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Supersymmetry breaking in four and more dimensions

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Why supersymmetry?

- A possibility within local relativistic QFT
- Fits naturally within superstrings/M-theory
- Hierarchy problem: $M_{weak}/M_P \sim 10^{-16}$
- Vacuum energy: $\Lambda_{cosm}/M_P \sim 10^{-30}$
- Unification of coupling constants
- Fits naturally with EW precision tests
- Natural candidates for cold dark matter
- Can provide a framework for inflation

But:

- No SUSY particle found (yet)
- No light Higgs found (yet)
- Flavor problems: B, L, FCNC, CP
- Hierarchy only partially solved
- No insight on vacuum energy

SUPERSYMMETRY BREAKING

crucial open problem to clarify the puzzle

(theoretically and experimentally)

Plan

supersymmetry breaking within:

1. N=1 D=4 global supersymmetry (SUSY)
2. N=1 D=4 supergravity (SUGRA)
3. Compactified extra dimensions (XDIM)
4. String effective supergravities (STRING)

1.

SUSY

N=1, D=4 SUSY algebra: $\{Q_\alpha, \overline{Q}_{\dot\alpha}\} = 2\sigma^\mu_{\alpha\dot\alpha} P_\mu$

(N>1, D=4 ➔ no chiral fermions)

Superfields: $\Phi(x,\theta,\overline{\theta}) = \varphi(x) + \ldots + \theta\theta\overline{\theta\theta}\, d(x)$

↑ anticommuting coordinates

Chiral superfields (chiral representation):

$$C^i = \varphi^i(x) + \sqrt{2}\theta\,\psi^i(x) + \theta\theta\, F^i(x)$$

complex spin-0 ↗ left-handed Weyl spinor ↑ (auxiliary) complex spin-0 ↗

Vector superfields (Wess-Zumino gauge):

$$V^a = -\theta\sigma^\mu\overline{\theta}\, A^a_\mu(x) + i\theta\theta\overline{\theta}\,\overline{\lambda}^a(x) + i\overline{\theta\theta}\theta\,\lambda^a(x) + \frac{1}{2}\theta\theta\overline{\theta\theta}\, D^a(x)$$

real spin-1 ↗ ↑ Weyl spinors ↑ (auxiliary) ↑ real spin-0

renormalizable N=1, D=4 global supersymmetry
assumed to be familiar from previous lectures

Renormalizable gauge-invariant Lagrangians

$$\mathcal{L} = [W(\phi) + \frac{1}{16 k g^2} tr \mathcal{W}\mathcal{W}]_F + \text{h.c.} + [\phi^\dagger e^V \phi + \xi_a V^a]_D$$

U(1) factors only

$$\mathcal{W}_\alpha = -\frac{1}{4}\overline{DD}e^{-V} D_\alpha e^V \qquad V = V^a T_a \qquad tr T^a T^b = k\delta_{ab}$$

generalized SUSY field strengths — Lie algebra valued — normalization

1. Choose a gauge group G (vector multiplets)
2. Choose a chiral multiplet content [Rep(G)]
3. Choose a gauge-invariant superpotential W

$$W(\phi) = a_i \phi^i + b_{ij}\phi^i\phi^j + c_{ijk}\phi^i\phi^j\phi^k$$

4. Choose the constants ξ_a for U(1) factors

Component form of the Lagrangian

after superspace integration, rescaling V➔2gV
and eliminating the F and D auxiliary fields:

$$\mathcal{L} = -(\overline{D_\mu\varphi})_i(D^\mu\varphi)^i - (1/4)F^a_{\mu\nu}F^{a\mu\nu}$$
$$-i\overline{\psi}_i\overline{\sigma}^\mu(D_\mu\psi)^i - i\overline{\lambda}^a\overline{\sigma}^\mu(D_\mu\lambda)^a - (1/2)(W_{ij}\psi^i\psi^j + \text{h.c.})$$
$$+[i\sqrt{2}g\overline{\varphi}_i(T^a)^i{}_j\psi^j\lambda^a + \text{h.c.}] - V(\varphi,\overline{\varphi})$$

where:

$$V = \overline{F}^i F_i + \frac{1}{2}D^a D^a = \overline{W}^i W_i + \sum_a \frac{g_a^2}{2}[\overline{\varphi}_i(T^a)^i{}_j\varphi^j + \xi_a]^2$$

The MSSM

- Gauge group SU(3) x SU(2) x U(1)

  gauginos $(\tilde{q}, \tilde{W}, \tilde{B})$

- 3 SM generations, 2 Higgs doublets

  squarks $(\tilde{q})$, sleptons $(\tilde{l})$, higgsinos $(\tilde{H}_{1,2})$

- R-parity conserving superpotential

$$W = Qh^U U^c H_2 + Qh^D D^c H_1 + Lh^E E^c H_1 + \mu H_1 H_2$$

- Explicit soft supersymmetry breaking

$$-\mathcal{L}_{soft} = \varphi^{\dagger} m^2 \varphi + \Big[(1/2)\lambda M \lambda + m_3^2 H_1 H_2 + \tilde{q} A^U \tilde{u}^c H_2 + \tilde{q} A^D \tilde{d}^c H_1 + \tilde{l} A^E \tilde{e}^c H_1 + h.c.\Big]$$

MSSM vs. Standard Model

improves hierarchy $(M_{weak} \sim \Delta m_{SUSY})$ but:

why $\frac{\Delta m_{SUSY}}{M_P} \sim 10^{-15}$ and $\mu \sim \Delta m_{SUSY}$?

irrelevant improvement on vacuum energy

typically $\Lambda_{cosm} \sim \sqrt{\Delta m_{SUSY} M_P} \quad (< M_P)$

B,L problem solved by R-parity but

new severe flavour problem (FCNC, CP)

need universality or equivalent conditions

move to spontaneous supersymmetry breaking

Spontaneous SUSY Breaking

$$Q_\alpha|0\rangle \neq 0 \quad \Leftrightarrow \quad \langle \delta_\eta \chi \rangle \neq 0 \quad (\chi = \psi^i, \lambda^a)$$

$$\delta_\eta \psi^i = \ldots + 2\eta F^i \qquad \delta_\eta \lambda^a = \ldots + i\eta D^a$$

$$broken\ SUSY \quad \Leftrightarrow \quad \langle F^i \rangle \neq 0 \ and/or \ \langle D^a \rangle \neq 0$$

a necessary and sufficient condition:

(global SUSY, constant bosonic background)

$$V = \overline{F}^i F_i + \frac{1}{2} D^a D^a \geq 0$$

$$broken\ SUSY \quad \Leftrightarrow \quad \langle V \rangle > 0$$

goldstino theorem:

broken SUSY ➔ massless spin-1/2 fermion

$$\widetilde{G} = \langle F_i \rangle \psi^i + \frac{i}{\sqrt{2}} \langle D_a \rangle \lambda^a$$ GOLDSTINO

some more explicit formulae:

(valid only in the renormalizable case)

$$\overline{F}_i = -W_i\,, \quad D_a = g_a [\overline{\varphi}_i (T_a)^i{}_j \varphi^j + \xi_a]$$

U(1) factors only ➚
Fayet-Iliopoulos term

$$Str\,\mathcal{M}^2 = \sum_i (-1)^{2J_i} (2J_i + 1)\, M_i^2 = 2\, g_a \langle D_a \rangle (tr\, T^a)$$

$\Rightarrow \; Str\,\mathcal{M}^2 = 0$ in the absence of anomalous U(1)s

violated by quantum corrections
& non-renormalizable interactions

Exercise n.1 (O'Raifeartaigh model):

$$W = \lambda X(Z^2 - m^2) + \mu Y Z \quad [0 < m^2 < \mu^2/(2\lambda^2)]$$

find the classical vacuum state and the spectrum

Exercise n.2 (Fayet-Iliopoulos model):

G=U(1) and one chiral superfield of charge e

find the classical vacuum state and the spectrum

Exercise n.3: prove goldstino theorem

for simplicity: classical level, renormalizable case

Exercise n.4: prove supertrace formula

for simplicity: renormalizable case

Supersymmetric effective theories

two-derivative effective Lagrangian:

(with gauge symmetry linearly realized on fields)

$$\mathcal{L} = \left[W(\phi) + \frac{1}{4} f_{ab}(\phi) \mathcal{W}^a \mathcal{W}^b \right]_F + h.c. + \left[K(\phi^\dagger, e^V \phi) + \xi_a V^a \right]_D$$

analytic gauge-invariant ↗ ; ↑ analytic symmetric product of adjoints ; ↖ real gauge-invariant ; ↖ FI terms

renormalizable case:

$$f_{ab}(\phi) = \frac{\delta_{ab}}{g_a^2}, \quad K(\phi^\dagger, \phi) = \phi^\dagger \phi, \quad W(\phi) =$$ degree-3 polynomial

generic case:

dim > 4 interactions with scale $\Lambda < M_P$

(gravitation consistently neglected)

Component form of the effective Lagrangian

$$\mathcal{L}_{SUSY} = \mathcal{L}_B + \mathcal{L}_{F,K} + \mathcal{L}_{F,2} + \mathcal{L}_{F,4}$$

$$\mathcal{L}_B = -\frac{1}{4}(Re\, f)_{ab} F^a_{\mu\nu} F^{b\mu\nu} + \frac{1}{4}(Im\, f)_{ab} F^a_{\mu\nu} \tilde{F}^{b\mu\nu} + K_{\bar{k}i} \overline{(D_\mu \varphi)}^{\bar{k}} (D^\mu \varphi)^i - V(\varphi, \bar{\varphi})$$

<Re f> = 1/g^2 <Im f> = theta-angle

$$\mathcal{L}_{F,K} = \frac{i}{2}(Re\, f)_{ab}[\lambda^a \sigma^\mu \overline{(D_\mu \lambda)}^b - (D_\mu \lambda)^a \sigma^\mu \bar{\lambda}^b] - \frac{1}{2}(Im\, f)_{ab} D_\mu (\lambda^a \sigma^\mu \bar{\lambda}^b)$$
$$-\frac{1}{2\sqrt{2}}[f_{abi} \psi^i \sigma^{\mu\nu} \lambda^a F^b_{\mu\nu} + \text{h.c.}] + \frac{i}{2} K_{i\bar{k}}[\psi^i \sigma^\mu \overline{(D_\mu \psi)}^{\bar{k}} - (D_\mu \psi)^i \sigma^\mu \bar{\psi}^{\bar{k}}]$$

D = gauge-covariant and Kahler-covariant derivative

$\mathcal{L}_{F,2}$ and $\mathcal{L}_{4,F}$ will be given when needed

What changes for SUSY breaking?

Auxiliary fields:

$$F^i = -K^{i\bar{k}}\overline{W}_{\bar{k}} + \frac{1}{2}K^{i\bar{k}}K_{\bar{k}lm}\psi^l\psi^m + \frac{1}{4}K^{i\bar{k}}\overline{f}_{ab\bar{k}}\overline{\lambda}^a\overline{\lambda}^b$$

$$D^a = -Re f^{ab}[\xi_b + K_i(T_b\varphi)^i] - [\frac{i}{2\sqrt{2}}Re f^{ab} f_{bci}\psi^i\lambda^c + h.c.]$$

Potential:

$$V = F_i K^{i\bar{k}}\overline{F}_{\bar{k}} + \frac{(Re f)^{ab}}{2}D_a D_b \geq 0$$

includes new interactions with 2 and 4 fermions…

new possibilities for spontaneous breaking associated with fermion condensates, e.g. gaugino condensation

Modified classical mass formulae:

$$Str\,\mathcal{M}^2 = -2\overline{F}^{\bar{k}}(R_{\bar{k}i} + S_{\bar{k}i})F^i + \text{D-term contributions}$$

$$R_{\bar{k}i} = \partial_{\bar{k}}\partial_i \log\det(K_{\bar{m}n}) \qquad S_{\bar{k}i} = \partial_{\bar{k}}\partial_i \log\det(Re\,f_{ab})$$

Realistic models?

no reliable models with MSSM fields only

an interesting failure: $W \ni \Lambda^2_{SUSY}\sqrt{H_1 H_2}$

(requires $\Lambda \sim \Lambda_{SUSY} \sim M_{weak}$)

need at least a goldstino multiplet

simplest choice: $T \equiv (z, \chi, F^z)$

gauge singlet chiral superfield

Supersymmetry breaking scale:

$$\Lambda_{SUSY}^4 = \langle ||F||^2 + ||D||^2 \rangle = \langle V \rangle$$

always ↗ (first equality) global SUSY only ↗ (second equality)

Supersymmetry breaking mass splittings:

$$(\Delta m^2_{SUSY})_{IJ} \sim \gamma_{IJ} \cdot \frac{\Lambda^4_{SUSY}}{\Lambda^2}$$

γ_{IJ} = O(1) effective T-I-J coupling

no viable model with dim≤4 couplings (supertrace formula!)
between goldstino & MSSM multiplets
only dim>4 couplings to obtain a realistic spectrum
(classical or quantum origin)

Examples of SUSY-breaking masses

$$\left[\gamma_{ij}\frac{|T|^2\phi_i^\dagger\phi_j}{\Lambda^2}\right]_D \Rightarrow (m^2)_{ij} \sim \gamma_{ij}\frac{\Lambda_{SUSY}^4}{\Lambda^2}$$

$$\left[\beta\frac{T\mathcal{W}^A\mathcal{W}^A}{\Lambda}\right]_F \Rightarrow M_A \sim \beta\frac{\Lambda_{SUSY}^2}{\Lambda}$$

$$\left[\beta'\frac{T^\dagger H_1 H_2}{\Lambda}\right]_D \Rightarrow \mu \sim \beta'\frac{\Lambda_{SUSY}^2}{\Lambda}$$

$$\left[\beta''\frac{|T|^2 H_1 H_2}{\Lambda^2}\right]_D \Rightarrow m_3^2 \sim \beta''\frac{\Lambda_{SUSY}^4}{\Lambda^2}$$

$$\left[\gamma'_{ij}\frac{T\phi_i\phi_j H}{\Lambda}\right]_F \Rightarrow A_{ij} \sim \gamma'_{ij}\frac{\Lambda_{SUSY}^2}{\Lambda}$$

The flavour problem again:

How can the special $\gamma_{ij}, \gamma'_{ij}$ needed to avoid the SUSY flavor problem arise?

Must know more about the symmetries of the underlying microscopic theory

SUSY breaking dynamics not essential, transmission mechanism may be enough

➔ Models for the mediation of SUSY breaking from a hidden to the observable MSSM sector

Exercise n.5 (MSSSM):

write down a MSSM with spontaneous breaking of both supersymmetry and SU(2) x U(1)

Hints (including some simplifying assumptions):

- Include a gauge-singlet goldstino chiral multiplet Z
- Aim only at a local minimum with spontaneous breaking
- Aim at $v_1=v_2$ and no goldstino components along $H_{1,2}$
- Aim at mu and A terms from the superpotential
- Aim at no mixing between sgoldstinos and Higgs bosons
- Aim at a vanishing VEV for the complex scalar z in Z
- For the Higgses, use gauge-invariant variables with vanishing VEVs, such as H_1H_2 or $|H_1|$^2 + $|H_2|$^2 suitably shifted, that give rho=1 at tree-level thanks to a custodial symmetry
- Make sure that also the sgoldstinos get acceptable masses

Gauge mediation

observable sector $\leftrightarrow$ messenger sector $\leftrightarrow$ hidden sector

gauge interactions (observable–messenger); superpotential interactions (messenger–hidden)

MSSM $\qquad \Phi(5)+\Phi^c(\bar{5}) \qquad$ T

$$W = kT\Phi\Phi^c + \dots \qquad \langle z\rangle \neq 0 \quad \langle F^z\rangle \neq 0$$

$$\mathcal{M}^2 = k^2\langle z\rangle^2 > \Delta m^2_{SUSY} = k\langle F^z\rangle$$

gaugino (1-loop) and scalar (2-loop) masses:

$$M \sim \frac{\alpha}{4\pi}\frac{\langle F^z\rangle}{\langle z\rangle} \qquad m^2 \sim \left(\frac{\alpha}{4\pi}\frac{\langle F^z\rangle}{\langle z\rangle}\right)^2 \qquad \mathcal{R} = \frac{\langle F^z\rangle}{\langle z\rangle}$$

SM gauge interactions ➔ universality

Effective theory of gauge mediation:

$$\Lambda \sim \frac{4\pi\langle z\rangle}{\alpha} \quad \text{and} \quad \Lambda_{SUSY}^2 \sim \langle F^z\rangle$$

$\Lambda_{SUSY} \geq O(10)\ TeV$ for a realistic spectrum

(but can be much higher for very large <S>)

$\mu\ ,\ m_3^2$ not generated by gauge interactions

$U(1)_{PQ}$ and $U(1)_R$ must be broken

require rather contrived modifications

phenomenological parametrization (minimal GMSB):

$$\mathcal{M} \quad \mathcal{R} \quad n_5 \qquad \tan\beta \quad sign(\mu)$$

R-symmetry and PQ-symmetry in the MSSM

U(1)$_R$ symmetries (of N=1 supersymmetry) act as

$$C^i(\theta,x) \to e^{iq_i\alpha} C^i(e^{-i\alpha}\theta,x), \quad V^a(\theta,\bar\theta,x) \to V^a(e^{-i\alpha}\theta, e^{i\alpha}\bar\theta,x)$$

$$R(\varphi^i) = q_i,\ R(\psi^i) = q_i - 1,\ R(F^i) = q_i - 2;\ \ R(A_\mu^a) = R(D^a) = 0,\ \ R(\lambda^a) = +1$$

R-invariance ➔ W must have R-charge R(W)=+2

R-parity: discrete Z_2 subroup of U(1)$_R$ $(\alpha = \pi)$

$R(H_1)=R(H_2)=0$, $R(Q)=R(U^c)=R(D^c)=R(L)=R(E^c)=+1$

➔ $R[W^{(3)}]=+2$, mu-term and gaugino masses break continuous R-symmetry but preserve R-parity

U(1)$_{PQ}$: ordinary symmetry acting on MSSM superfields as

$q(H_1)=q(H_2)=+1$, $q(Q,U^c,D^c,L,E^c)$ such that $W^{(3)}$ invariant

$\mu,\ m_3^2$ not invariant under U(1)$_{PQ}$

Dynamical SUSY Breaking

global N=1 SUSY: laboratory for non-perturbative breaking

controllable models of DSB do exist

simplest example: the 3-2 model

$$G = SU(3) \times SU(2) \quad \text{and} \quad [Q(3,2), \overline{U}(\bar{3},1), \overline{D}(\bar{3},1), L(1,2)]$$

$$W = W_{cl} + W_{np} \qquad W_{cl} = \lambda Q \overline{U} L$$

no T.L. flat directions, non-anomalous U(1)xU(1)$_R$ symmetry

$$(\Lambda_3 \gg \Lambda_2): \qquad W_{np} = \frac{\Lambda_3^7}{(Q\overline{U})(Q\overline{D})}$$

spontaneously broken supersymmetry!

(also for generic ratio of dynamical scales)

… but we should not forget gravity …

Effective goldstino couplings

Noether theorem ➔ conserved current of SUSY
assume F-breaking and canonical kinetic terms at vacuum

$$J^{\mu}_{\alpha} = i\overline{W}_{\overline{k}}(\sigma^{\mu}\overline{\psi}^{\overline{k}})_{\alpha} + j^{\mu}_{\alpha}$$

$$j^{\mu}_{\alpha} = (\partial_{\nu}\overline{\varphi}^{\overline{k}})(\sigma^{\nu}\overline{\sigma}^{\mu}\psi^{k}) - \frac{1}{2\sqrt{2}}(\sigma^{\nu}\overline{\sigma}^{\rho}\sigma^{\mu}\overline{\lambda}^{a})_{\alpha}F^{a}_{\nu\rho} + \dots$$

$$0 = \partial_{\mu}J^{\mu}_{\alpha} = i\langle F\rangle(\sigma^{\mu}\partial_{\mu}\widetilde{G})_{\alpha} + \partial_{\mu}j^{\mu}_{\alpha}$$

↓

$$\mathcal{L}_{gold} = -i\overline{\widetilde{G}}\overline{\sigma}^{\mu}\partial_{\mu}\widetilde{G} - \frac{1}{\langle F\rangle}(\widetilde{G}\partial_{\mu}j^{\mu} + \text{h.c.})$$

↓

$$\mathcal{L}^{1\widetilde{G}}_{int} = \frac{m^2_{\varphi} - m^2_{\psi}}{F}\widetilde{G}\psi\overline{\varphi} + \text{h.c.} + \frac{i}{\sqrt{2}}\frac{M_{\lambda}}{F}\widetilde{G}\sigma^{\mu\nu}\lambda F_{\mu\nu} + \text{h.c.} + \dots$$

Limitations of the effective theory

$$\text{particle interpretation} \Rightarrow \Delta m^2_{SUSY} < O(\text{few})\,\Lambda^2_{SUSY}$$

Example:

$$\Gamma(\tilde{f} \to f\,\tilde{G}) = \frac{\tilde{m}^5}{16\pi F^2} < \tilde{m} \quad \Rightarrow \quad \tilde{m}^2 < \sqrt{16\pi}\,F$$

(simplifying assumptions of pure F-breaking and massless fermion)

$$\text{perturbative unitarity} \Rightarrow E^2 < O(\text{few})\,\frac{\Lambda^4_{SUSY}}{\Delta m^2_{SUSY}}$$

Example:

$$\mathcal{A}(f\bar{f} \to \tilde{G}\bar{\tilde{G}}) \sim \frac{s\tilde{m}^2}{F^2} < O(1) \quad \rightarrow \quad s < O\left(F^2/\tilde{m}^2\right)$$

The MSSM cutoff depends on the SUSY breaking scale!

Low-energy theorems

$$\sqrt{s} \ll \Delta m_{SUSY}$$

Effective interactions among light particles described by a non-linearly realized supersymmetry: no more dependence on the susy-breaking masses, only on the susy-breaking scale, as a result of supersymmetric cancellations

Example: two fermions and two goldstinos

$$\mathcal{L}_{eff} = -\frac{1}{F^2}\left[(\widetilde{G}\sigma^\mu\partial^\nu\overline{\widetilde{G}})(\overline{f}\overline{\sigma}_\nu\partial_\mu f) + \frac{\alpha}{4}(\widetilde{G}\sigma^\mu\partial^\nu\overline{f})(\overline{\widetilde{G}}\overline{\sigma}_\nu\partial_\mu f)\right]$$

α = residual ambiguity from higher-derivative four-fermion interactions in the linear theory

Solution to exercise 1 (F-breaking)

$$W = \lambda X(Z^2 - m^2) + \mu YZ \quad [0 < m^2 < \mu^2/(2\lambda^2)]$$

$$F_x^\dagger = \lambda(z^2 - m^2) \quad F_y^\dagger = \mu z \quad F_z^\dagger = \mu y + 2\lambda xz$$

$$V = \lambda^2|z^2 - m^2|^2 + \mu^2|z|^2 + |\mu y + 2\lambda xz|^2$$

minimized for <x> arbitrary and <y>=<z>=0

$$\langle F_y \rangle = \langle F_z \rangle = 0 \quad \langle F_x \rangle = \lambda m^2 \neq 0 \quad \langle V \rangle = \lambda^2 m^4 > 0$$

Spectrum (around <x>=0 for simplicity):

field	x	y	z	ψ_x	(ψ_y, ψ_z)
$(\text{mass})^2$	0	μ^2	$\mu^2 \pm 2\lambda^2 m^2$	0	μ^2

$$\psi_x = \text{goldstino} \quad \Delta m^2_{SUSY} \sim \lambda \cdot \lambda m^2 \quad Str\mathcal{M}^2 = 0$$

Solution to exercise 2 (D-breaking)

G=U(1) and one chiral superfield of charge e

no gauge-invariant superpotential W

invariant FI-term: $\xi \int d^4\theta\, V \rightarrow F = 0 \quad D = \xi + e|\varphi|^2$

$$e\xi < 0 \Rightarrow \langle|\varphi|^2\rangle = -\xi/e\,, \quad \langle D\rangle = 0$$

gauge symmetry broken, supersymmetry unbroken

$$e\xi > 0 \Rightarrow \langle\varphi\rangle = 0\,, \quad \langle D\rangle = \xi$$

gauge symmetry unbroken, supersymmetry broken

Spectrum:

field	ψ	λ	A_μ	φ
$(\text{mass})^2$	0	0	0	$e\xi$

$$\lambda = \text{goldstino} \quad \Delta m^2_{SUSY} = e\cdot\xi \quad Str\mathcal{M}^2 = 2e\xi$$

Solution to exercise 3 (goldstino theorem)

(simplified: classical level, renormalizable case)

Minimization of the potential:

$$V_j = W_{ij} F^i + g^2 \varphi_i^\dagger (T^a)^i{}_j D^a = 0$$

Gauge-invariance of the superpotential

$$\delta W = 0 \;\Rightarrow\; \varphi_j^\dagger (T_a)^j{}_i F^i = 0$$

$$\Downarrow$$

$$(F^i \;\; D^a) \begin{pmatrix} W_{ij} & g\varphi_j^\dagger (T_a)^j{}_i \\ g\varphi_j^\dagger (T_a)^j{}_i & 0 \end{pmatrix} = (0 \;\; 0)$$

fermion mass matrix

q.e.d.

Solution to exercise 4 (supertrace formula)

(simplified: renormalizable case)

$$(\mathcal{M}_1^2)^{ab} = 2 D_i^a D^{b\,j} \;\Rightarrow\; 3\,tr(\mathcal{M}_1^2) = 6\, D_i^a D^{a\,j}$$

$$\mathcal{M}_{1/2} = \begin{pmatrix} W_{ij} & \sqrt{2} D_i^b \\ \sqrt{2} D_j^a & 0 \end{pmatrix} \qquad \mathcal{M}_0^2 = \begin{pmatrix} V_i^{\,j} & V_{il} \\ V^{kj} & V_l^{\,k} \end{pmatrix}$$

$$-2\,tr(\mathcal{M}_{1/2}\mathcal{M}_{1/2}^\dagger) = -2\,\overline{W}^{ij} W_{ij} - 8\, D_i^a D^{a\,i}$$

$$tr\,\mathcal{M}_0^2 = 2\,\overline{W}^{ij} W_{ij} + 2\, D_i^a D^{a\,i} + 2\, D^a D_i^{a\,i}$$

$$Str\,\mathcal{M}^2 = 2\, g_a \langle D_a \rangle\, (tr\, T^a)$$

vanishes in the absence of anomalous U(1)s

Solution to exercise 5 (MSSSM)

$$\langle H_1^0 \rangle = \langle H_2^0 \rangle = v/\sqrt{2} \qquad Y = H_1^0 H_2^0 - H_1^- H_2^+ - v^2/2$$

$$T = |H_1^0|^2 + |H_2^0|^2 + |H_1^-|^2 + |H_2^+|^2 - v^2$$

$$K = K_{can} - \frac{\alpha_Z}{4\Lambda^2}|Z|^4 - \frac{\alpha_Q}{\Lambda^2}|Z|^2|Q|^2 - \ldots - \frac{\alpha_L}{\Lambda^2}|Z|^2|L|^2 - \ldots$$

$$-\frac{\gamma}{2\Lambda}[(Z+\overline{Z})T - (Y\overline{Z}+\overline{Y}Z)] - \frac{\beta}{2\Lambda^2}|Z|^2[T-(Y+\overline{Y})]$$

$$W = FZ + \frac{\sigma}{6}Z^3 + \frac{\delta}{2\Lambda}Y^2 + \left(h_t + \frac{k_t}{\Lambda}Z\right) T^c Q H_2 + \ldots$$

$$f_{AB} = \frac{\delta_{AB}}{g_A^2}\left(1 + \frac{2\eta_A}{\Lambda}Z\right)$$

acceptable vacuum & spectrum for suitable parameter choices

2.

SUGRA

SUGRA: general considerations

local supersymmetry = super-gravity

$\varepsilon = \varepsilon(x)$ fermionic parameter $\left[\varepsilon Q, \overline{\varepsilon Q}\right] = 2\varepsilon\sigma^{\mu}\bar{\varepsilon} P_{\mu}$

1) local supersymmetry ➔ spin-3/2 gauge fermion (gravitino)
2) GCT Invariance ➔ inevitable inclusion of Einstein gravity

N=1, D=4 gravitational multiplet

$\begin{pmatrix} e_{\mu}^{m} \\ \psi_{\mu} \end{pmatrix}$ spin-2, spin-3/2 + auxiliary fields

interactions controlled by $\kappa = 1/M_P$

$$M_P = \frac{G_N^{-1/2}}{\sqrt{8\pi}} \simeq 2.4 \times 10^{18}\ GeV$$

SUGRA units: $M_P = 1$

Local supersymmetry breaking

A crucial difference with global supersymmetry:

$$V_{sugra} = ||F||^2 + ||D||^2 - ||H||^2$$

aux(chi.) aux(vec.) aux(grav.)

$$\Lambda_{SUSY}^4 = \langle ||F||^2 + ||D||^2 \rangle > M_{weak}^4$$

no sparticle observed

$$\Lambda_{cosm} = \langle V_{sugra} \rangle^{1/4} < M_{weak}^2 / M_P$$

limits on vacuum energy

phenomenology ➔

gravitational effects crucial for vacuum selection

only afterwards one can take the global limit

The super-Higgs effect (flat space)

$$\psi_\mu(\pm 3/2) \oplus \widetilde{G}(\pm 1/2)$$ massive gravitino

(goldstino expression similar to the global case)

$$\Lambda_{cosm} = \langle V \rangle^{1/4} \simeq 0 \quad \& \quad ||H||^2 = 3\, m_{3/2}^2 M_P^2$$

$$\rightarrow \quad \Lambda_{SUSY}^4 = 3\, m_{3/2}^2 M_P^2$$

One-to-one correspondence $\Lambda_{SUSY} \leftrightarrow m_{3/2}^2$

Minimal multiplet content for a realistic model:

MSSM (chiral+vector) + gravitational + goldstino

$m_{3/2}(\Lambda_{SUSY})$ model-dependent parameter even after choosing $\Delta m_{SUSY} \sim M_{weak}$

The two flat limits $(M_P \to \infty)$

$$m_{3/2} \text{ fixed}\,, \quad \Lambda_{SUSY} \to \infty \,:$$

explicitly broken global SUSY with soft terms

$$\Delta m^2_{SUSY} = O(m^2_{3/2})$$

and decoupled goldstino in the limit

$$\Lambda_{SUSY} \text{ fixed}\,, \quad m_{3/2} \to 0 \,:$$

spontaneously broken global SUSY

interacting goldstino multiplet with effective couplings

$$\lambda_G \sim \frac{\Delta m^2_{SUSY}}{\Lambda^2_{SUSY}}$$

Gravitino mass vs. phenomenology

heavy		light		very light
$O(M_{weak})$	$\gg$	$m_{3/2}$	$\gg$	$O(M_{weak}^2/M_P)$
$O(M_{weak}/M_P)$	$\ll$	λ_G	$\ll$	$O(1)$
$O(\sqrt{M_{weak}M_P})$	$\gg$	Λ_{SUSY}	$\gg$	$O(M_{weak})$
$O(M_P)$	$\gg$	Λ	$\gg$	$O(M_{weak})$

Heavy gravitino: MSSM + soft terms with cutoff $O(M_P)$

- MSSM LSP stable (dark matter)
- Fits nicely with grand unification

Light gravitino: MSSM + goldstino multiplet with cutoff $\ll M_P$

- MSSM LSP ➔ particle + (goldstino)

Very light gravitino: MSSM + goldstino multiplet but cutoff $O(M_{weak})$

- Unsuppressed (s)goldstino interactions
- May avoid Higgs bound $m_h < 130$ GeV

Some SUGRA formalism (part I)

For reasons of time, we will present some standard results on supergravity without giving their derivation

pure supergravity Lagrangian:

$$\mathcal{L}_{pure\ sugra} = -\frac{1}{2} e R + e\, \varepsilon^{\rho\lambda\mu\nu} \overline{\psi}_\rho \overline{\sigma}_\lambda \mathcal{D}_\mu \psi_\nu$$

couplings to chiral superfield (no gauge group for now) fully specified by the real dimensionless function

$$G(\phi^\dagger, \phi) = \frac{K(\phi^\dagger, \phi)}{M_P^2} + \log \left| \frac{W(\phi)}{M_P^3} \right|^2$$

Natural supergravity units: $M_P=1$

Classical Kahler invariance of G:

$$K \to K + \eta(\phi) + \eta^\dagger(\phi^\dagger) \qquad W \to W e^{-\eta(\phi)}$$

Some selected terms in the Lagrangian

Field-dependent gravitino mass term:

$$-e W e^{K/2} \overline{\psi}_\mu \overline{\sigma}^{\mu\nu} \overline{\psi}_\nu + \text{h.c.} \;\rightarrow\; m_{3/2}^2 = e^G = |W|^2 e^K$$

Matter fermion mass term:

$$-e\, e^{G/2} \left(G_{ij} - G_{ij\bar{k}} G^{\bar{k}} + G_i G_i\right) \psi^i \psi^j + \text{h.c.}$$

Matter fermion-gravitino mixing:

$$-(ie/\sqrt{2})\, e^{G/2} G_i \psi^i \sigma^\mu \overline{\psi}_\mu + \text{h.c.}$$

Scalar potential:

$$V = e^G [G_i G^{i\bar{k}} G_{\bar{k}} - 3] = F_i F^i - 3 m_{3/2}^2$$

$\langle G_i \rangle = 0 \quad \langle V \rangle = 0$ unbroken SUSY in Minkowski

$\langle G_i \rangle \neq 0 \quad \langle V \rangle = 0$ broken SUSY in Minkowski

$\langle G_i \rangle = 0 \quad \langle V \rangle = -3 m_{3/2}^2 < 0$ unbroken SUSY in adS

Hidden-sector supergravity models

Their minimal realization consists of a hidden sector
(gravitational multiplet + singlet goldstino superfield Z)
and an observable sector containing the MSSM multiplets
talking only via $O(1/M_P)^n$ non-renormalizable T.L. couplings

The goldstino is the fermion in the Z multiplet

$$\Lambda^2_{SUSY} \sim M_{weak} M_P \quad m_{3/2} \sim M_{weak} \quad \Delta m_{SUSY} \sim m_{3/2}$$

hidden and observable

additional contributions to Str M^2 of the order of the gravitino mass, even in the case of canonical Kahler potential

Appropriate flat limit: $M_P \to \infty \quad (m_{3/2} \text{ fixed})$

leading to the MSSM with explicit soft SUSY breaking

Example: the Polonyi model

just one chiral multiplet Z, with canonical Kahler potential $K=|Z|^2$ and the Polonyi superpotential

$$W = m^2(Z+b)$$

$|b|<2$ ➔ no solutions to $G_Z=0$ ➔ broken SUGRA

$b=2-3^{1/2}$ ➔ broken SUGRA with $\langle V\rangle=0$

$$\langle z\rangle = \sqrt{3}-1, \quad m_{3/2}^2 = m^4 e^{(\sqrt{3}-1)^2}, \quad m_A^2 = 2\sqrt{3}m_{3/2}^2, \quad m_B^2 = 2(2-\sqrt{3})m_{3/2}^2$$

Unsatisfactory features:

- $\langle V\rangle=0$ by fine-tuning the value of the b parameter
- gravitino mass at the weak scale by tuning the scale of the explicit mass parameter m^2 to $O(M_{weak}M_P)$

Coupling Polonyi to the observable sector

(Z, Y^i) $K=|Z|^2+|Y^i|^2$ $W=W_{Pol}(Z) + W_0(Y^i)$

↖ charged fields cubic ↗

$<y^i>=0$ local minimum with $m_{3/2} \neq 0$ and $<V>=0$

$$(M_0^2)_{ij} = \langle V_{ij} \rangle = 0 \quad (M_0^2)_{i\bar{k}} = \langle V_{i\bar{k}} \rangle = \delta_{i\bar{k}}\, m_{3/2}^2$$

$$Str\,\mathcal{M}^2 = m_{3/2}^2\,[-4+4+2(N_{TOT}-1)]$$

grav z y

(general result for models with canonical kinetic terms)

Good news:

- universal, positive scalar masses $m_0=m_{3/2}$
(in contrast with global renormalizable SUSY)

Generic problems of N=1 D=4 supergravity

- Classical vacuum energy

$$V_{cl} = O(m_{3/2}^2 M_P^2) \quad \overset{?}{\longrightarrow} \quad \langle V_{cl} \rangle \simeq 0$$

- ($m_{3/2}/M_P$) hierarchy

$$m_{3/2} = O(M_P) \quad \overset{?}{\longrightarrow} \quad m_{3/2} < O(10^{-15} M_P)$$

- Stability of the classical vacuum

$$\Delta V = O(m_{3/2}^2 M_P^2) \quad \overset{?}{\longrightarrow} \quad \Delta V < O(m_{3/2}^4)$$

Str $M^2 \neq 0$ in a generic N=1 supergravity model

➔ field-dependent 1-loop quadratic divergences

- Universality of squark/slepton mass terms

(or equivalent condition to suppress FCNC)
not guaranteed with general kinetic terms

generic N=1 supergravity is not enough: too flexible!
more insight from symmetries/dynamics?
look first at some special supergravities

No-scale supergravity models

Illustrate the idea with the simplest example

$$K = -3\log(T+\overline{T})$$

(can be justified with XDIM or N>1 SUGRA)

SU(1,1)/U(1) Kahler invariance (T-duality)

$$T \rightarrow \frac{aT - ib}{icT + d} \qquad (ad - bc = 1)$$

$$\text{if } \; W(T) \rightarrow (icT+d)^3 \, W\left(\frac{aT-ib}{icT+d}\right)$$

A stable class of superpotentials (N>1 gaugings):

$$W = m_0 - i m_1 T + 3 n^1 T^2 + i n^0 T^3$$

No-scale (continued)

W = k ≠ 0 (T-independent)

(breaks T ➔ 1 /T but preserves T ➔ T + i a)

➔ special no-scale properties:

$V \equiv 0$ classically flat potential

$F_T \neq 0 \; (\forall T)$ broken supersymmetry

$m_{3/2}^2 = \frac{|k|^2}{(T+\overline{T})^3}$ sliding scale Λ_{SUSY}

can be coupled to charged chiral multiplets C^i via:

$$K \to K + \sum_i |C^i|^2 (T+\overline{T})^{n_i} + \ldots \qquad W \to W + d_{ijk} C^i C^j C^k$$

SU(1,1) duality extends via $C^i \longrightarrow (icT + d)^{n_i} \, C^i$

Coupling no-scale to an observable sector

still local minima of V with $\langle C^i \rangle = 0$ and all no-scale properties

universal supersymmetry-breaking mass terms:

$$\tilde{m}_i^2 = (1+n_i) m_{3/2}^2 \quad A_{ijk} = (3+n_i+n_j+n_k) m_{3/2}$$

assuming no terms $O[(m_{3/2}M_P)^2]$ in $V_{eff} = V_{cl} + \Delta V$ may allow for a dynamical generation of the hierarchy $m_{3/2} \ll M_P$

interplay of gauge vs. Yukawa renormalization effects

➔ effective infrared fixed point of $V_{eff}[m_{3/2}(T), H_1, H_2]$

problems at this N=1, D=4 level:

- Unexplained origin of K and W
- No control over UV quantum corrections

Help from XDIM or STRINGS?

Some SUGRA formalism (part II)

general couplings to gauge superfields (including axionic symmetries and R-symmetry)

coupling of supergravity to chiral multiplets controlled by

$G = K + \log |W|^2$

gauge symmetries must be isometries of Kahler manifold

couplings to vector multiplets $V = V^a T_a$ described by

$f_{ab}(\phi)$ gauge kinetic function

$$X_a(\phi) = X_a^i(\phi)(\partial/\partial\phi^i)$$ holomorphic Killing vectors

concentrate for simplicity on the scalar fields z^i

$$\delta z^i = \varepsilon^a X_a^i \qquad D_\mu z^i = \partial_\mu z^i - A_\mu^a X_a^i$$

general D-terms in supergravity

Linear gauge symmetry: $X_a^i = i\,(T_a)^i{}_k z^k$

Axionic shift symmetry: $X_a^i = i\,q_a^i$

$$V_D = \frac{1}{2} D_a D^a$$ Killing potentials D_a solution of

$$X_a^i = -i\,G^{i\bar{k}} \frac{\partial D_a}{\partial \bar{z}^k}$$ complex Killing equations

G gauge invariant ➔ $K' = K + H + \bar{H} \quad W' = W e^{-H}$

$$D_a = i\,G_i X_a^i = i\,K_i X_a^i + i\frac{W_i}{W} X_a^i$$

Not restrictive to take K gauge-invariant. Then find:

$$D_a = i\,K_i X_a^i + \xi_a$$ ← constant FI term R-symmetry gauging

some immediate consequences

$$D_a = i G_i X_a^i$$

•There is never pure D-breaking in supergravity (unless $m_{3/2}$=0 and V_D is uncancelled, as in the unphysical limit of global supersymmetry)

•If V_F admits a supersymmetric adS_4 vacuum with all $\langle G_i \rangle = 0$ and $W \neq 0$, such a configuration automatically minimizes also V_D at zero: there Is no uplifting of susy adS_4 vacua to dS_4 vacua as an effect of D-terms from gauge interactions

gaugino condensation

In an asymptotically free N=1 SYM theory ("SQCD"):

$$\langle \lambda\lambda \rangle \sim \Lambda^3_{SQCD} \sim \mu^3 e^{-\frac{3}{2\beta_0 g^2(\mu)}}$$

$\mu =$ renormalization scale $\qquad \beta_0 = \frac{1}{16\pi^2}[3C(G) - T(R)]$

easily deduced from $\qquad \frac{1}{g^2(\mu')} = \frac{1}{g^2(\mu)} + \beta_0 \log\frac{\mu^2}{\mu'^2}$

For a field-dependent gauge coupling $\quad f_{ab} = \delta_{ab} S$

can deduce the form of the effective superpotential:

$$W_{np} = W_0\, e^{-kS} \qquad k = 3/(2\beta_0)$$

More rigorous derivations possible but not given here

A simple model with metastable dS vacua

$$K = -p \log(S+\bar{S}) + K_0 \quad (0 < p \in \mathbf{R})$$

Can gauge the U(1) isometry acting as a shift of Im(S):

$$X^S = iq \quad (q \in \mathbf{R})$$

Most general superpotential compatible with gauged U(1):

$$W = W_0 e^{-kS} \quad (k \in \mathbf{R})$$

A gauge kinetic function compatible with gauged U(1):

$$f = S$$

but a more general form f = a S + b would still be OK

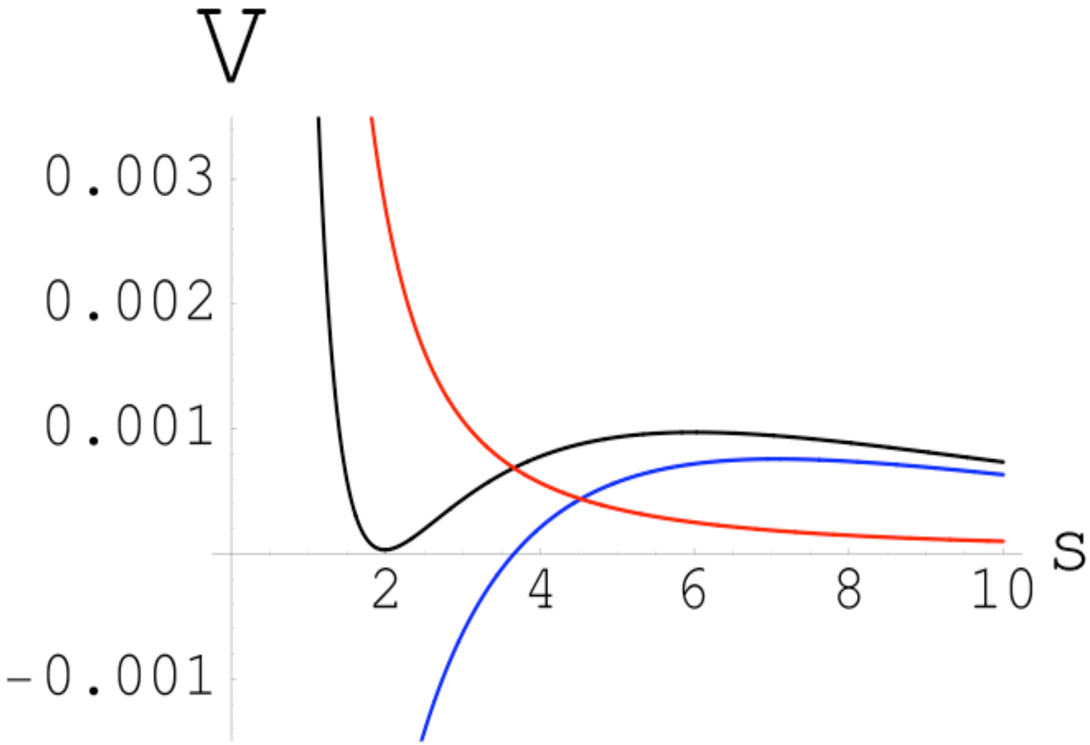
…more on the model…

$$V = \frac{e^{G_0} e^{-2ks}}{(2s)^p} \left[\frac{(2s)^2}{p} \left(k + \frac{p}{2s} \right)^2 - 3 \right] + \frac{q^2}{2s} \left(k + \frac{p}{2s} \right)^2$$

can obtain metatstable adS for suitable parameter choices

$$S = s + i\sigma$$

Example:
p=1
q=0.3
exp(G_0)=1/64
k=0.1



3.

XDIM

SUSY breaking & extra dimensions

Most present activity on supersymmetry breaking (theory and phenomenology) has to do with extra spatial dimensions

- Motivated by superstring theories
- Constrain effective d=4 SUGRA

Bottom-up approach (this lecture):
build simple toy-models to identify qualitatively new features of phenomenological relevance
will give here some examples
in particular Scherk-Schwarz compactifications

Top-down approach (next lecture):
identify the possibilities allowed by the consistency constraints of string/M-theory compactifications
supersymmetry breaking by general fluxes

Preamble: free massless D=5 scalar

$\mathcal{L} = (\partial^M \phi^\dagger)(\partial_M \phi) \qquad x^M \equiv (x^\mu, y) \quad (flat)$

symmetry: $\phi' = e^{-i\beta}\phi \quad \beta \in \mathbf{R}$ (constant)

circle compactification: $y \equiv y + 2\pi R \ (\forall y)$

Strict periodicity conditions: $\phi(x, y+2\pi R) = \phi(x,y)$

$\phi(x,y) \propto \sum_n \varphi_n(x) e^{\frac{iny}{R}} \qquad (\partial^y \phi^\dagger)(\partial_y \phi) \Rightarrow$ D=4 masses

standard Kaluza-Klein spectrum: $m_n^2 = \frac{n^2}{R^2} \quad (n \in \mathbf{Z})$

Twisted periodicity conditions: $\phi(x, y+2\pi R) = e^{-i\beta}\phi(x,y)$

$$\phi(x,y) \propto e^{\frac{-i\beta y}{2\pi R}} \sum_n \varphi_n(x) e^{\frac{iny}{R}}$$

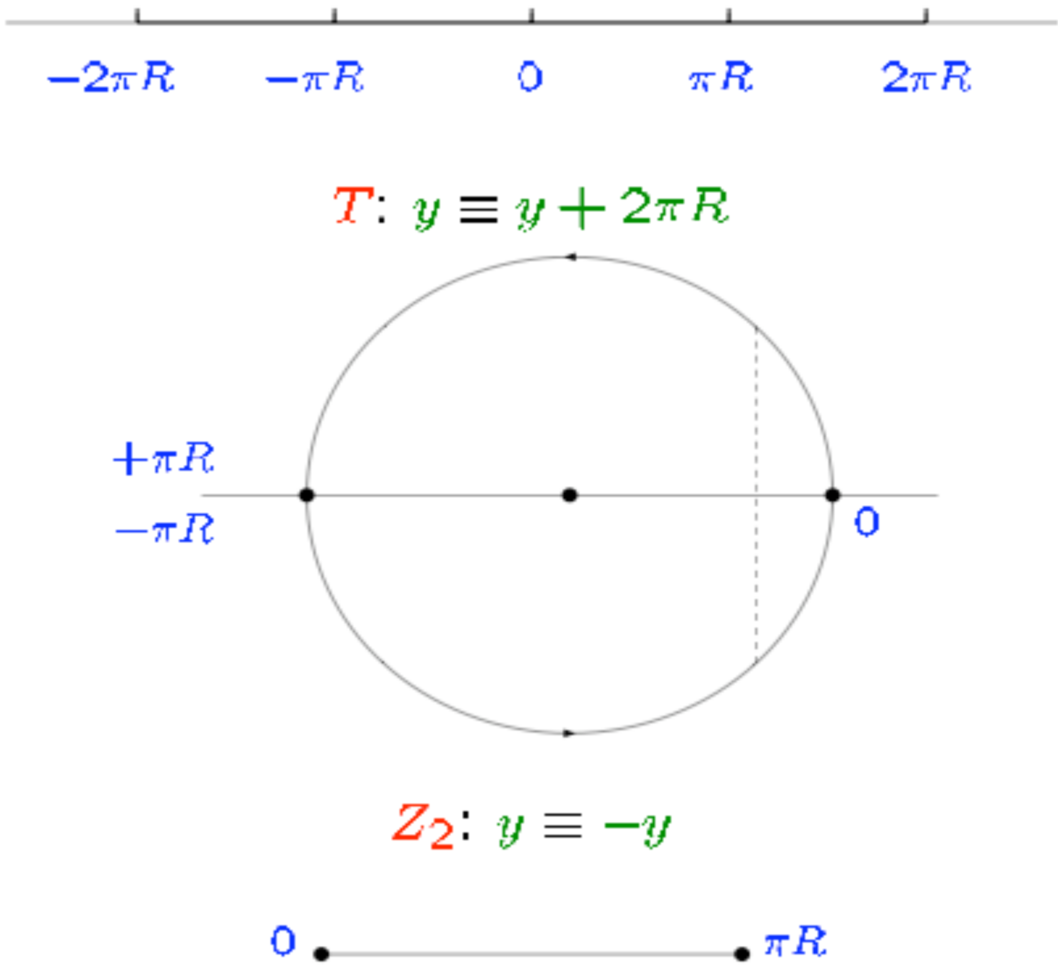
shifted Kaluza-Klein spectrum: $m_n^2 = \left(\frac{n}{R} - \frac{\beta}{2\pi R}\right)^2 \quad (n \in \mathbf{Z})$

A useful case study: the orbifold S^1/Z_2

R:

S^1:

S^1/Z$_2$:



Upstairs approach: work on covering space S^1

The Scherk-Schwarz twist for a spinor

(interacting) D=5 massless spinor $\Psi(x^\mu, y)$

$$\Psi = \begin{pmatrix} \psi_1 \\ \psi_2 \end{pmatrix} \quad \Psi(-y) = Z\,\Psi(y) \quad Z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

Two D=4 Weyl spinors, index-1=even and index-2=odd

$$\mathcal{L} = \mathcal{L}_0 + \mathcal{L}_{int}$$

invariant under Z_2
and a global SU(2)

$$\mathcal{L}_0 = i\overline{\Psi}^T \overline{\sigma}^\mu \partial_\mu \Psi - \frac{1}{2}\left(i\Psi^T \widehat{\sigma}^2 \partial_y \Psi + \text{h.c.}\right)$$

$$\Psi'(y) = U\,\Psi(y) \quad U \in SU(2)$$

Twist: $\Psi(y + 2\pi R) = U_\beta\,\Psi(y)$ $\quad (U_\beta Z U_\beta = Z)$

not restrictive to take: $U_\beta = exp(i\beta\,\widehat{\sigma}^2) \quad (0 < \beta < \pi)$

Effects of the Scherk-Schwarz twist

move to a basis of periodic fields by a local field redefinition:

$$\Psi(y+2\pi R) = V(y)\,\widetilde{\Psi}(y) \qquad \widetilde{\Psi}(y+2\pi R) = \widetilde{\Psi}(y)$$

The choice of V(y) is not unique (physics is fully determined by the original Lagrangian and by the twist) . The simplest one is:

$$V(y) = \exp\left(i\beta\widehat{\sigma}^2 \frac{y}{2\pi R}\right)$$

Can easily check that this induces a universal shift in the KK spectrum

$$m_n = \frac{n}{R} - \frac{\beta}{2\pi R} \qquad (n \in \mathbf{Z})$$

Can apply the mechanism to theories with global SUSY: obtain D=4 theories with explicit but soft SUSY breaking

The superHiggs effect (flat case)

The simplest case is just minimal D=5 Poincare' supergravity

supergravity multiplet: $\left(E_M^A,\ \Psi_M,\ B_M\right)$

$$\kappa\mathcal{L} = i\varepsilon^{MNOPQ}\overline{\Psi}_M\Sigma_{NO}D_P\Psi_Q + \ldots$$

$$\delta\Psi_M = \frac{2}{\kappa}D_M\eta + \ldots \qquad D_M\Psi = \left(\partial_M + \frac{1}{2}\omega_{MAB}\Sigma^{AB}\right)\Psi$$

local SUSY transf. param. ↖ any spinor field ↖ ↑ spin connection

$$\Psi_M = \begin{pmatrix}\psi_{1M}\\ \psi_{2M}\end{pmatrix} \quad (M = \mu, 5)$$

D=5 gravitino

Flat background solution of the D=5 equations of motion

$$\langle G_{\mu\nu}\rangle \propto \eta_{\mu\nu},\ \langle G_{55}\rangle = \text{const},\ \langle B_5\rangle = \text{const},\ \langle G_{\mu 5}\rangle = \langle B_\mu\rangle = \langle\Psi_M\rangle = 0$$

S^1/Z_2 compactification without twist

$$Z = \hat{\sigma}^3 \ \text{for} \ \Psi_\mu \qquad Z = -\hat{\sigma}^3 \ \text{for} \ \Psi_5$$

$$even: \ E_\mu^\alpha, \, E_5^5, \, B_5 \qquad odd: \ E_\mu^5, \, E_5^\alpha, \, B_\mu$$

↓

"dilaton" and "axion" zero modes in the physical spectrum

The complete spectrum:

$$M_{(0)} = 0 \qquad (E_\mu^\alpha, \, \psi_\mu^1, \, \psi_5^2, \, E_5^5, \, B_5)$$

$$M_{(n)} = \frac{n}{R} \quad (n \neq 0) \qquad (2, 3/2, 3/2, 1)$$

Effective N=1 D=4 no-scale supergravity

$$E_M^A = \begin{pmatrix} \phi^{-1/2}\hat{e}_\mu^a & \phi A_\mu \\ 0 & \phi \end{pmatrix} \quad B_M = \begin{pmatrix} B_\mu \\ B \end{pmatrix} \quad \Psi_M \longrightarrow \psi_\mu^1, \psi_\mu^2, \psi_5^1, \psi_5^2$$

Odd fields have no zero modes: $(A_\mu, B_\mu, \psi_\mu^2, \psi_5^1)$

Even fields recombine into $(\hat{e}_\mu^a, \psi_\mu)$ and (z, χ)

where $\chi \propto \psi_5^2$ and $T = E_5^5 + i\sqrt{2/3}\,B_5$

By dimensional reduction of the D=5 action we obtain

$$\hat{e}_4^{-1}\mathcal{L} = -\frac{1}{2}R(\hat{e}) - \frac{3}{4}\phi^{-2}\hat{g}^{\mu\nu}(\partial_\mu\phi)(\partial_\nu\phi) - \frac{1}{2}\phi^{-2}\hat{g}^{\mu\nu}(\partial_\mu B)(\partial_\nu B) + \ldots$$

➔ $K_{\overline{T}T}\hat{g}^{\mu\nu}(\partial_\mu\overline{T})(\partial_\nu T)$ with $K = -3\log(T+\overline{T})$

S^1/Z_2 compactification with a Scherk-Schwarz twist

The discussion is essentially the same as for the spinor

$$\Psi_M(y+2\pi R) = U_\beta \Psi_M(y), \quad \Psi_M(y) = V(y)\widetilde{\Psi}_M(y), \ \ldots$$

sufficient to look at the derivative terms only in the D=5 Lagrangian and SUSY transformation laws

can redefine also the local SUSY parameter

$$\eta = (\eta_1, \eta_2)^T \quad \eta(y) = V(y)\widetilde{\eta}(y)$$

to show that SUSY breaking is spontaneous:

$\beta \neq 0$ ➔ unitary gauge where $\widetilde{\Psi}_5$ disappears

The non-locality of SUSY-breaking order parameter improves the ultraviolet behaviour of symmetry-breaking quantities, e.e. finite ($1/R^4$) one-loop vacuum energy

This property would be missed working in the reduced theory

Higher-dimensional "mediation" models

1. Gaugino mediation

Tree-level: MSSM gaugino masses with vanishing scalar masses
One-loop: induced scalar masses sufficiently universal
Easy to implement in orbifold or brane-world constructions

2. Anomaly mediation

Arrange for no tree-level masses and no light scalars
(not as easy as it sounds in higher-dimensional supergravities)
Then 1-loop contributions to gaugino and scalar masses fixed by beta-function coefficients: problem of negative slepton squared masses can be corrected only by introducing additional mediation mechanisms

3. Split supersymmetry

Higher-dimensional mechanisms exist to realize the split supersymmetry scenario, but no time is left to describe them

4.

STRINGS

Preamble and disclaimer

Superstring theory is the present best candidate for a quantum theory unifying gravity with all other interactions: effective theories below M_{string} are D=10,11 supergravities, an appropriate framework to study supersymmetry breaking

Will now discuss simple N=1 compactifications to show

- What the potential perturbative sources are for supersymmetry breaking (and moduli stabilization)
- How to make contact with the formalism of N=1 D=4 supergravity via some effective K, W and f_{ab}

Less simple and technically more challenging studies can address issues such as soft terms or realistic models

String effective supergravities in D≥10

(describe for simplicity only bulk bosonic degrees of freedom)

- "M-theory" (D=11 ➔ N=8): $g_{\widehat{M}\widehat{N}}$ $A^{(3)}$
- Type-IIA (D=10 ➔ N=8):
  $g_{MN},\ \Phi,\ B_{MN};\ A^{(1)} \leftrightarrow A^{(7)},\ A^{(3)} \leftrightarrow A^{(5)},\ A^{(9)}$ non-dynamical
- Type-IIB (D=10 ➔ N=8):
  $g_{MN},\ \Phi,\ B_{MN};\ A^{(0)} \leftrightarrow A^{(8)},\ A^{(2)} \leftrightarrow A^{(6)},\ A^{(4)}$ self-dual
- Heterotic (D=10 ➔ N=4): $g_{MN},\ \Phi,\ B_{MN};\ A_M^a$ E_8xE_8 SO(32)
- Type-I (D=10 ➔ N=4): $g_{MN},\ \Phi,\ A^{(2)} \leftrightarrow A^{(6)};\ A_M^a$ SO(32)

+ possible additional degrees of freedom localized on branes

orbifold/orientifold projections to N=1

compactifications on flat T^6 (or T^6 x S^1)) would give theories with N=4 or N=8 supersymmetry in D=4

simple way of obtaining N=1 in D=4: $Z_2 \times Z_2$ orbifold

	x^5	x^6	x^7	x^8	x^9	x^{10}
Z_2 :	$-$	$-$	$-$	$-$	$+$	$+$
Z_2' :	$+$	$+$	$-$	$-$	$-$	$-$

plus, for N=8 theories, additional Z_2 "orientifold"

Discuss here (qualitatively) three simple examples:

- Heterotic (without YM fields)
- IIA with additional I_3 orientifold
- IIB with additional I_6 orientifold

Heterotic on $T^6/(Z_2 x Z_2')$

(neglecting Yang-Mills fields for simplicity)

Z_2 x Z_2 invariant (bulk) fields:

$$e^{-2\Phi} = s\,(t_1 t_2 t_3)^{-1} \qquad g_{\mu\nu} = s^{-1}\,\tilde{g}_{\mu\nu} \qquad B_{\mu\nu} \leftrightarrow \sigma$$

$$B_{56} = \tau_1 \qquad B_{78} = \tau_2 \qquad B_{910} = \tau_3$$

$$g_{i_A j_A} = \frac{t_A}{u_A}\begin{pmatrix} u_A^2 + \nu_A^2 & \nu_A \\ \nu_A & 1 \end{pmatrix} \qquad (A = 1, 2, 3)$$

D=10 heterotic (bosonic) Lagrangian:

$$L_{ED} = \frac{1}{2\,\kappa_{10}^2}\, e^{-2\Phi}\, e_{10}\, \left[R_{10} + 4\, g^{MN}(\partial_M \Phi)(\partial_N \Phi)\right]$$

$$L_H = -\frac{1}{4\;3!\;\kappa_{10}^2}\, e^{-2\Phi}\, e_{10}\, g^{MM'} g^{NN'} g^{PP'}\, \tilde{H}_{MNP}\, \tilde{H}_{M'N'P'}$$

$$\left[\; L_{YM} = -\frac{1}{4\, g_{10}^2}\, e^{-2\Phi}\, e_{10}\, g^{MM'}\, g^{NN'}\, F^a_{MN}\, F^a_{M'N'} \;\right]$$

D=4, N=1 effective Lagrangian

$$S = s + i\sigma \quad T_A = t_A + i\tau_A \quad U_A = u_A + i\nu_A \quad (A = 1,2,3)$$

$$K = -\log(S+\overline{S}) - \sum_A \log(T_A + \overline{T}_A) - \sum_A \log(U_A + \overline{U}_A)$$

$W = 0$ ➔ unbroken SUSY, 7 complex moduli

could add what survives from D=10 Yang-Mills sector:

[$$f_{ab} = \delta_{ab} S \quad W = d_{ijk} C^i C^j C^k \quad \Delta K = \ldots$$

but for simplicity we will neglect this sector]

Z_2 x Z_2 invariant (bulk) fluxes:

$$\widetilde{H}_{579}\,, \widetilde{H}_{679}\,, \widetilde{H}_{589}\,, \widetilde{H}_{689}\,, \widetilde{H}_{5710}\,, \widetilde{H}_{6710}\,, \widetilde{H}_{5810}\,, \widetilde{H}_{6810}\,. \quad (8)$$

$$\omega^{i_A}{}_{i_B i_C} \quad [(ABC) = (123), (231), (312)] \quad (24)$$

(geometrical fluxes equivalent to Scherk-Schwarz twists)

Effective superpotential from fluxes

(assuming for simplicity plane-interchange symmetry)

$$\tilde{H}_{579} \leftrightarrow 1 \quad \tilde{H}_{689} = \tilde{H}_{5810} = \tilde{H}_{6710} \leftrightarrow -(U_1 U_2 + U_2 U_3 + U_1 U_3)$$

$$\tilde{H}_{679} = \tilde{H}_{589} = \tilde{H}_{5710} \leftrightarrow i\,(U_1 + U_2 + U_3) \quad \tilde{H}_{6810} \leftrightarrow -i\,U_1 U_2 U_3$$

$$C_{679} = C_{895} = C_{1057} \leftrightarrow i\,(T_1 + T_2 + T_3) \quad C_{579} = C_{957} = C_{795} \leftrightarrow (T_1 U_1 + T_2 U_2 + T_3 U_3)$$

$$C_{6810} = C_{8106} = C_{1068} \leftrightarrow -i\,(T_1 U_2 U_3 + T_2 U_1 U_3 + T_3 U_1 U_2) \quad C_{5810} = C_{7106} = C_{968} \leftrightarrow -(T_1 + T_2 + T_3) U_1 U_2 U_3$$

$$C_{896} = C_{1067} = C_{689} = C_{1058} = C_{6710} = C_{8105} \leftrightarrow -(T_1 U_2 + T_1 U_3 + T_2 U_1 + T_2 U_3 + T_3 U_1 + T_3 U_2)$$

$$C_{589} = C_{796} = C_{7105} = C_{958} = C_{5710} = C_{967} \leftrightarrow i\,(T_1 U_1 U_2 + T_2 U_2 U_3 + T_3 U_3 U_1 + T_1 U_1 U_3 + T_2 U_2 U_1 + T_3 U_3 U_2)$$

$C_{i_A i_B i_C} \equiv \omega^{i_A}{}_{i_B i_C}$ are the parameters of the geometrical fluxes

geometrical expression: $W \propto \int_{X_6} (H + i\,dJ) \wedge \Omega$

Consistency conditions:

$$\omega \cdot \omega = 0 \quad \omega \cdot \tilde{H} = 0$$

(generalized Bianchi identities, also N=4 Jacobi identities)

Physics possibilities (heterotic)

Impossible to generate an S-dependence in W via perturbative fluxes: S stabilized non-perturbatively? Analogous to volume modulus stabilization in type IIB

$$e^{-K}V = \sum_{i=1}^{7} |W - W_i(z_i + z_i)|^2 - 3|W|^2$$

Can obtain no-scale vacua with spontaneously broken supersymmetry in flat Minkowski space

Also runaway (cosmological) solution with strictly positive potential, but no stable adS vacua with all moduli stabilized

We can complete the discussion adding the D=10 Yang-Mills fields and the associated two-form fluxes, but the general qualitative conclusions do not change

N=1 superpotentials from fluxes in type IIA/B

similar analyses can be carried out in type-II theories

additional Z_2 orientifold needed to obtain N=1 in D=4
with non trivial action on fields and on coordinates
discussion complicated by the presence of branes
and orientifold planes with associated DBI+WZ actions

IIB bulk fluxes (O3): $H_3 \quad F_3 \quad (\text{no } \omega_3,\ F_1,\ F_5\ !)$
$W(S, U_A)$ is generated, but no T-dependence!

IIA bulk fluxes (O6): $\omega_3 \quad H_3 \quad F_0, F_2, F_4, F_6$
$W(S, T_A, U_A)$ can be generated (linear in U_A)

➔ can fix all bulk moduli on stable SUSY adS vacua

➔ can obtain no-scale models or other possibilities

Towards realistic type-II models

(a lot of activity in the recent literature)

- Inclusion of brane fluctuations (DBI+WZ actions)
- Localized magnetic fluxes (with generalized Bianchi id.)
- Consistent inclusion of D-terms in effective supergravity
- Search for stable non-adS_4 vacua with stabilized moduli
- Calculation of soft terms around semirealistic vacua
- Perturbative and NP corrections to classical results
- …