAT, since W does not depend on volume ρ and

$K(\rho) = -3 \ln[-i(\rho - \bar\rho)] \Rightarrow$ $K_\rho = -\frac{3}{\rho - \bar\rho}$

$K_{\rho\bar\rho} = -\frac{3}{(\rho-\bar\rho)^2}$

dependence on ρ drops out from V:

$$V = e^K \left[\cancel{K^{\rho\bar\rho} D_\rho W D_{\bar\rho} W} + K^{ij} D_i W D_j W - \cancel{3|W|^2} \right]$$

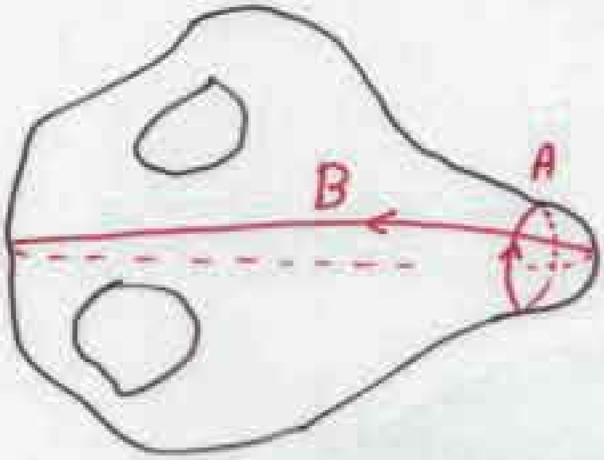
$D_\rho W = \partial_\rho K W = -\frac{3}{\rho - \bar\rho} W$

$$= e^K \left[K^{ij} D_i W D_j W \right]$$

NO-SCALE SUPERPOTENTIAL

SPECIFIC EXAMPLE: to be more precise, we need to choose the CY and the fluxes.

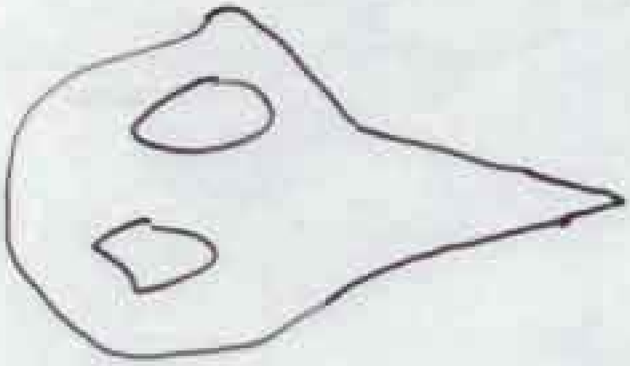
- choose CY near a "conifold" singularity
- one pair of fluxes



$$\int_A F_{(3)} = M$$

$$\int_B H_{(3)} = K$$

$$Z = \int_A \Omega$$



WHEN $Z=0$, A COLLAPSES: CONICAL SINGULARITY; MUCH STUDIED:

$$\mathcal{F}(Z) \sim \frac{Z^2}{2\pi i} \log Z/e + \text{hol.}$$

we have the constraint:

$$\int H \wedge F + \mu_3 = MK + \text{sources} = 0$$

Now

$$W = \int_M G_{(3)} \wedge \Omega = M \int_B \Omega - K\tau \int_A \Omega$$

since $\int_B \Omega = \frac{\partial F}{\partial z} \sim \frac{z}{2\pi i} \ln z$ + holomorphic terms

$$W \cong \left(\frac{Mz}{2\pi i} \ln z - K\tau z + \text{hol.} \right)$$

We take $M, K \gg 1$ and $z \ll 1$

$$0 = D_z W \sim \partial_z W = \frac{M}{2\pi i} \ln z - i \frac{K}{g_s} + O(1)$$

$$\Downarrow$$

$$\boxed{z \sim e^{-\frac{2\pi K}{M g_s}}}$$

- z is the size of the cycle A compared with the typical size of the CY :

EXPONENTIAL HIERARCHY OF SCALES

with reasonable values of M, K

Examples:

$M = 1$

$K/g_s \sim 5$

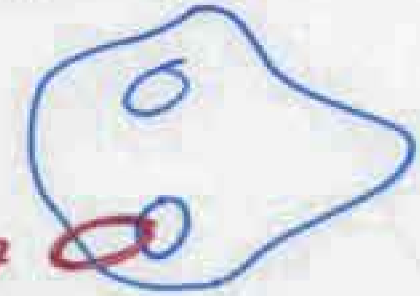
$$z \sim e^{-10\pi} \sim 10^{-15}$$

CAN WE STABILIZE THE VOLUME ALSO ?

- In GKP we need to assume some non-perturbative effects :

$$W = a_0 + a_1 e^{i\rho} + \dots\dots$$

some are natural in string theory :
suppose you have extra branes



D7

a non-perturbative effect (gaugino condensate, instantons) on the D7 give the desired term in W

$$\int_{D7} d^8x \; \frac{1}{g_{YM}^2} F_{\mu\nu}^2 \sqrt{g} \;=\; R^4 \int d^4x \sqrt{g}$$

$\sim 1/g_s$

average radius for the CY

Since $\rho = b + ie^{4u}$

$$\frac{1}{g_{(4)}^2} \sim \frac{1}{g_{YM}^2} e^{4u}$$

$$\tau_{YM} \sim \rho/g_s$$

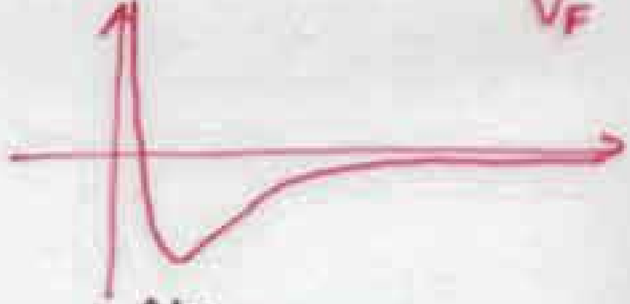
Gaugino condensate : $\langle\lambda\lambda\rangle \sim e^{i\tau/N} \Rightarrow W = e^{i\rho/g_s N}$

- Many other mechanisms exist in the literature. Also examples of complete perturbative stabilizations in IIA

KKLT PROPOSAL

Metastable de-Sitter vacuum:

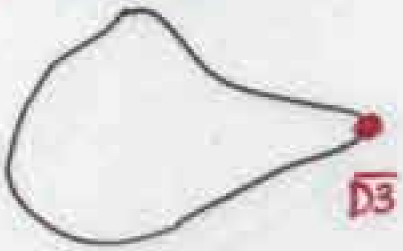
- First, get a susy vacuum with all moduli stabilized taking



$$W = W_0 + a e^{i\rho}$$

$$V_F = e^K\left(|DW|^2 - 3|W|^2\right)$$

- Introduce $\overline{D3}$ branes which break susy.



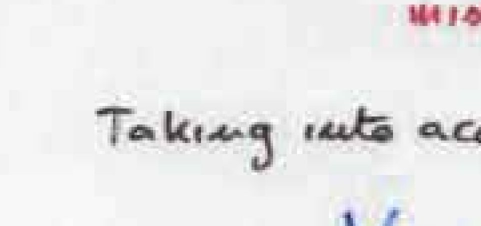
CONSTRAINTS:

charge: $MK + Q_{\overline{D3}} = Q_{O3}$

mismatch in tension: $V_D = MK + T_{\overline{D3}} - T_{O3} = 2T_{\overline{D3}}$

Taking into account the warping

$$V_{TOT} = V_F + V_D = V_F + \frac{c}{\rho^2}$$



$$\boxed{\text{METASTABLE de-Sitter VACUUM WITH ALL MODULI STABILIZED}}$$

and potentially small cosmological constant (choose K, M ...)

IV

THE PROBLEM OF THE VACUUM

- OLD PARADIGM : STRING THEORY HAS ONLY ONE PHENOMENOLOGICALLY ACCEPTABLE VACUUM.

$$\underset{\text{HETEROTIC}}{E_8 \times E_8} \xrightarrow[\mathcal{M}']{} \cdots\cdots \xrightarrow[\text{SUSY BREAKING}]{} \underset{\text{SM}}{SU(3)\times SU(2)\times U(1)}$$

- NEW PARADIGM : IT IS EXTREMELY EASY TO CONSTRUCT ACCEPTABLE VACUA

$$\mathcal{M}_{\text{VACUA}}\,(\underset{\uparrow}{N_i}) \longrightarrow \#\,\text{vacua} \sim C^N$$

INTEGER PARAMETERS

VACUA ARE CHARACTERIZED BY

- COSMOLOGICAL CONSTANT Λ_0
- SCALE OF SUSY BREAKING M_{SUSY}
- ...

we typically have vacua with free integer parameters that allow for all ranges of values for Λ_0, M_{SUSY}, ...

– new point of view motivated by

- new mechanism for moduli stabilizations
- de Sitter vacua (KKLT)

(all coming from fluxes...)

With large number of vacua:

[Weinberg; but see also "split susy"] • SHOULD WE APPEAL TO THE ANTHROPIC PRINCIPLE?

[Douglas] • SHOULD WE STUDY THE STATISTICAL PROPERTIES OF THE ENSEMBLE OF VACUA?

$\mu(P)$ = # vacua with property P

$$\mu(M_{susy} \sim \mathrm{TeV}) \lesseqgtr \mu(M_{susy} \gg \mathrm{TeV})$$

?

(even if we have constructed only a small fraction of possible vacua)

The overall picture is that of a LANDSCAPE:
[Susskind]

Potential with many (infinite) minima with various features ($V(\bar{\phi}_i) = \Lambda_0 =$ c.c.): all of that (or many) are METASTABLE VACUA

Not much more than a "phylosophical" debate ...

Hawking
Brown - Teitelboim
Weinberg
Bousso - Polchinski

Let us see how fluxes may influence our understanding of the cosmological constant:

$$\boxed{\lambda = 10^{-120} M_{PL}^4}$$

SMALL BUT NOT ZERO

λ can be generated by background fluxes:

$$S = \int d^4x \sqrt{g} \left(\frac{1}{2k^2} R - \lambda_{bare} - \frac{Z}{2} \frac{F_{(4)}^2}{4!} \right)$$

$$F_{(4)} = d A_{(3)}$$

eq motion:

$$D_\mu F^{\mu\nu\tau\rho} = 0 \Rightarrow F^{\mu\nu\tau\rho} = c\, \varepsilon^{\mu\nu\tau\rho} \Rightarrow F^2 = -24c^2$$

this contribution of a BACKGROUND LORENTZ INVARIANT VEV FOR $F_{(4)}$ may cancel $\lambda_{bare} \sim M_P^4$

(Hawking)

USING EQS MOTION:

$$\boxed{\lambda = \lambda_{bare} - \frac{Z}{2} \frac{F^2}{4} = \lambda_{bare} + \frac{Zc^2}{2}}$$

EXERCISE: derive λ from Einstein eqs. of motion. To substitute F^2 back in the action gives wrong result!

Now, the string theory input is that c is quantized:

Four-form $F_{(4)}$ $\longrightarrow$ $\int_{X_4} F_{(4)} = \frac{2\pi n}{e}$

electric membrane

$e\int A_{(3)}$

magnetic charge.
(take X_4 euclidean)

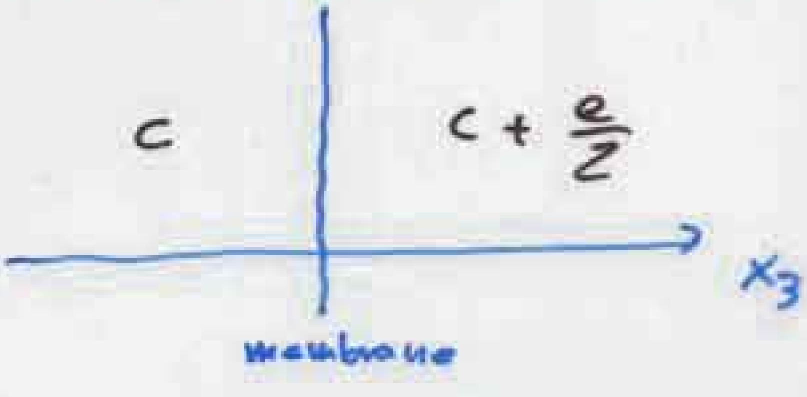
$\Downarrow$

since $F_{(0)} = * F_{(4)}$

$c \sim F_{(0)}$ quantized $\frac{en}{Z}$

NOTE: it easy to take care of normalization factors like Z with a trick:

membrane in 4d = domain wall



$$Z\, d * F_{(4)} = e\, \delta(\text{membrane}) = e\, \delta(x_3)$$

$$Z \int_{[-\varepsilon,\varepsilon]} d * F_{(4)} = Z F_0(\text{right}) - Z F_0(\text{left}) = e$$

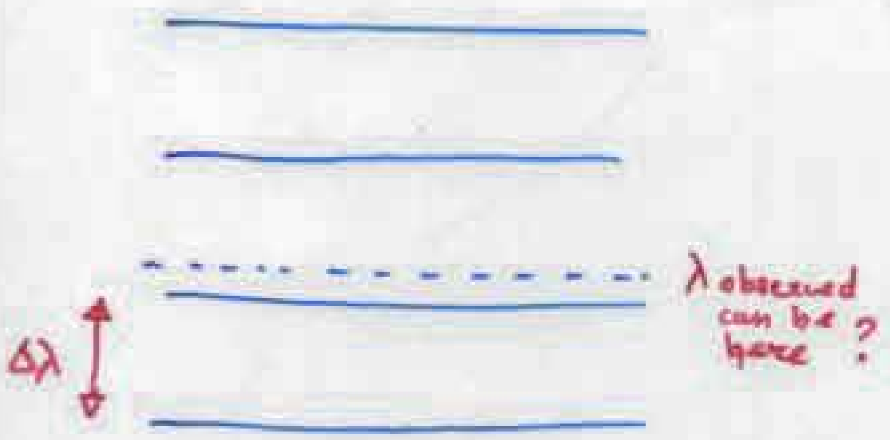
so the jump induced by a 3-brane is $\Delta F_0 = \frac{e}{Z}$

So $c = \frac{en}{Z}$

$$\lambda = \lambda_{bare} + \frac{Zc^2}{2} = \lambda_{bare} + \frac{e^2 n^2}{2Z}$$

- Since $n \in \mathbb{Z}$ we have a large number of allowed values for λ and consistent vacua with discretized spacing : DISCRETUUM OF VACUA

It is important, for statistical or antropic reasonings, that



$$\boxed{\underset{\text{spacing}}{\Delta\lambda} \ll \lambda_{observed}}$$

This means :

$$\begin{cases} d\lambda = \frac{e^2 n}{Z}\, dn \\ \lambda_{bare} + \frac{e^2 n^2}{2Z} \sim 0 \end{cases} \rightarrow \quad \bar{n} \sim \sqrt{\frac{2\lambda_{bare} Z}{e^2}}$$

$$\boxed{e\sqrt{\frac{\lambda_{bare}}{Z}} < 10^{-120} M_{pl}^2}$$

now, this is difficult to obtain with a single flux:

EXAMPLE: type IIA with $A_{(3)}$ and 3-branes

- $M_S^8 \int F_{(4)}^2 d^{10}x \rightarrow M_S^8 V_6 \int F^2 d^4x \Rightarrow \boxed{Z = M_S^8 V_6}$
- $e \sim M_S^3$
- $\lambda_{bare} \sim M_S^4$

$$e\sqrt{\frac{\lambda_{bare}}{Z}} = \frac{M_S}{\sqrt{V_6}}$$

STRING: $V_6 \sim R^6$ with $R \sim M_S$

$$e\sqrt{\frac{\lambda_{bare}}{Z}} \sim M_S^4$$ enormously large!

LARGE EXTRA DIM: we can lower $M_S \sim$ TeV since $M_S^8 R^6 = M_{Pl}^2 \rightarrow V_6 = M_{Pl}^2 / M_S^8$

$$e\sqrt{\frac{\lambda_b}{Z}} \sim M_S / \sqrt{V_6} \sim \frac{M_S^5}{M_{Pl}} \sim 10^{-80} M_{Pl}^4$$

still too large

But it is easy to get a better result with many fluxes.

We actually HAVE many fluxes in our previous example:

II A on T^6

$F_{(7)} \longrightarrow F_{\mu\nu\rho\tau ijk}$ $\quad \frac{6\cdot5\cdot4}{3\cdot2} = 20$ fluxes

$F_{(4)}$

also $F_{(9)}$ ....

we have much more than 20 fluxes... and it is easy to get models with more 4-forms in 4d.

Now
$$\lambda = \lambda_{bare} + \frac{1}{2}\sum_{i=1}^{J} n_i^2 q_i^2$$

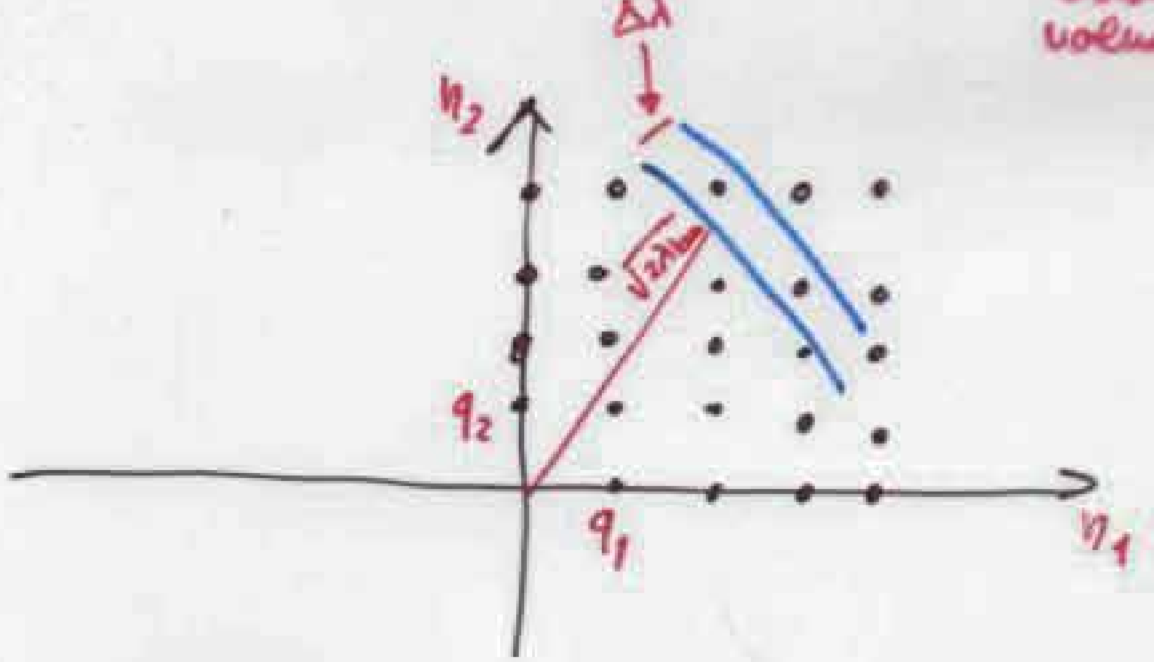
J \# fluxes

$q_i = e_i/\sqrt{Z}$

we now want

$$2|\lambda_{bare}| < \sum^{J} n_i^2 q_i^2 < 2\left(|\lambda_{bare}| + \underbrace{\Delta\lambda}_{\text{observed value } 10^{-120} M_p^4}\right)$$

$J=2$



volume fund. cell in q_i $\geq$ volume shell

$\Downarrow$

$$\prod q_i \leq \Omega_{J-1}\, r^{J-1}\, \Delta r$$

$r = \sqrt{2\lambda_{bare}}$ $\qquad \dfrac{\Delta\lambda}{\sqrt{2\lambda_{bare}}}$

$$\Delta\lambda \sim \frac{\prod_{i=1}^{J} q_i}{\Omega_{J-1}\,|\lambda_{bare}|^{J-1}}$$

easy to satisfy if J is large

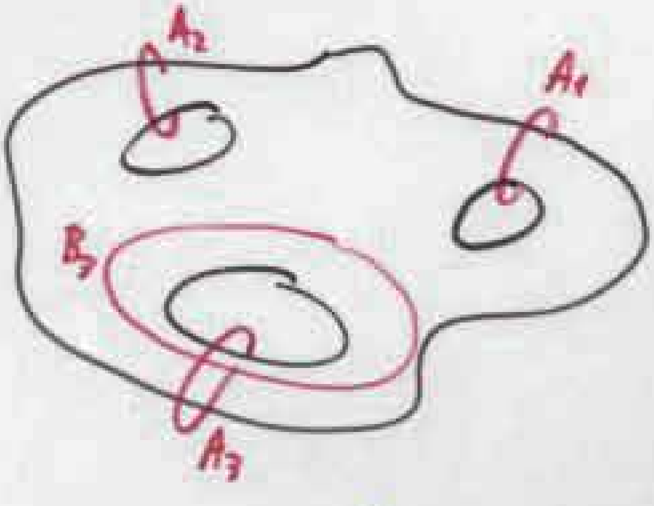
Example: $\lambda_b \sim M_p^4$ $\qquad \Delta\lambda \sim \alpha^J \lambda_{bare} = \alpha^J M_p^4$
$q \sim \alpha M_p^2$
$J \sim 100$ $\qquad$ Take $\alpha = 1/10$

- So it is easy to construct modes with a densely distributed discretuum of equivalent vacua which only differ for the value of the cosm. constant.

We can add structure and find densely distributed discretuum of vacua with varying

- cosm. constant
- susy breaking scale
- gauge group
- with more (semi) realistic models: Higgs mass
- ....

Example : GKP type of models, including KKLT etc ...



$$\int_{A_i} F_{(3)} = M_i$$

$$\int_{B_i} H_{(3)} = K_i$$

only constraints from charge conservation:

$$\sum N_i K_i = \text{O3-charge}$$

Generically we have a constraint of the form:

$$L = N_i \eta_{ij} N_j \le L_*$$

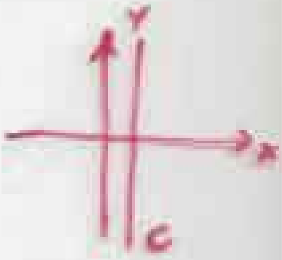
vector of fluxes

total O3 charge - tadpole

EXAMPLE: SUSY VACUA:

$$\mathcal{N}^{\text{SUSY}}_{\text{vacua}}(L<L_*) = \sum_{\substack{\text{SUSY}\\ \text{VACUA}}} \theta(L_*-L) =$$

$$\sum \frac{1}{2\pi i}\int_C \frac{d\alpha}{\alpha}\, e^{\alpha(L_*-L)} =$$



$$\frac{1}{2\pi i}\int_C \frac{d\alpha}{\alpha}\, e^{\alpha L_*}\left(\sum_{\text{vac}} e^{-\frac{\alpha}{2} N\eta N}\right)$$

approximating N as a continuum

$$\int dz \int dN\, e^{-\frac{\alpha}{2} N\eta N}\,\delta(DW)\,|\det D^2 W|$$

IF WE ARE INTERESTED IN OTHER VACUA (NON-SUSY, WITH GIVEN HIGGS MASS, etc...) we must change the delta function in this expression

A roughly estimate is easy

$$\sum e^{-\frac{\alpha}{2} N \eta N} \xrightarrow[N \to N/\sqrt{\alpha}]{} \alpha^{-2m} \sum e^{-\frac{N \eta N}{2}}$$

where m is the number of fluxes

$$\mathcal{N}^{susy}_{vacua} \sim \frac{1}{2\pi i} \int \frac{d\alpha}{\alpha^{2m+1}} e^{\alpha L_*} C \sim \frac{(L_*)^{2m}}{(2m)!}$$

The real formula is just a mild modification

$$\mathcal{N}_{vacua} \sim \frac{L^K}{K!} [c_n]$$

- K number of fluxes = number of 3-cycles
- L tadpole
- c_n geometrical factor, mostly irrelevant

For typically CY :

$K \sim 100$

$L \sim 1000$

$\mathcal{N}_{vacua} \sim 10^{500}$ even too large one can do more refined analysis for a given model (i.e CY):

- Large uniform component of the vacuum distribution
- Enhanced number of vacua near conifold points (but with tachions)
- Hierarchically small scales are common
- many susy ~~par~~ breaking parameters favor high scale of susy breaking

Anyhow, this is just a toy model... and many other exists.